AP Calculus BC Name: _____

Unit 4 – Optimization Problems Review

#1) There are 320 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is as large as possible? (answer: 80 yd x 80 yd)

#2) A rectangular area of 10,000 m^2 must be enclosed by a fence. What should the dimensions of the rectangle be if we want to enclose this area using the minimum amount of fencing?
(answer: 100m x 100m)

#3) A rectangular area of 10,000 m^2 must be enclosed by a fence. One side of the fencing must be a high-security style fence which costs \$10/m, and the remaining 3 sides will use lower-cost \$6/m fencing. What are the dimensions of the rectangle which will minimize the cost of the fencing required?
(answer: expensive side = $\sqrt{7500} \approx 86.6025\, m$, other dimension: $\frac{10000}{\sqrt{7500}} \approx 115.4701\, m$)

#4) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (answer: V_{max} = 4000 cm^3, with base side = 20cm, height = 30cm).

#5) The rate (in mg carbon/m^3/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function $P = \frac{100I}{I^2 + I + 4}$ where I is the light intensity measured in thousands of foot-candles). For what light intensity if P a maximum? (answer: I = 2 thousand foot-candles)

#6) A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder. (answer: $\frac{4\pi}{27} r^2 h$)

#7) A poster is to have an area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?
(answer: $\sqrt{120} = 2\sqrt{30}\ in.$ x $\frac{180}{\sqrt{120}} = \frac{90}{\sqrt{30}}\ in.$)

#8) A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$15 per foot along three sides and fencing costing \$10 per foot along the fourth side. Find the minimum total cost. (answer: \$3000 with fence side = 50 ft, other sides = 60 ft)

#9) A boat leaves a dock at 2:00 pm and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 pm. At what time were the two boats closest together? (answer: 21.6 minutes after 2:00 pm)

#10) [Challenge problem] A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How much of the journey should be walking and how much boating? (answer: she should walk all the way and not boat at all)