AP Calculus BC Name: ___________________________________

4.1 Worksheet (odds and #14)

Find the value of the derivative (if it exists) at each indicated extremum. Check your answers by graphing the function in your calculator and comparing the slope of the tangent line at these points.

1. $f(x) = \frac{x^2}{x^2+4}$ at (0,0)

2. $f(x) = 4 - |x|$ at (0,1)

Find the critical numbers of the given function.

3. $g(t) = t\sqrt{4-t} \quad (t < 3)$

4. $h(x) = sin^2x + cosx \quad (0 < x < 2\pi)$

5. $f(x) = x^2 log_2(x^2 + 1)$

6. $g(t) = 2tlnt$

Find the absolute extrema of the function on the closed interval.

7. $f(x) = x^3 - \frac{3}{2}x^2, \quad [-1, 2]$

8. $y = 3x^{2/3} - 2x, \quad [-1, 1]$

9. $f(x) = sinx, \quad [\frac{5\pi}{6}, \frac{11\pi}{6}]$

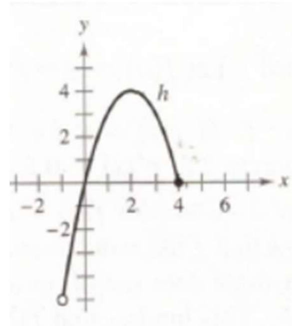
10. $f(x) = 3cosx, \quad [0, 2\pi]$

Use a graphing calculator to graph the function and find the absolute extrema of the function on the given interval.

11. $f(x) = \frac{3}{x-1}, \quad (1, 4]$

12. $f(x) = \sqrt{x + 4}e^{(x^2/10)}, \quad [-2, 2]$

13. (MULTIPLE-CHOICE) Which is the following statements is true about the graph of *h*?



(A) *h* has a minimum at (-1, -5) and a maximum at (2, 4).

(B) *h* has an absolute minimum at (4, 0) and a maximum at (2, 4).

(C) *h* has a maximum at (2, 4).

(D) *h* has no extrema.

14. (FREE-RESPONSE) The function *f* is defined $f(x) = \frac{(4lnx)}{x^3}$ for all x > 0.

(a) Find $f'(x)$.

(b) Find an equation of the tangent line to the graph of *f* at *x* = *e*.

(c) Find the critical numbers of *f*. Does the function have a relative maximum, a relative minimum, or neither at each critical number?

(d) Find $\lim_{x \to 0^+} f(x)$.

15. (MULTIPLE-CHOICE) The critical numbers for $f(x) = x^2(3x-1)^3$ are:

(A) *x = 0* and *x* = $\frac{1}{3}$

(B) *x = 0* and *x* = $\frac{2}{15}$

(C) *x* = $\frac{2}{15}$ and *x* = $\frac{1}{3}$

(D) *x = 0, x* = $\frac{2}{15}$ and *x* = $\frac{1}{3}$

4.2 Worksheet (odds and #16)

Explain why Rolle's Theorem does not apply to the function even though there exist *a* and *b* such that $f(a) = f(b)$.

1. $f(x) = 1 - |x - 1|$, [0, 2]

2. $f(x) = \left|\frac{1}{x}\right|$, [-1, 1]

Determine whether Rolle's Theorem can be applied to *f* on [a,b]. If Rolle's Theorem can be applied, find all values of *c* in the (a,b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

3. $f(x) = (x - 1)(x - 2)(x - 3)$, [1, 3]

4. $f(x) = -x^2 + 3x$, [0, 3]

5. $f(x) = tanx$, $[0, \pi]$

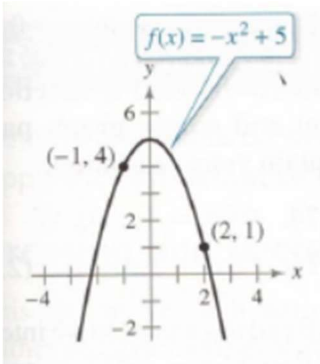
6. $f(x) = \frac{x^2-1}{x}$, [-1, 1]

Use a graphing calculator to graph the function on [a,b]. Determine whether Rolle's Theorem can be applied, and, if so, find all values of c in (a,b) such that $f'(c) = 0$.

7. $f(x) = 2 + \arcsin(x^2 - 1)$, [-1, 1]

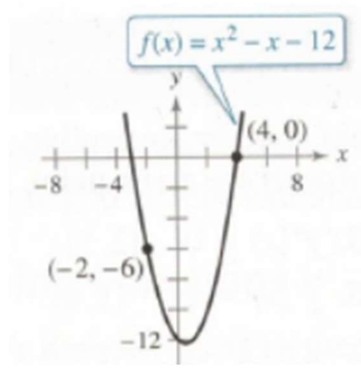
8. $f(x) = \frac{x}{2} - sin\frac{\pi x}{6}$, [-1, 0]

9. Consider the graph of the function $f(x) = -x^2 + 5$:



a) Find the equation of the secant line joining the points (-1, 4) and (2, 1).
b) Use the Mean Value Theorem to determine a point *c* in the interval (-1, 2) such that the tangent line at *c* is parallel to the secant line.
c) Find the equation of the tangent line through *c*.

10. Consider the graph of $f(x) = x^2 - x - 12$:



a) Find the equation of the secant line joining the points (-2, -6) and (4, 0).
b) Use the Mean Value Theorem to determine a point *c* in the interval (-2, 4) such that the tangent line at *c* is parallel to the secant line.
c) Find the equation of the tangent line through *c*.

Determine whether the Mean Value Theorem can be applied to *f* on [a,b] and if it can be applied, find all values of *c* in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

11. $f(x) = x log_2 x$, [1, 2]

12. $f(x) = (x+3)\ln(x+3)$, [-2, -1]

Use a graphing calculator to (a) graph the function *f* on [a,b], (b) find and graph the secant line through the endpoints of the interval, and (c) find and graph any tangent lines to the graph of *f* that are parallel to the secant line.

13. $f(x) = 2e^{(x/4)}\cos(\frac{\pi x}{4})$, [0, 2]

14. $f(x) = x^4 - 2x^3 + x^2$, [0, 6]

15. (MULTIPLE-CHOICE) Consider the function $f(x) = \begin{cases} 0, & x = 0 \\ 1 - x, & 0 < x \leq 1 \end{cases}$

Which of the following statements is *false*?

(A) *f* is differentiable on (0,1).

(B) *f*(0) = *f*(1).

(C) *f* is continuous on [0,1].

(D) The derivative of *f* is never equal to zero on the interval (0,1).

16. (MULTIPLE-CHOICE) If the Mean Value Theorem is applied to the function $f(x) = x^3 - 4x$ on the interval [0,2], then the number *c* that must exist in the interval (0,2) is

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{3}$

(D) $\frac{2\sqrt{3}}{3}$

17. (MULTIPLE-CHOICE) Which of the following functions do not satisfy the conditions of the Mean Value Theorem on the interval [-1, 1]?

(A) $f(x) = \sqrt[5]{x}$

(B) $g(x) = 2xarccosx$

(C) $h(x) = \frac{x}{x-3}$

(D) $p(x) = \sqrt{x+1}$

4.3 Worksheet

Find the open intervals analytically where the function is increasing and decreasing. (You can use your calculator to graph the function and verify your results).

1. $f(x) = \frac{x^3}{4} - 3x$

2. $f(x) = x^4 - 2x^2$

3. $f(x) = \frac{1}{(x+1)^2}$

4. $f(x) = \frac{x^2}{2x-1}$

5. $f(x) = x^2 - 2x - 8$

6. $f(x) = 12x - x^3$

7. $f(x) = x\sqrt{16 - x^2}$

8. $f(x) = \frac{lnx}{\sqrt{x}}$

9. $f(x) = x - 2cosx,\ \ 0 < x < 2\pi$

10. $f(x) = 3sinx,\ \ 0 < x < 2\pi$

For the following functions: (a) find the critical numbers of *f*, (b) find the open interval(s) on which the function is increasing or decreasing, and (c) apply the First Derivative Test to identify all relative extrema. (You can verify your result by graphing the function in your calculator)

11. $f(x) = 2x^3 + 3x^2 - 12x$

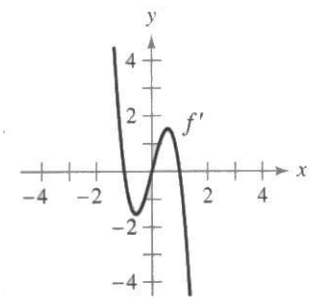
12. $f(x) = \frac{x^5 - 5x}{5}$

13. $f(x) = e^{-1/(x-2)}$

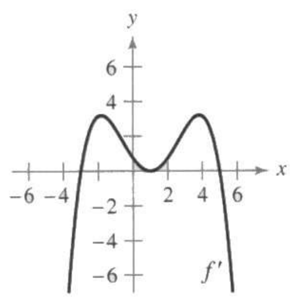
14. $f(x) = x - log_4(x)$

Use the graph of *f′(x)* to (a) identify the critical number of *f*, (b) identify the open interval(s) on which *f* is increasing or decreasing, and (c) determine whether *f* has a relative maximum, relative minimum, or neither at each critical number.

15.



16.



17. [Multiple-Choice] The derivative *h′* of a function *h* is continuous and has exactly two zeros. Selected values of *h′* are shown in the table:

x	−3	−2	−1	0	1	2	3
$h'(x)$	12	5	0	−3	−4	−3	0

If the domain of *h* is the set of all real numbers, then *h* is decreasing on which of the following intervals?

4.4 Worksheet

Find the open intervals analytically where the function is concave up and concave down. (You can use your calculator to graph the function and verify your results).

1. $f(x) = -x^3 + 6x^2 - 9x - 1$

2. $h(x) = x^5 - 5x + 2$

3. $f(x) = \frac{24}{x^2+12}$

4. $g(x) = \frac{x^2+4}{4-x^2}$

5. $y = 2x - tanx, \quad (-\frac{\pi}{2}, \frac{\pi}{2})$

6. $y = x + \frac{2}{sinx}$

Find the point(s) of inflection and discuss the concavity of the graph of the function.

7. $f(x) = x^3 - 9x^2 + 24x - 18$

8. $f(x) = -x^3 + 6x^2 - 5$

9. $f(x) = x\sqrt{x+3}$

10. $f(x) = x\sqrt{9-x}$

11. $f(x) = \frac{4}{x^2+1}$

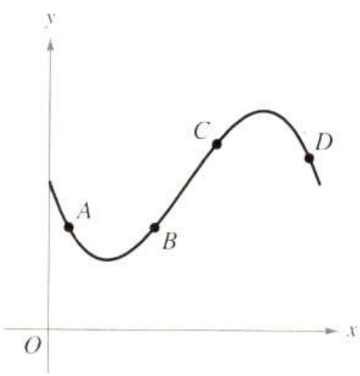
12. $f(x) = \frac{x+3}{\sqrt{x}}$

Use the First and Second Derivative Tests to find all relative extrema.

13. $f(x) = \frac{e^x + e^{-x}}{2}$

14. $f(x) = x^2 e^{-x}$

15. [Multiple-Choice] At which of the four points on the graph shown is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ negative?



16. [Multiple-Choice] The graph of the function $h(x) = 4xe^{-x}$ is

(A) decreasing and concave upward on $(2, \infty)$

(B) increasing and concave downward on $(-\infty, 2)$

(C) increasing and concave upward on $(2, \infty)$

(D) decreasing and concave upward on $(1, \infty)$

4.5 Worksheet

Evaluate the limit, using L'Hôpital's Rule if necessary.

1. $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

2. $\lim_{x \to 0} \frac{\sqrt{25 - x^2} - 5}{x}$

3. $\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)}$

4. $\lim_{x \to 0^+} \frac{e^x - (1 + x)}{x^3}$

5. $\lim_{x \to \infty} \frac{x^3}{e^{x/2}}$

6. $\lim_{x \to \infty} \frac{x^3}{e^{x^2}}$

7. $\lim_{x \to 0} \frac{\arctan(x)}{\sin(x)}$

8. $\lim_{x \to 0} \frac{x}{\arctan(2x)}$

Convert the function in the limit to an indeterminant form, then evaluate the limit using L'Hôpital's Rule.

9. $\lim_{x \to \infty} xsin\left(\frac{1}{x}\right)$

10. $\lim_{x \to \infty} xtan\left(\frac{1}{x}\right)$

11. $\lim_{x \to \infty} x^{(1/x)}$

12. $\lim_{x \to 0^+} (1 + x)^{(1/x)}$

13. $\lim_{x \to 1^+} (lnx)^{x-1}$

14. $\lim_{x \to 4^+} \left(3(x-4)\right)^{x-4}$

For each part of 15 and 16, determine if the limit can be evaluated using L'Hôpital's Rule by showing the indeterminant form (but do not evaluate the limit)

15. (i) $\lim_{x \to 2} \frac{x-2}{x^3-x-6}$

16. (i) $\lim_{x \to 0} \frac{x^2-4x}{2x-1}$

(ii) $\lim_{x \to \infty} \frac{x^3}{e^x}$

(ii) $\lim_{x \to 1} \frac{1+x(lnx-1)}{(x-1)lnx}$

(iii) $\lim_{x \to 1} \frac{\cos(\pi x)}{\ln(x)}$

(iii) $\lim_{x \to 3} \frac{e^{x^2}}{x-3}$

For 17 and 18, show that the indeterminate forms $0^0, \infty^0, and\ 1^\infty$ do not always have a value of 1 by evaluating each limit.

17. $\lim_{x \to 0^+} x^{\left(\frac{ln2}{1+lnx}\right)}$

18. $\lim_{x \to \infty} x^{\left(\frac{ln2}{1+lnx}\right)}$

4.6 Worksheet - Odd plus #2 and #4.

Use the First and Second Derivative Tests as well as intercepts and limits to evaluate asymptotes, so fully analyze and sketch the given function. You must show all work and label all intercepts, relative extrema, points of inflection, and asymptotes. (You can verify your work by graphing the function in a calculator).

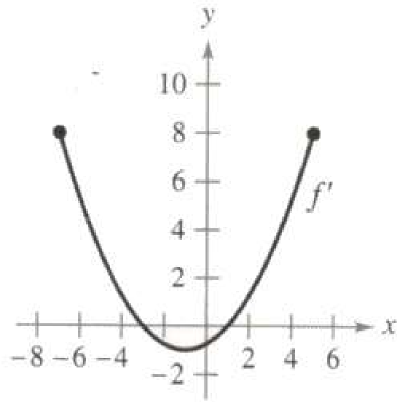
1. $f(x) = \frac{x^2-6x+12}{x-4}$

2. $f(x) = 3x^{(2/3)} - 2x$

3. $f(x) = 2 - x - x^3$

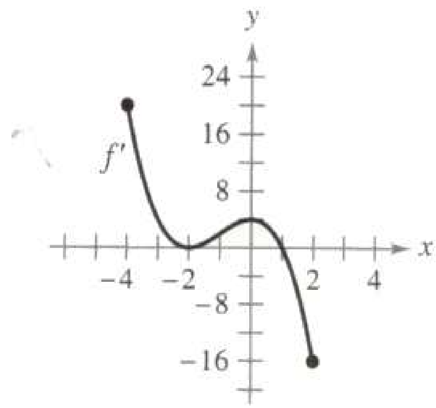
4. $f(x) = x^5 - 5x$

5. The graph of the **first derivative** of a function *f* on the interval [-7, 5] is shown:



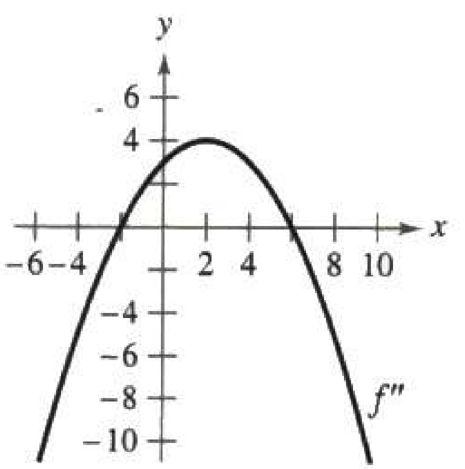
(a) On what interval(s) is *f* increasing and decreasing?
(b) On what interval(s) is *f* concave up and down?
(c) At what x-value(s) does *f* have relative extrema?
(d) At what x-value(s) does *f* have points of inflection?

6. The graph of the **first derivative** of a function *f* on the interval [-4, 2] is shown:



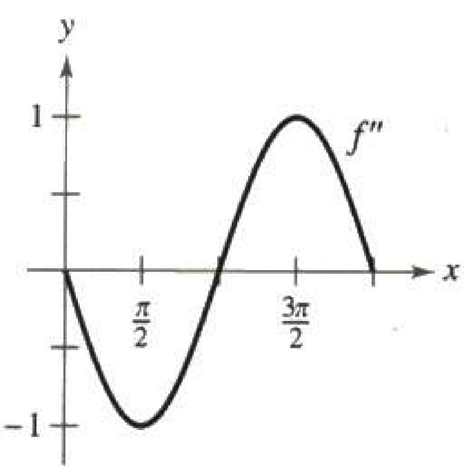
(a) On what interval(s) is *f* increasing and decreasing?
(b) On what interval(s) is *f* concave up and down?
(c) At what x-value(s) does *f* have relative extrema?
(d) At what x-value(s) does *f* have points of inflection?

7. The graph of the **second derivative** of a function *f* is shown:



(a) On what interval(s) is *f* concave up and down?
(b) On what interval(s) is f' increasing and decreasing?
(c) At what x-value(s) does *f* have points of inflection?

8. The graph of the **second derivative** of a function *f* is shown:



(a) On what interval(s) is *f* concave up and down?
(b) On what interval(s) is f' increasing and decreasing?
(c) At what x-value(s) does *f* have points of inflection?

4.7 Worksheet - Odds plus #2 and #4.

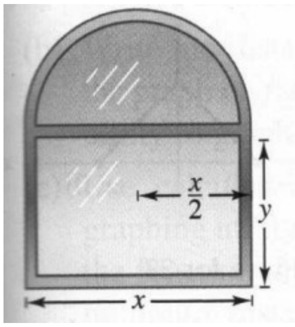
1. Find two positive numbers such that the product of the numbers is 147 and the sum of the first number plus the three times the second number is a minimum.

2. Find the length and width of a rectangle that has a perimeter of 80 m and will maximize the area enclosed.

3. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 245,000 square meters, and no fencing is needed along the one side which is along the river. What dimensions will require the least amount of fencing?

4. A rectangular page is to contain 30 square inches of print. The margins on each side are 1 inch. Find the dimensions of the paper such that the least amount of paper is used.

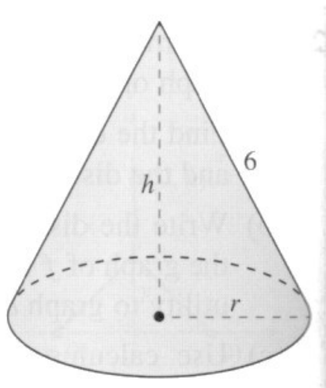
5. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions which provides maximum total window area if the total perimeter is 16 ft (assume the crossbar in the middle is not part of the perimeter).



6. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid must be 14 cm^3.
Find the radius of the cylinder that requires minimum surface area.

7. A rectangular package to be sent by a postal service can have a maximum combined length and girth of 108 inches (girth is the distance around all sides of a cross-section). Find the dimensions of a package which encloses the maximum possible volume. (assume the cross-section is a square)

8. A right cone has a slant height of 6:



(a) write the volume of the cone as a function of the radius. ($V_{cone} = \frac{1}{3}\pi r^2 h$)

(b) What are the dimensions that maximize the volume of the cone?

9. A rectangle is bounded by the x-axis and the graph $y = 25 - x^2$.

(a) What are the dimensions of the rectangle that maximize the rectangle's area?

(b) What are the dimensions of the rectangle that maximize the rectangle's perimeter?

10. (Challenge problem) Let $f(x) = 2 - 2sinx$.

(a) Sketch the graph of *f* on the interval $\left[0, \frac{\pi}{2}\right]$

(b) Find the distance from the origin to the y-intercept and the distance from the origin to the x-intercept.

(c) Write the distance *d* from the origin to a point on the graph *f* as a function of *x*. Use your calculator to graph *d* and find the minimum distance.

(d) Use calculus and the zero feature of your calculator to find the value of *x* that minimizes the function *d* on the interval $\left[0, \frac{\pi}{2}\right]$.

What is the minimum distance?