AP Calculus BC

Name: answers / solutions

3.1 Worksheet (Odds and 14)

Find the derivative of the function.

1. $g(t) = t^2 - \frac{4}{t^3}$

$\boxed{g'(t) = 2t + \frac{12}{t^4}}$

2. $f(x) = 8x + \frac{3}{x^2} = 8x + 3x^{-2}$

$f'(x) = 8 - 6x^{-3}$

$\boxed{f'(x) = 8 - \frac{6}{x^3}}$

3. $f(x) = \frac{4x^3+3x^2}{x}$

$\boxed{f'(x) = 8x + 3}$

4. $f(x) = \frac{2x^4-4}{x^3} = \frac{2x^4}{x^3} - \frac{4}{x^3} = 2x - 4x^{-3}$

$f'(x) = 2 + 12x^{-4}$

$\boxed{f'(x) = 2 + \frac{12}{x^4}}$

5. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{\sqrt[3]{x^2}}}$

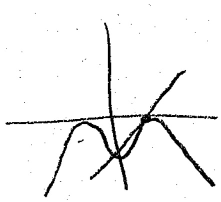
6. $f(t) = t^{2/3} - t^{\frac{1}{3}} + 4$

$\boxed{f'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{3}t^{-2/3}}$

Find an equation of the tangent line to the graph of f at the given point. Graph f and the tangent line on your calculator and use the *tangent* feature on your calculator to confirm the results.

7. $y = -2x^4 + 5x^2 - 3, \quad (1,0)$

$\boxed{(y-0) = 2(x-1)}$



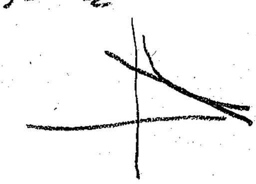
8. $f(x) = \frac{2}{\sqrt[4]{x^3}}, \quad (1,2)$

$f(x) = 2x^{-3/4}$

$f'(x) = -\frac{3}{2}x^{-7/4}, \quad f'(1) = -\frac{3}{2}$

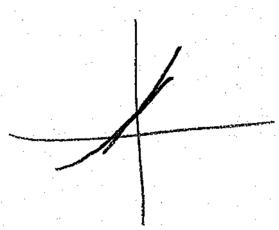
$\boxed{(y-2) = -\frac{3}{2}(x-1)}$

$y = -3/2x + 7/2$



9. $g(x) = x + e^x, \quad (0,1)$

$\boxed{(y-1) = 2(x-0)}$

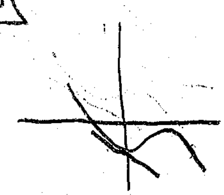


10. $h(t) = \cancel{4}t^2 - \frac{1}{2}e^t, \quad \left(0, -\frac{1}{2}\right)$

(typo)

$h'(t) = 2t - \frac{1}{2}e^t, \quad h'(0) = 0 - \frac{1}{2}e^0 = -\frac{1}{2}$

$\boxed{(y+1/2) = -\frac{1}{2}(x-0)}$



Determine the points (if any) at which the graph of the function has a horizontal tangent line.

11. $y = x^4 - 2x^2 + 3$

$\boxed{(0,3), (1,2), (-1,2)}$

12. $y = x^3 + x$

$y' = 3x^2 + 1 \Rightarrow 0$

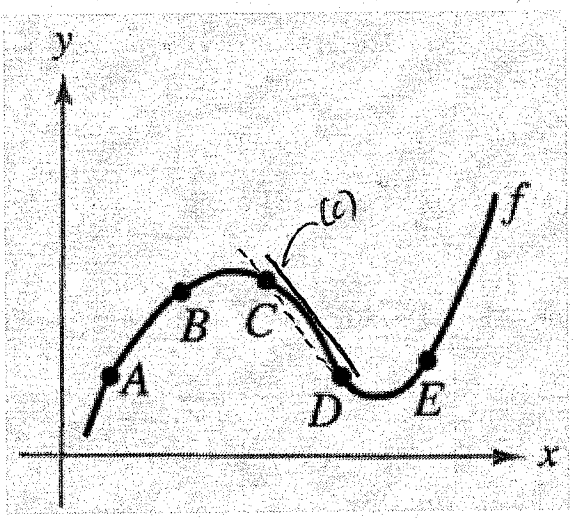
$3x^2 = -1$

$x^2 = -1/3$ not possible

so $\boxed{\text{no points}}$

where tangent line is horizontal

13. Use the graph of f to answer each question.



a) Between which two consecutive points is the average rate of change of the function greatest?

$\boxed{AB}$

b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

Avg rate of change AB is greater than instantaneous rate of change at B.

c) Sketch a tangent line to the graph between C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

(on the sketch)

14. Find equations of the two lines that are tangent to the graphs of both $y = x^2$ and $y = -x^2 + 6x - 5$.

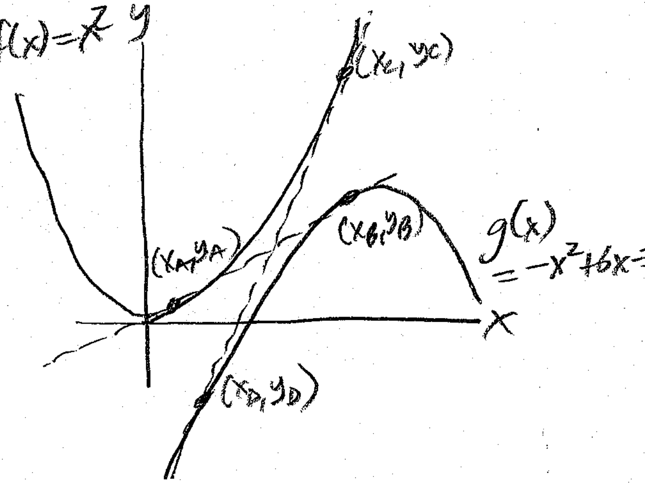
(on next page) →

(14) $y = x^2$ $\quad y = -x^2+6x-5$

rename to $\quad$ rename to

$f(x) = x^2 \qquad g(x) = -x^2+6x-5$

$f'(x) = 2x \qquad g'(x) = -2x+6$



For each tangent line, the slopes (derivatives) must be equal on both curves:

A–B: $m = \dfrac{y_B - y_A}{x_B - x_A} = \dfrac{(-(x_B)^2+6x_B-5)-(x_A)^2}{x_B - x_A}$

but $f'(x_A) \overset{\text{must}}{=} g'(x_B)$

$2x_A = -2x_B+6 \rightarrow \underline{x_A = -x_B+3}$

so $m = \dfrac{(-x_B^2+6x_B-5)-(-x_B+3)^2}{x_B-(-x_B+3)} = \dfrac{-x_B^2+6x_B-5-x_B^2+6x_B-9}{2x_B-3}$

$m = \dfrac{-2x_B^2+12x_B-14}{2x_B-3} \overset{\text{must}}{=} g'(x_B) = -2x_B+6$

so $-2x_B^2+12x_B-14 = (-2x_B+6)(2x_B-3)$

$2x_B^2-6x_B+4=0$

$x_B^2-3x_B+2=0$

$(x_B-1)(x_B-2)=0$

$\boxed{x_B=1}$ or $x_B=2$

(actually, this is x_D)

so $\boxed{x_D=2}$

Now, for $\underline{x_B=1}$:

$y_B = g(1) = -(1)^2+6(1)-5 = 0$

so $(1,0)$ is on the tangent line

and $x_A = -x_B+3 = -(1)+3 = 2$

$y_A = f(2) = 2(2) = 4$

so $(2,4)$ is also on the tangent line

so $\underline{\text{1st tangent line}}$: $m = \frac{4-0}{2-1} = 4$

$\boxed{(y-0) = 4(x-1)}$

and for $\underline{x_D=2}$:

$y_D = g(2) = -(2)^2+6(2)-5 = 3$

so $(2,3)$ is on the tangent line

$x_C = -x_D+3 = -2+3 = 1$

$y_C = f(1) = (1)^2 = 1$

so $(1,1)$ is also on the tangent line

so $\underline{\text{2nd tangent line}}$: $m = \frac{3-1}{2-1} = 2$

$\boxed{(y-1) = 2(x-1)}$

Find the average rate of change of the function over the given interval. Compare the average rate of change with the instantaneous rates of change at the endpoints of the interval.

15. $g(x) = x^2 + e^x$, $[0,1]$

avg rate of change $= \boxed{e}$

$g'(0) = \boxed{1}$ $g'(1) = \boxed{1+e}$

(comparison description)

16. $h(x) = x^3 - \frac{1}{2}e^x$, $[0,2]$

avg rate of change $= \dfrac{h(2)-h(0)}{2-0}$

$= \dfrac{(8-\frac{1}{2}e^2)-(0-\frac{1}{2}e^0)}{2}$

$\approx \boxed{2.4027}$

$h'(x) = 3x^2 - \frac{1}{2}e^x$

$h'(0) = 0 - \frac{1}{2}e^0 = \boxed{-\frac{1}{2}}$

$h'(2) = 2^3 - \frac{1}{2}e^2 \approx \boxed{4.305}$

The avg rate of change of $h(x)$ is between the instantaneous rates of change at the endpoints.

17. The volume of a cube with sides of length s is given by $V = s^3$. Find the rate of change of the volume with respect to s when s = 6 centimeters.

$\frac{dV}{ds} = \boxed{108\ cm^3/sec}$

18. The area of a square with sides of length s is given by $A = s^2$. Find the rate of change of the area with respect to s when s = 6 meters.

$\frac{dA}{ds} = 2s\Big|_{s=6} = 2(6) = \boxed{12\ m^2/sec}$

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3.2 Worksheet (Odds)

Use the Product Rule to find the derivative of the function.

1. $g(x) = (2x - 3)(1 - 5x)$

$\boxed{g'(x) = -20x + 17}$

2. $y = (3x - 4)(x^3 + 5)$

$y' = (3x-4)(3x^2) + (x^3+5)(3)$

$= 9x^3 - 12x^2 + 3x^3 + 15$

$= \boxed{12x^3 - 12x^2 + 15}$

Use the Quotient Rule to find the derivative of the function.

3. $h(x) = \frac{\sqrt{x}}{x^3+1}$

$\boxed{\dfrac{\frac{1}{2}x^{1/2} - \frac{5}{2}x^{5/2}}{(x^3+1)^2}}$

4. $f(x) = \frac{x^2}{2\sqrt{x}+1}$

$f'(x) = \dfrac{(2\sqrt{x}+1)(2x) - x^2(x^{-1/2})}{(2\sqrt{x}+1)^2}$

$= \dfrac{4x^{3/2} + 2x - x^{3/2}}{(2\sqrt{x}+1)^2} = \boxed{\dfrac{3x^{3/2} + 2x}{(2\sqrt{x}+1)^2}}$

Find f '(x) and then use it to find f '(c).

5. $f(x) = (x^3 + 4x)(3x^2 + 2x - 5), \quad c = 0$

$\boxed{f'(0) = -20}$

6. $f(x) = (x^2 - 3x + 2)(x^3 + 1), \quad c = 2$

$f'(x) = (x^2 - 3x + 2)(3x^2) + (x^3+1)(2x-3)$

$f'(2) = (2^2 - 3(2) + 2)(3 \cdot 2^2) + (2^3+1)(2(2)-3)$

$= (4 - 6 + 2)(12) + (9)(1)$

$= 0 + 9$

$= \boxed{9}$

7. $f(x) = \frac{x^2-4}{x-3}, \quad c = 1$

$\boxed{f'(1) = -\frac{1}{4}}$

8. $f(x) = \frac{x-4}{x+4}, \quad c = 3$

$f'(x) = \dfrac{(x+4)(1) - (x-4)(1)}{(x+4)^2}$

$= \dfrac{x+4-x+4}{(x+4)^2} = \dfrac{8}{(x+4)^2}$

$f'(3) = \dfrac{8}{(3+4)^2} = \boxed{\dfrac{8}{49}}$

Complete the table to find the derivative of the function without using the Quotient Rule.

	Function	Rewrite	Differentiate	Simplify
9.	$y = \frac{x^3 + 6x}{3}$			$\boxed{y' = x^2 + 2}$
10.	$y = \frac{5x^2 - 3}{4}$	$y = \frac{5}{4}x^2 - \frac{3}{4}$	$y' = \frac{5}{2}x$	$\boxed{y' = \frac{5}{2}x}$
11.	$y = \frac{4x^{3/2}}{x}$			$\boxed{y' = \frac{2}{\sqrt{x}}}$
12.	$y = \frac{2x}{x^{1/3}}$	$y = 2x^{2/3}$	$y' = \frac{4}{3}x^{-1/3}$	$\boxed{y' = \frac{4}{3\sqrt[3]{x}}}$

Find the derivative of the algebraic function.

13. $f(x) = \frac{4-3x-x^2}{x^2-1}$

$$\boxed{f'(x) = \frac{3}{(x+1)^2}}$$

14. $f(x) = \frac{x^2+5x+6}{x^2-4}$

$$f'(x) = \frac{(x^2-4)(2x+5) - (x^2+5x+6)(2x)}{(x^2-4)^2}$$

$$= \frac{2x^3 - 8x + 5x^2 - 20 - 2x^3 - 10x^2 - 12x}{(x-2)(x+2)(x-2)(x+2)}$$

$$= \frac{-5x^2 - 20x - 20}{(x-2)^2(x+2)^2} = \frac{-5(x^2+4x+4)}{(x-2)^2(x+2)^2}$$

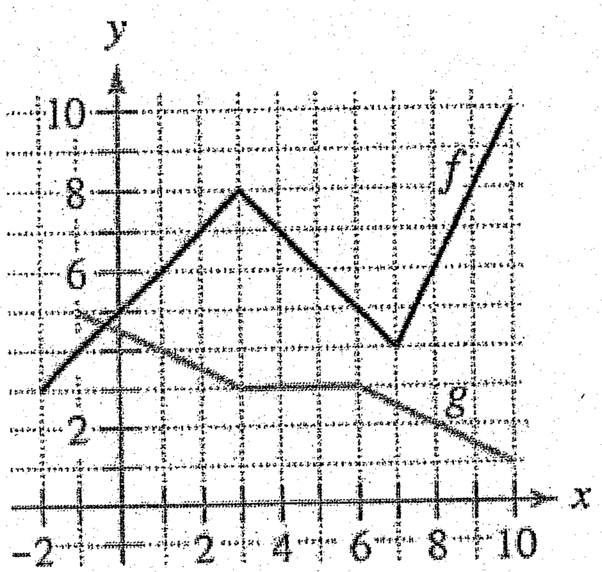
$$= \frac{-5(x+2)(x+2)}{(x-2)^2(x+2)^2} = \boxed{\frac{-5}{(x-2)^2}}$$

$p'(x) = f(x)g'(x) + g(x)f'(x)$

Use the graphs of f and g. Let p(x) = f(x)g(x) and q(x) = f(x)/g(x).

$q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

15.



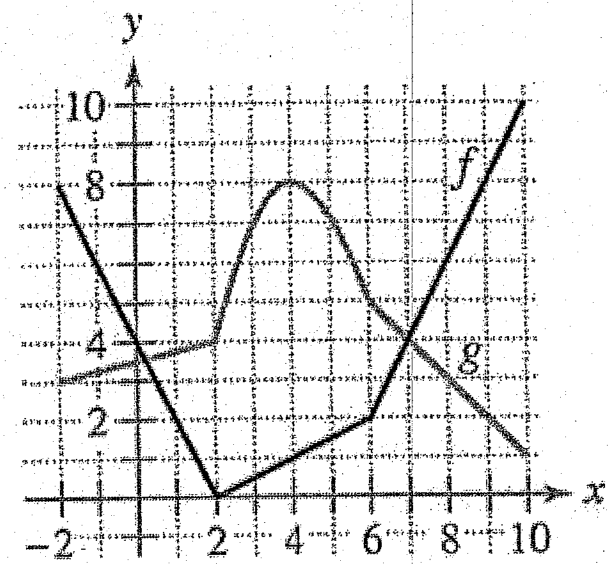
a) Find p'(1)

$\boxed{1}$

b) Find q'(4)

$\boxed{-\frac{1}{3}}$

16.



a) Find p'(4)

$p'(4) = f(4)g'(4) + g(4)f'(4)$

$= (8)(\frac{1}{2}) + (1)(0)$

$= 4 + 0$

$= \boxed{4}$

b) Find q'(7)

$q'(7) = \dfrac{g(7)f'(7) - f(7)g'(7)}{[g(7)]^2}$

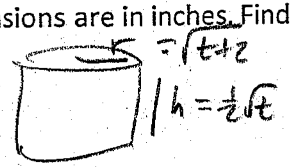
$= \dfrac{(4)(2) - (4)(-1)}{[4]^2}$

$= \dfrac{8+4}{16} = \dfrac{12}{16} = \boxed{\frac{3}{4}}$

17. The length of a rectangle is given by 6t + 5 and its height is $\sqrt{t}$, where t is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time.

$$\boxed{\frac{dA}{dt} = 9\sqrt{t} + \frac{5}{2\sqrt{t}}}$$

18. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.



$V = \pi r^2 h$

$V = \pi(\sqrt{t+2})^2(\frac{1}{2}\sqrt{t}) = \pi(t+2)\frac{1}{2}\sqrt{t} = \frac{\pi}{2}(t+2)t^{1/2}$

$V = \frac{\pi}{2}(t \cdot t^{1/2} + 2t^{1/2}) = \frac{\pi}{2}(t^{3/2} + 2t^{1/2})$

$\frac{dV}{dt} = \frac{\pi}{2}(\frac{3}{2}t^{1/2} + t^{-1/2}) = \frac{\pi}{4}(3t^{1/2} + 2t^{-1/2})$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{4}\left(3\sqrt{t} + \frac{2}{\sqrt{t}}\right)}$$

AP Calculus BC

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3.3 Worksheet (Odds)

Use rules of differentiation to find the derivative of the function.

1. $y = \frac{\pi}{2}\sin(x) - \cos(x)$

$\boxed{y' = \frac{\pi}{2}\cos x + \sin x}$

2. $g(t) = \pi\cos(t)$

$\boxed{g'(t) = -\pi\sin t}$

3. $y = x^2 - \frac{1}{2}\cos(x)$

$\boxed{y' = 2x + \frac{1}{2}\sin x}$

4. $y = 7 + \sin(x)$

$\boxed{y' = \cos x}$

5. $y = \frac{1}{2}e^x - 3\sin(x)$

$\boxed{y' = \frac{1}{2}e^x - 3\cos x}$

6. $y = \frac{3}{4}e^x + 2\cos(x)$

$\boxed{y' = \frac{3}{4}e^x - 2\sin x}$

Use the Product Rule to find the derivative of the function.

7. $f(x) = e^x\cos(x)$

$f'(x) = -e^x\sin x + e^x\cos x$

or

$e^x(\cos x - \sin x)$

8. $g(x) = \sqrt{x}\sin(x)$

$g'(x) = \sqrt{x}\frac{d}{dx}[\sin x] + \sin x\frac{d}{dx}[x^{1/2}]$

$= \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right)$

$\boxed{g'(x) = \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}}$

Use the Quotient Rule to find the derivative of the function.

9. $g(x) = \frac{\sin(x)}{e^x}$

$\boxed{g'(x) = \frac{\cos x - \sin x}{e^x}}$

10. $f(t) = \frac{\cos(t)}{t^3}$

$f'(t) = \frac{t^3\frac{d}{dt}[\cos t] - \cos t\frac{d}{dt}[t^3]}{(t^3)^2}$

$= \frac{t^3(-\sin t) - \cos t(3t^2)}{t^6}$

$= \frac{-t^2(t\sin t + 3\cos t)}{t^2 \cdot t^4}$

$= \boxed{-\frac{(t\sin t + 3\cos t)}{t^4}}$

Find the derivative of the transcendental function.

11. $f(x) = -e^x + \tan(x)$

$\boxed{f'(x) = -e^x + \sec^2 x}$

12. $f(x) = e^x - \cot(x)$

$f'(x) = e^x - (-\csc^2 x)$

$\boxed{f'(x) = e^x + \csc^2 x}$

13. $y = \frac{3(1-\sin(x))}{2\cos(x)}$

$\boxed{y' = \frac{-3(1-\sin x)}{2\cos^2 x}}$

14. $y = \frac{\sec(x)}{x}$

$y' = \frac{(x)(\sec x \tan x) - \sec x(1)}{x^2}$

$\boxed{y' = \frac{\sec x(x\tan x - 1)}{x^2}}$

Determine the points (if any) at which the graph of the function has a horizontal tangent line.

15. $y = x + \sin(x)$, $0 \le x < 2\pi$

$\boxed{(\pi, \pi)}$

16. $y = \sqrt{3}x + 2\cos(x)$, $0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \frac{\sqrt{3}}{2}$ $x = \pi/3$ or $2\pi/3$

$f(\pi/3) = \sqrt{3}\,\pi/3 + 2\underbrace{\cos(\pi/3)}_{(1/2)} = \frac{\sqrt{3}\pi}{3} + 1$ $\boxed{\left(\frac{\pi}{3}, \frac{\sqrt{3}\pi}{3} + 1\right)}$

$f(2\pi/3) = \sqrt{3}\,2\pi/3 + 2\underbrace{\cos\left(\frac{2\pi}{3}\right)}_{(-1/2)} = \frac{2\sqrt{3}\pi}{3} - 1$ $\boxed{\left(\frac{2\pi}{3}, \frac{2\sqrt{3}\pi}{3} - 1\right)}$

Find an equation of the tangent line to the graph of f at the given point. Use your graphing calculator to graph f and the tangent line. Use your calculator's *tangent* feature to confirm your results.

17. $f(x) = \tan(x)$, $\left(\frac{\pi}{4}, 1\right)$

$\boxed{(y-1) = 2\left(x - \frac{\pi}{4}\right)}$

(graph)

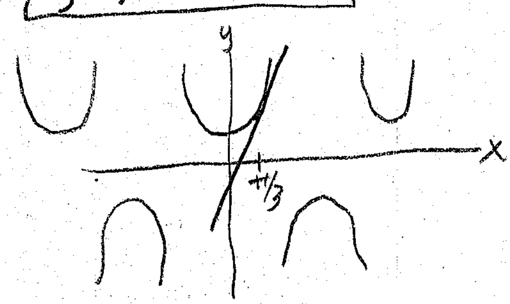
18. $f(x) = \sec(x)$, $\left(\frac{\pi}{3}, 2\right)$

$f'(x) = \sec x \tan x$

$f'(\pi/3) = \sec(\pi/3)\tan(\pi/3) = \frac{1}{\cos(\pi/3)} \frac{\sin(\pi/3)}{\cos(\pi/3)}$

$= \frac{1}{(\frac{1}{2})} \frac{(\sqrt{3}/2)}{(\frac{1}{2})} = \frac{\sqrt{3}}{2} \cdot \frac{4}{1} = 2\sqrt{3}$

$\boxed{(y-2) = 2\sqrt{3}(x - \pi/3)}$ $\left(y = 2\sqrt{3}x - \frac{2\sqrt{3}\pi}{3} + 2\right)$



Find the second derivative of the function.

19. $f(x) = x^2 + 7x - 4$

$\boxed{f''(x) = 2}$

20. $f(x) = 4x^5 - 2x^3 + 5x^2$

$f'(x) = 20x^4 - 6x^2 + 10x$

$\boxed{f''(x) = 80x^3 - 12x + 10}$

21. $f(x) = \frac{x}{x-1}$

$\boxed{f''(x) = \frac{2}{(x-1)^3}}$

22. $f(x) = \frac{x^2+3x}{x-4}$

$f'(x) = \frac{(x-4)(2x+3) - (x^2+3x)(1)}{(x-4)^2} = \frac{2x^2-5x-12-x^2-3x}{(x-4)^2}$

$= \frac{x^2-8x-12}{(x-4)^2}$

$f''(x) = \frac{(x-4)^2(2x-8) - (x^2-8x-12)(2(x-4)(1))}{(x-4)^4}$

continued...

Find the given higher order derivative.

23. $f'''(x) = 2\sqrt{x}, \quad f^{(4)}(x)$

$\boxed{f^{(4)}(x) = \frac{1}{\sqrt{x}}}$

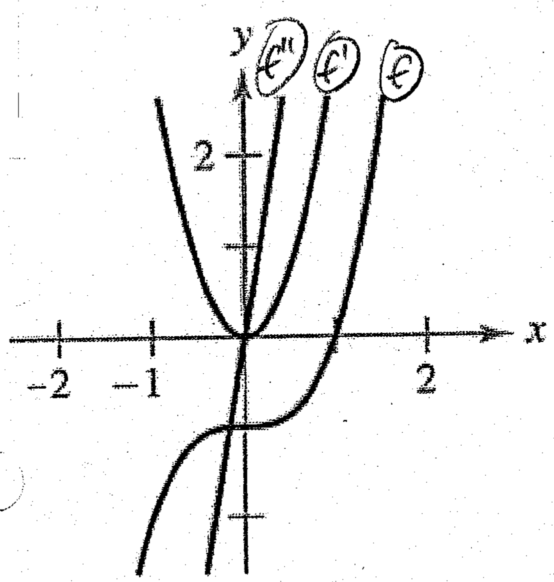
24. $f^{(4)}(x) = 2x + 1, \quad f^{(6)}(x)$

$f^{(5)}(x) = 2$

$\boxed{f^{(6)}(x) = 0}$

The graphs of f, f ′, and f ″ are shown on the same set of coordinate axes. Identify each graph. Explain your reasoning.

25.



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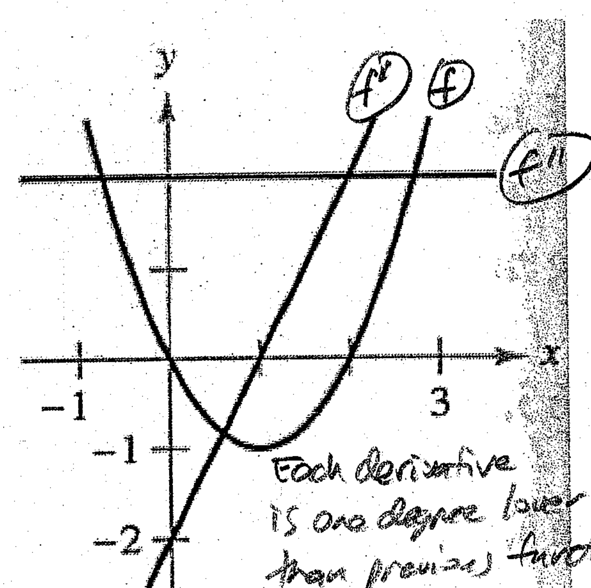


Each derivative is one degree lower than previous function

22 continued...

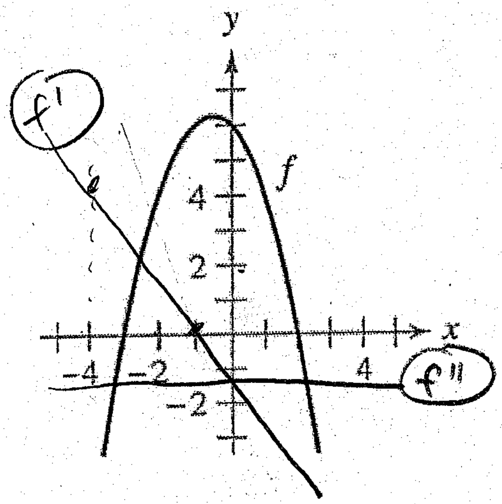
$$f''(x) = \frac{(x-4)\left[(x-4)(2x-8) - (x^2-8x-12)2\right]}{(x-4)^4}$$

$$= \frac{(x-4)\left[2x^2-16x+32-2x^2+16x+24\right]}{(x-4)^4}$$

$$= \frac{(x-4)\left[56\right]}{(x-4)^4} = \boxed{\frac{56}{(x-4)^3}}$$

The graph of f is shown. Sketch the graphs of f ' and f ''.

27.



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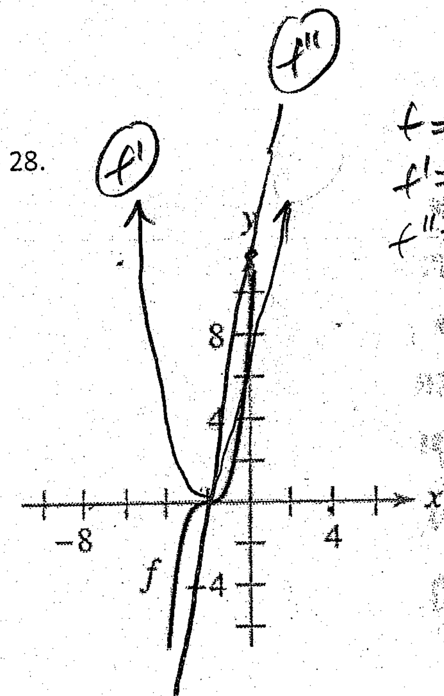
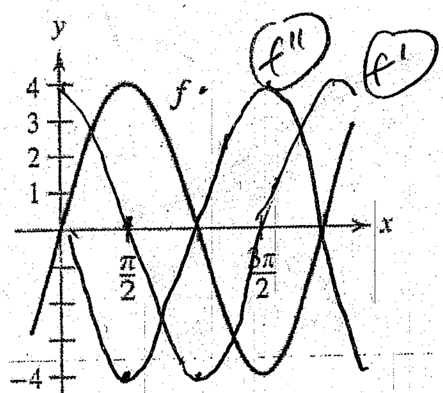


$f=(x+2)^3$

$f'=3(x+2)^2$

$f''=6(x+2)=6x+12$

29.

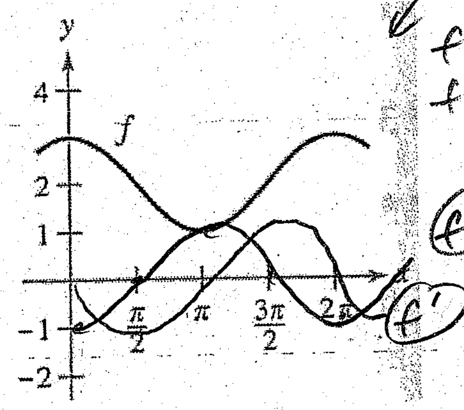


30.

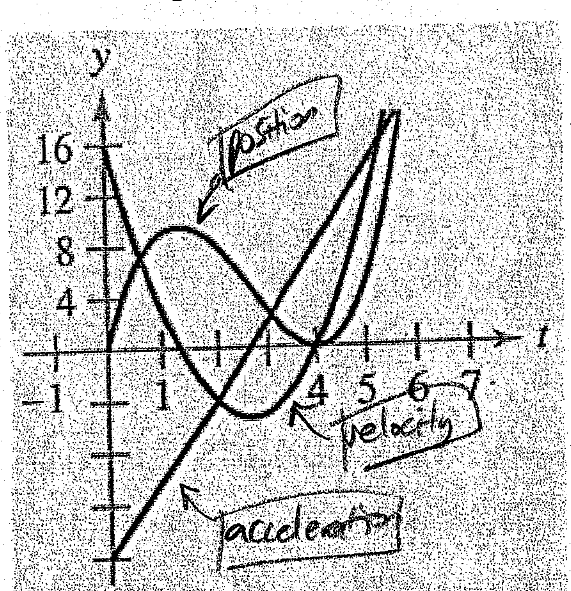
$f=\cos x+2$

$f'=-\sin x$

$f''=-\cos x$



31. The figure shows the graphs of the position, velocity, and acceleration functions of a particle.



a. Which graph is position? Which is velocity? Acceleration? Explain your reasoning.

(explanation)

b. On the graph, identify when the particle is speeding up or slowing down.

Speeding up: $1.3<t<2.7$ and $4<t<5.5$

Slowing down: $0<t<1.3$ and $2.7<t<4$

AP Calculus BC

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3.4 Worksheet (Odds and 28)

Find the derivative of the function.

1. $y = (2x-7)^3$

$\boxed{y' = 6(2x-7)^2}$

2. $y = 5(2-x^3)^4$

$y' = 5\frac{d}{dx}\left[(2-x^3)^4\right] = 5\left[4(2-x^3)^3\frac{d}{dx}(2-x^3)\right]$

$= 5(4)(2-x^3)^3(-3x^2)$

$= \boxed{-60x^2(2-x^3)^3}$

3. $f(t) = \sqrt{5-t}$

$\boxed{f'(t) = -\frac{1}{2}(5-t)^{-1/2} = \frac{-1}{2\sqrt{5-t}}}$

4. $g(x) = \sqrt{4-3x^2}$

$g'(x) = \frac{1}{2}(4-3x^2)^{-1/2}(-6x)$

$= \frac{-6x}{2\sqrt{4-3x^2}} = \boxed{\frac{-3x}{\sqrt{4-3x^2}}}$

5. $f(t) = \left(\frac{1}{t-3}\right)^2$

$\boxed{f'(t) = \frac{-2}{(t-3)^3}}$

6. $y = -\frac{3}{(t-2)^4} = -3(t-2)^{-4}$

$y' = 12(t-2)^{-5}(1) = \boxed{\frac{12}{(t-2)^5}}$

7. $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

$\boxed{g'(x) = \frac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}}$

8. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$

$h'(t) = 2\left(\frac{t^2}{t^3+2}\right)^1\left[\frac{(t^3+2)(2t)-(t^2)(3t^2)}{(t^3+2)^2}\right]$

$= 2\left(\frac{t^2}{t^3+2}\right)\left[\frac{2t^4+4t-3t^4}{(t^3+2)^2}\right]$

$= 2\left(\frac{t^2}{t^3+2}\right)\left(\frac{-t^4+4t}{(t^3+2)^2}\right) = \boxed{\frac{-2t^3(t^3-4)}{(t^3+2)^3}}$

Find the slope of the line tangent to the graph of the function at the given point.

9. $y = \ln(x^3)$, (1,0)

$\boxed{3}$

10. $y = \ln\left(x^{\frac{3}{2}}\right)$, (1,0)

$y' = \frac{1}{x^{3/2}}\frac{d}{dx}\left[x^{3/2}\right] = \frac{1}{x^{3/2}}\left[\frac{3}{2}x^{1/2}\right]$

$= \frac{3x^{1/2}}{2x^{3/2}} = \frac{3}{2x}\Big|_{x=1} = \frac{3}{2(1)} = \boxed{\frac{3}{2}}$

Find the derivative of the function.

11. $y = \frac{2}{e^x + e^{-x}}$

this one requires quotient rule

$$\boxed{y' = \frac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}}$$

can't split "a common numerator"

12. $y = \frac{e^x - e^{-x}}{2} = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$y' = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

$$\boxed{y' = \frac{1}{2}(e^x + e^{-x})}$$

ok to split a common denominator

13. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$$\boxed{f'(x) = \frac{-x^2+1}{x(x^2+1)} = \frac{-1(x^2-1)}{x(x^2+1)}}$$

14. $f(x) = \ln\left(\frac{2x}{x+3}\right)$

$f'(x) = \frac{1}{\left(\frac{2x}{x+3}\right)} \frac{d}{dx}\left[\frac{2x}{x+3}\right] = \frac{x+3}{2x}\left(\frac{(x+3)(2) - (2x)(1)}{(x+3)^2}\right)$

$= \frac{(x+3)}{2x} \frac{(2x+6-2x)}{(x+3)^2}$

$= \frac{(x+3)(6)}{2x(x+3)^2} = \boxed{\frac{3}{x(x+3)}}$

15. $y = \ln\left(\sqrt{\frac{x+1}{x-1}}\right)$

$$\boxed{y' = \frac{-1}{(x+1)(x-1)}}$$

16. $y = \ln\left(\sqrt[3]{\frac{x-2}{x+2}}\right)$

$y' = \frac{1}{\sqrt[3]{\frac{x-2}{x+2}}} \frac{d}{dx}\left[\left(\frac{x-2}{x+2}\right)^{1/3}\right] = \frac{1}{\sqrt[3]{\frac{x-2}{x+2}}} \frac{1}{3}\left(\frac{x-2}{x+2}\right)^{-2/3} \frac{d}{dx}\left[\frac{x-2}{x+2}\right]$

$= \frac{1}{\left(\frac{x-2}{x+2}\right)^{1/3} 3\left(\frac{x-2}{x+2}\right)^{2/3}}\left(\frac{(x+2)(1)-(x-2)(1)}{(x+2)^2}\right)$

$= \frac{1}{3\left(\frac{x-2}{x+2}\right)} \frac{(x+2-x+2)}{(x+2)^2} = \frac{(x+2)(4)}{3(x-2)(x+2)^2} = \boxed{\frac{4}{3(x-2)(x+2)}}$

17. $y = \ln|\sin(x)|$

$$\boxed{y' = \cot x}$$

18. $y = \ln|\csc(x)|$

$y' = \frac{1}{\csc x} \frac{d}{dx}[\csc x] = \frac{1}{\csc x}(-\csc x \cot x)$

$$\boxed{y' = -\cot x}$$

Find and evaluate the derivative of the function at the given point. Use a calculator to verify.

19. $y = \sqrt{x^2 + 8x}$, $(1,3)$

$f'(1) = \boxed{\frac{5}{3}}$

20. $y = \sqrt[5]{3x^3 + 4x}$, $(2,2)$

$y = (3x^3+4x)^{1/5}$

$y' = \frac{1}{5}(3x^3+4x)^{-4/5}(9x^2+4)$

$f'(2) = \frac{1}{5}(3(2)^3+4(2))^{-4/5}(9(2)^2+4)$

$= \frac{1}{5}(32)^{-4/5}(40)$

$= \boxed{\frac{1}{2}}$

Find an equation of a line tangent to the graph of f at the given point. Graph both f and the tangent line on your graphing calculator.

21. $f(x) = \tan^2(x)$, $\left(\frac{\pi}{4}, 1\right)$

$\boxed{(y-1) = 4\left(x - \frac{\pi}{4}\right)}$

(graph)

22. $y = 2\tan^3(x)$, $\left(\frac{\pi}{4}, 2\right)$

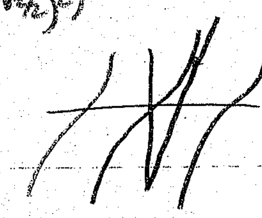
$y' = 6(\tan x)^2(\sec^2 x)$

$y'\Big|_{x=\pi/4} = 6\left(\frac{\sin \pi/4}{\cos \pi/4}\right)^2\left(\frac{1}{\cos \pi/4}\right)^2$

$= 6\left(\frac{\sqrt{2}/2}{\sqrt{2}/2}\right)^2\left(\frac{1}{(\sqrt{2}/2)^2}\right) = 6(1)\left(\frac{4}{2}\right) = 12$

$\boxed{(y-2) = 12(x - \pi/4)}$

$(y = 12x - 3\pi + 2)$



Find the second derivative of the function.

23. $f(x) = 5(2 - 7x)^4$

$\boxed{f''(x) = 2940(2-7x)^2}$

24. $f(x) = 6(x^3 + 4)^3$

$f'(x) = 18(x^3+4)^2(3x^2)$ (product rule)

$f''(x) = 18(x^3+4)^2(6x) + (3x^2)(36(x^3+4)^1(3x^2))$

$= \boxed{108x(x^3+4)^2 + 324x^4(x^3+4)}$

$= x(x^3+4)(108(x^3+4) + 324x^3)$

$= x(x^3+4)(432x^3+432)$

$= \boxed{432x(x^3+4)(x^3+1)}$

25. $f(x) = \sin(x^2)$

$\boxed{f''(x) = -4x\sin(x^2) + 2\cos(x^2)}$

26. $f(x) = \sec^2(\pi x)$

$f'(x) = 2(\sec(\pi x))^1(\sec(\pi x)\tan(\pi x))$

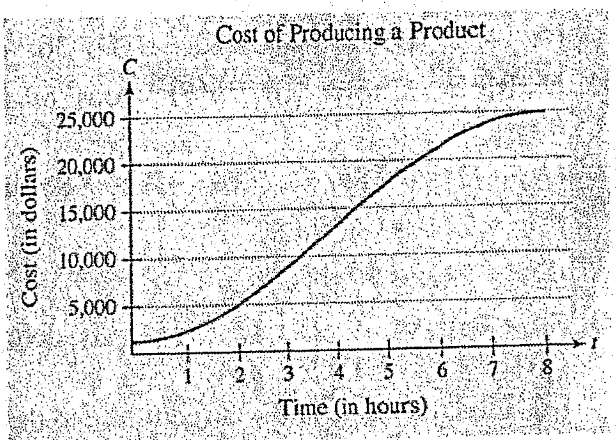
$= 2\sec^2(\pi x)\tan(\pi x)$ (product rule)

$f''(x) = 2\sec^2(\pi x)(\sec^2(\pi x)) + \tan(\pi x)(2(\sec(\pi x))^1(\sec x\tan x))$

$f''(x) = \boxed{2\sec^3(\pi x) + 2\sec^2(\pi x)\tan^2(\pi x)}$

$= \boxed{2\sec^2(\pi x)[\sec(\pi x) + \tan^2(\pi x)]}$

27. The cost C (in dollars) of producing x units of a product is $C = 60x + 1350$. For one week, management determined that the number of units produced x at the end of t hours can be modeled by $x = -1.6t^3 + 19t^2 - 0.5t - 1$. The graph shows the cost C in terms of the time t.



a. Using the graph, which is greater, the rate of change of the cost after 1 hour, or the rate of change of the cost after 4 hours?

4 hours (explanation)

b. Explain why the cost function is not increasing at a constant rate during the 8-hour shift.

(explanation)

28. Show that the derivative of an odd function is even. That is, if f(-x) = -f(x), then f '(-x) = f '(x).

(proof)

29. Show that the derivative of an even function is odd. That is, if f(-x) = f(x), then f '(-x) = -f '(x).

If $f(x)$ is even, $f(-x) = f(x)$

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[f(x)]$ derivatives are also equal

chain rule $f'(-x)\frac{d}{dx}[-x] = f'(x)$

$f'(-x)(-1) = f'(x)$

$f'(-x) = -f'(x)$

So derivative of an even function is odd.

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3.5 Worksheet (Odds)

Find the derivative of the function using implicit differentiation.

1. $x^{1/2} + y^{1/2} = 16$

$$\boxed{\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}}$$

2. $2x^3 + 3y^3 = 64$

$$\frac{d}{dx}[2x^3] + \frac{d}{dx}[3y^3] = \frac{d}{dx}[64]$$

$$6x^2 + 9y^2\frac{dy}{dx} = 0$$

$$9y^2\frac{dy}{dx} = -6x^2, \quad \frac{dy}{dx} = \frac{-6x^2}{9y^2} = \boxed{\frac{-2x^2}{3y^2}}$$

3. $x^3 - xy + y^2 = 7$

$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}}$$

4. $x^2y + y^2x = -2$

$$x^2\frac{d}{dx}[y] + y\frac{d}{dx}[x^2] + y^2\frac{d}{dx}[x] + x\frac{d}{dx}[y^2] = \frac{d}{dx}[-2]$$

$$x^2\frac{dy}{dx} + y(2x) + y^2(1) + x\left(2y\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2xy} = \frac{-y(y+2x)}{x(x+2y)}}$$

5. $x^3y^3 - y = x$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}}$$

6. $\sqrt{xy} = x^2y + 1$

$$\frac{d}{dx}[(xy)^{1/2}] = x^2\frac{d}{dx}[y] + y\frac{d}{dx}[x^2] + \frac{d}{dx}[1]$$

$$\frac{1}{2}(xy)^{-1/2}\left[x\frac{d}{dx}[y] + y\frac{d}{dx}[x]\right] = x^2\frac{dy}{dx} + y(2x) + 0$$

$$\frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y(1)\right) = x^2\frac{dy}{dx} + 2xy$$

$$\frac{dy}{dx}\left(\frac{x}{2\sqrt{xy}} - x^2\right) = 2xy - \frac{y}{2\sqrt{xy}}$$

...continued

7. $xe^y - 10x + 3y = 0$

$$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

8. $e^{xy} + x^2 - y^2 = 10$

$$\frac{d}{dx}[e^{(xy)}] + \frac{d}{dx}[x^2] - \frac{d}{dx}[y^2] = \frac{d}{dx}[10]$$

$$e^{(xy)}\frac{d}{dx}[xy] + 2x - 2y\frac{dy}{dx} = 0$$

$$e^{(xy)}\left(x\frac{d}{dx}[y] + y\frac{d}{dx}[x]\right) + 2x - 2y\frac{dy}{dx} = 0$$

$$e^{xy}\left(x\frac{dy}{dx} + y(1)\right) + 2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(xe^{xy} - 2y) = -2x - ye^{xy}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$$

6 continued...

$$\frac{dy}{dx} = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2} \frac{(2\sqrt{xy})}{(2\sqrt{xy})}$$

$$\boxed{\frac{dy}{dx} = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}}$$

Find the derivative by implicit differentiation and evaluate the derivative at the given point.

9. $xy = 6, \quad (-6,-1)$

$\boxed{\frac{dy}{dx} = \frac{-y}{x}}$ $\boxed{-\frac{1}{6}}$

10. $y^3 - x^2 = 4, \quad (2,2)$

$\frac{d}{dx}[y^3] - \frac{d}{dx}[x^2] = \frac{d}{dx}[4]$

$3y^2\frac{dy}{dx} - 2x = 0$

$\boxed{\frac{dy}{dx} = \frac{2x}{3y^2}} \Bigg|_{(2,2)} = \frac{2(2)}{3(2)^2} = \frac{4}{12} = \boxed{\frac{1}{3}}$

11. $3e^{xy} - x = 0, \quad (3,0)$

$\boxed{\frac{dy}{dx} = \frac{1-3ye^{xy}}{3xe^{xy}}}$ $\boxed{\frac{1}{9}}$

12. $y^2 = \ln(x), \quad (e,1)$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[\ln x]$

$2y\frac{dy}{dx} = \frac{1}{x}$

$\boxed{\frac{dy}{dx} = \frac{1}{2xy}} \Bigg|_{(e,1)} = \frac{1}{2(e)(1)} = \boxed{\frac{1}{2e}}$

Find the second derivative in terms of x and y using implicit differentiation.

13. $x^2 + y^2 = 4$

$\boxed{\frac{d^2y}{dx^2} = \frac{-y^2-x^2}{y^3}}$

14. $x^2y - 4x = 5$

$x^2\frac{d}{dx}[y] + y\frac{d}{dx}[x^2] - \frac{d}{dx}[4x] = \frac{d}{dx}[5]$

$x^2\frac{dy}{dx} + y(2x) - 4 = 0, \quad x^2\frac{dy}{dx} = 4 - 2xy$

$\frac{dy}{dx} = \frac{4-2xy}{x^2}$ (quotient rule)

$\frac{d^2y}{dx^2} = \frac{x^2\frac{d}{dx}[4-2xy] - (4-2xy)\frac{d}{dx}[x^2]}{(x^2)^2}$

$= \frac{x^2[0 - (2x\frac{d}{dx}[y] + y\frac{d}{dx}[2x])] - (4-2xy)(2x)}{x^4}$

$= \frac{x^2(0-(2x\frac{dy}{dx} + y(2))) - (4-2xy)(2x)}{x^4}$

$= \frac{-2x^3\frac{dy}{dx} + 2x^2y - 8x + 4x^2y}{x^4}$

continued...

14 continued...

$$\frac{d^2y}{dx^2} = \frac{-2x^3\boxed{\frac{dy}{dx}} + 2x^2y - 8x + 4x^2y}{x^4} \quad \left(\frac{dy}{dx} = \frac{4-2xy}{x^2}\right.$$

$$= \frac{-2x^3\left(\frac{4-2xy}{x^2}\right) + 2x^2y - 8x + 4x^2y}{x^4}$$

$$= \frac{-8x + 4x^2y + 2x^2y - 8x + 4x^2y}{x^4}$$

$$= \frac{10x^2y - 16x}{x^4}$$

$$= \frac{2x(5xy-8)}{x(x^3)}$$

$$= \boxed{\frac{2(5xy-8)}{x^3}}$$

Continued...

15. $xy - 1 = 2x + y^2$

$$\boxed{\frac{d^2y}{dx^2} = \frac{8-4x+2xy-2y^2}{(x-2y)^3}}$$

16. $x^2 - y^2 = 36$

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[y^2] = \frac{d}{dx}[36]$$

$$2x - 2y\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y\frac{d}{dx}[x] - x\frac{d}{dx}[y]}{y^2}$$

$$= \frac{y(1) - x\boxed{\frac{dy}{dx}}}{y^2} = \frac{y(1) - x\left(\frac{x}{y}\right)}{y^2}\left(\frac{y}{y}\right)$$

$$= \boxed{\frac{y^2-x^2}{y^3}}$$

Find the equation for the tangent line and normal line to the circle at each given point. Use a graphing calculator to graph them out and see it all in action!

17. $x^2 + y^2 = 25, \quad (4,3), (-3,4)$

$(y-3) = -\frac{4}{3}(x-4)$ tangent lines

$(y-4) = \frac{3}{4}(x+3)$

$(y-3) = \frac{3}{4}(x-4)$ normal lines

$(y-4) = -\frac{4}{3}(x+3)$

18. $x^2 + y^2 = 36, \quad (6,0), \ (5, \sqrt{11})$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = \frac{d}{dx}[36]$$

$$2x + 2y\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$(6,0)$

$\frac{dy}{dx} = \frac{-6}{(0)}$ DNE (vertical line)

$\boxed{x=6}$

$(5, \sqrt{11})$

$\frac{dy}{dx} = \frac{-5}{\sqrt{11}}$

$\boxed{(y-\sqrt{11}) = \frac{-5}{\sqrt{11}}(x-5)}$

normals $\perp$ tangent lines

$\boxed{y=0}$

$\boxed{(y-\sqrt{11}) = \frac{\sqrt{11}}{5}(x-5)}$

Use logarithmic differentiation to find the derivative.

19. $y = x^{2/x}, \ x > 0$

$$\boxed{\frac{dy}{dx} = \left(\frac{2(1-\ln x)}{x^2}\right)\left(x^{\frac{2}{x}}\right)}$$

20. $y = x^{x-1}, \quad x > 0$

$$\ln(y) = \ln(x^{x-1})$$

$$\ln(y) = (x-1)\ln(x)$$

$$\frac{d}{dx}[\ln y] = (x-1)\frac{d}{dx}[\ln x] + \ln x \frac{d}{dx}[(x-1)]$$

$$\frac{1}{y}\frac{dy}{dx} = (x-1)\frac{1}{x} + \ln x(1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x-1}{x} + \ln x$$

$$\frac{dy}{dx} = \left(\frac{x-1}{x} + \ln x\right)y$$

$$\boxed{\frac{dy}{dx} = \left(\frac{x-1}{x} + \ln x\right)\left(x^{x-1}\right)}$$

Continued...

21. $y = (x-2)^{x+1}, \quad x > 2$

$$\boxed{\frac{dy}{dx} = \left(\frac{x+1}{x-2} + \ln(x-2)\right)\left((x-2)^{x+1}\right)}$$

22. $y = (1+x)^{1/x}, \quad x > 0$

$$\ln(y) = \ln\left((1+x)^{1/x}\right)$$

$$\ln(y) = \frac{1}{x}\ln(1+x)$$

$$\frac{d}{dx}[\ln y] = \left(\frac{1}{x}\right)\frac{d}{dx}[\ln(1+x)] + \ln(1+x)\frac{d}{dx}[x^{-1}]$$

$$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{x}\right)\frac{1}{1+x}(1) + \ln(1+x)(-x^{-2})$$

$$\frac{dy}{dx} = \left(\frac{1}{x(1+x)} - \frac{\ln(1+x)}{x^2}\right)y$$

$$\boxed{\frac{dy}{dx} = \left(\frac{1}{x(1+x)} - \frac{\ln(1+x)}{x^2}\right)\left((1+x)^{1/x}\right)}$$

23. $y = x^{\ln(x)}, \quad x > 0$

$$\boxed{\frac{dy}{dx} = \frac{2\ln x}{x}\left(x^{\ln x}\right)}$$

24. $y = (\ln(x))^{\ln(x)}, \quad x > 1$

$$\ln(y) = \ln\left(\ln x^{\ln x}\right)$$

$$\ln(y) = (\ln x)(\ln(\ln x))$$

$$\frac{d}{dx}[\ln y] = \ln x \frac{d}{dx}[\ln(\ln x)] + \ln(\ln x)\frac{d}{dx}[\ln x]$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x \frac{1}{\ln x}\frac{1}{x} + \ln(\ln(x))\frac{1}{x}$$

$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{\ln(\ln(x))}{x}\right)y$$

$$\boxed{\frac{dy}{dx} = \left(\frac{1+\ln(\ln(x))}{x}\right)\left(\ln x^{\ln x}\right)}$$

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3.6 Worksheet (Odds and 14)

Show that the slopes of the graphs of f and f^{-1} are reciprocals at the given points.

1. $f(x) = x^3$ $\left(\frac{1}{2}, \frac{1}{8}\right)$

$f^{-1}(x) = \sqrt[3]{x}$

(verification steps)

2. $f(x) = 3 - 4x$ $(1, -1)$

$f^{-1}(x) = \frac{3-x}{4} = \frac{3}{4} - \frac{1}{4}x$

$f'(x) = -4,\ f'(1) = -4$ ✓ (reciprocals)

$(f^{-1}(x))' = -\frac{1}{4},\ (f^{-1}(-1))' = -\frac{1}{4}$ ✓

the y value

3. $f(x) = \sqrt{x-4}$ $(5,1)$

$f^{-1}(x) = x^2 + 4, x \geq 0$

(verification steps)

4. $f(x) = \frac{4}{1+x^2}$ $(1,2)$

$f^{-1}(x) = \sqrt{\frac{4-x}{x}} = \left(\frac{4-x}{x}\right)^{1/2}$

$f'(x) = \frac{(1+x^2)(0) - 4(2x)}{(1+x^2)^2} = \frac{-8x}{(1+x^2)^2}$

$f'(1) = \frac{-8(1)}{(1+1^2)^2} = \frac{-8}{4} = -2$ ✓

$(f^{-1}(x))' = \frac{1}{2}\left(\frac{4-x}{x}\right)^{-1/2}\left(\frac{x(-1)-(4-x)(1)}{x^2}\right)$

$(f^{-1}(2))' = \frac{1}{2}\left(\frac{4-2}{2}\right)^{-1/2}\left(\frac{2(-1)-(4-2)(1)}{(2)^2}\right)$

$= \frac{1}{2}(1)(-1) = -\frac{1}{2}$ ✓ (reciprocals)

Find the derivative of the function.

5. $f(x) = \arcsin(x+1)$

$$\boxed{f'(x) = \frac{1}{\sqrt{1-(x+1)^2}}}$$

6. $f(x) = \arcsin(x^2)$ typo

$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}}(2x)$

$$\boxed{f'(x) = \frac{2x}{\sqrt{1-x^4}}}$$

Continued...

7. $g(x) = \frac{\arcsin(3x)}{x}$

$$\boxed{g'(x) = \frac{\frac{3x}{\sqrt{1-9x^2}} - \arcsin(3x)}{x^2}}$$

8. $g(x) = \frac{\arccos(x)}{x+1}$

$$g'(x) = \frac{(x+1)\frac{d}{dx}[\arccos x] - \arccos x \frac{d}{dx}[(x+1)]}{(x+1)^2}$$

$$= \frac{(x+1)\frac{-1}{\sqrt{1-x^2}}(1) - \arccos x\,(1)}{(x+1)^2}$$

$$\boxed{g'(x) = \frac{\frac{-(x+1)}{\sqrt{1-x^2}} - \arccos(x)}{(x+1)^2}}$$

9. $y = 2x\arccos(x) - 2\sqrt{1-x^2}$

$$\boxed{y' = 2\arccos x}$$

10. $y = \ln(t^2+4) - \frac{1}{2}\arctan\left(\frac{t}{2}\right)$

$$y' = \frac{1}{t^2+4}(2t) - \frac{1}{2}\frac{1}{1+\left(\frac{t}{2}\right)^2}\left(\frac{1}{2}\right)$$

$$= \frac{2t}{t^2+4} - \frac{1}{4\left(1+\frac{t^2}{4}\right)}$$

$$= \frac{2t}{t^2+4} - \frac{1}{4+t^2}$$

$$\boxed{y' = \frac{2t-1}{t^2+4}}$$

Find an equation of the tangent line to the graph of the function at the given point.

11. $y = 2\arcsin(x)$, $\left(\frac{1}{2}, \frac{\pi}{3}\right)$

$$\boxed{\left(y - \frac{\pi}{3}\right) = \frac{4}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$$

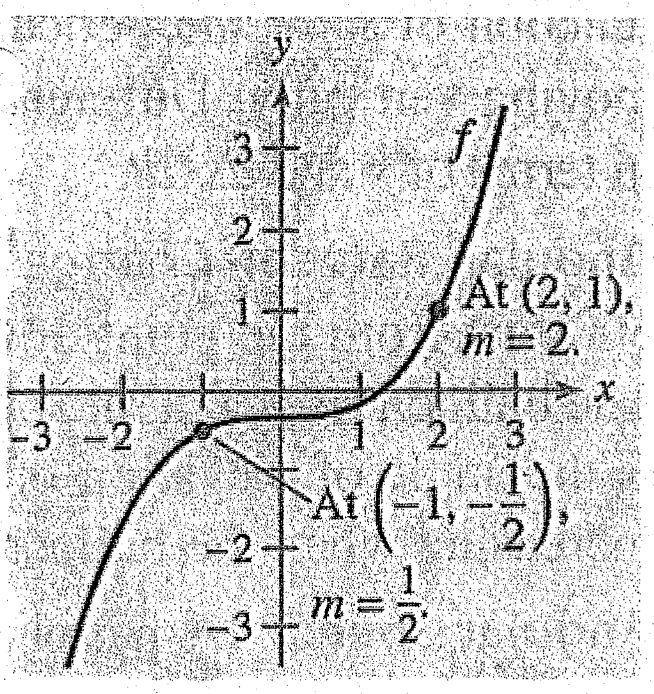
12. $y = \frac{1}{2}\arccos(x)$, $\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$

$$y' = \frac{1}{2}\frac{-1}{\sqrt{1-x^2}}\Big|_{x=-\frac{\sqrt{2}}{2}} = \frac{-1}{2\sqrt{1-\left(-\frac{\sqrt{2}}{2}\right)^2}} = \frac{-1}{2\sqrt{1-\frac{1}{2}}}$$

$$m = \frac{-1}{2\sqrt{\frac{1}{2}}} = \frac{-1}{\frac{2}{\sqrt{2}}} = \frac{-\sqrt{2}}{2}$$

$$\boxed{\left(y - \frac{3\pi}{8}\right) = \frac{-\sqrt{2}}{2}\left(x + \frac{\sqrt{2}}{2}\right)}$$

13. Use the information in the graph of f below.



a. What is the slope of the tangent line to the graph of f^{-1} at the point $\left(-\frac{1}{2}, -1\right)$? Explain.

$\boxed{2}$ (define $g(x) = f^{-1}(x)$

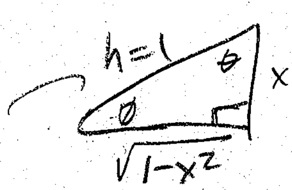
and use theorem:

$$g'(x) = \frac{1}{f'(g(x))}$$

b. What is the slope of the tangent line to the graph of f^{-1} at the point (1,2)? Explain.

$\boxed{\frac{1}{2}}$

14. Prove that $\arccos(x) = \frac{\pi}{2} - \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)$, $|x| < 1$.



$\tan\phi = \frac{opp}{adj} = \frac{x}{\sqrt{1-x^2}} \rightarrow \arctan\left(\frac{x}{\sqrt{1-x^2}}\right) = \phi$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} \rightarrow \arccos(x) = \theta$

$\phi + \theta = \pi/2 \ (90°)$

$\arctan\left(\frac{x}{\sqrt{1-x^2}}\right) + \arccos(x) = \pi/2$

so $\arccos x = \frac{\pi}{2} - \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)$ ✓

$h^2 = (\sqrt{1-x^2})^2 + x^2$

$h^2 = 1 - x^2 + x^2$

$h^2 = 1$

$h = 1$