These problems provide an overview of the unit but we recommend that you also review all homework problems from the unit.

#1) If $f(x) = \dfrac{x}{1+2x}$, find the slope of the tangent line at $\left(-\dfrac{1}{4}, -\dfrac{1}{2}\right)$ and use it to write the equation of

the tangent line to the curve.

#2) Find the slope of the tangent line to the curve $y = 9 - 2x^2$ at the point $(2,1)$.

Find an equation of this tangent line.

#3) The displacement (in meters) of an object moving in a straight line is given by $s = 1 + 2t + \dfrac{t^2}{4}$,

where t is measured in seconds.
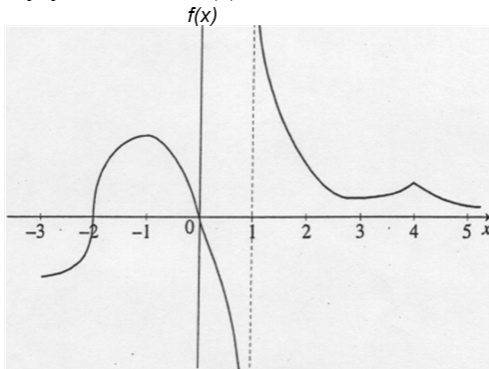 (a) Find the average velocity over the following time periods:
    (i) $[1,3]$ (ii) $[1,2]$ (iii) $[1,1.5]$ (iv) $[1,1.1]$

  (b) Find the instantaneous velocity when $t = 1$.

#4) Find values for a and b that will make f continuous everywhere, if $f(x) = \begin{cases} 3x+1, & x < 2 \\ ax+b, & 2 \le x < 5 \\ x^2, & 5 \le x \end{cases}$

#5) The graph of $f(x)$ is given below. For which value(s) of x is $f(x)$ not differentiable? Justify your answer(s).

#6) Given:

$$f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$$

(a) Find the right-hand and left-hand limits of f at x = -1, 0, and 1.
(b) Does f have a limit as x approaches -1? 0? 1?
    If so, what is it? If not, why not?
(c) Is f continuous at x = -1? 0? 1? Explain.

#7) Find the average rate of change of $f(x) = 1 + \sin x$ over the interval $[0, \frac{\pi}{2}]$.

#8) Let $f(x) = x^2 - 3x$ and $P = (1, f(1))$.

    Find (a) the slope of the curve $y = f(x)$ at P,

        (b) an equation of the tangent at P,

        (c) an equation of the normal at P.

#9) Is there a number that is exactly 4 more than its cube?

#10) Which of the following values is the average rate of change of $f(x) = \sqrt{x+1}$

    over the interval $[0, 3]$?

(multiple choice):   a) -3   b) -1   c) -1/3   d) 1/3   e) 3

#11) Which of the following statements is false for the function

$$f(x) = \begin{cases} \dfrac{3}{4}x, & 0 \le x < 4 \\ 2, & x = 4 \\ -x+7, & 4 < x \le 6 \\ 1, & 6 < x < 8 \end{cases}$$

(multiple choice):

a) $\lim\limits_{x \to 4} f(x)$ exists

b) $f(4)$ exists

c) $\lim\limits_{x \to 6} f(x)$ exists

d) $\lim\limits_{x \to 8^-} f(x)$ exists

e) f is continuous at $x = 4$

#12) Which of the following is an equation for the tangent line to $f(x) = 9 - x^2$ at $x = 2$?
(multiple choice):

a) $y = \dfrac{1}{4}x + \dfrac{9}{2}$

b) $y = -4x + 13$

c) $y = -4x - 3$

d) $y = 4x - 3$

e) $y = 4x + 13$

#13) (a) If $f(x) = e^{-x^2}$, estimate the value of $f'(1)$ graphically and numerically.

   (b) Find an approximate equation of the tangent line to the curve $f(x) = e^{-x^2}$

   at the point where $x = 1$

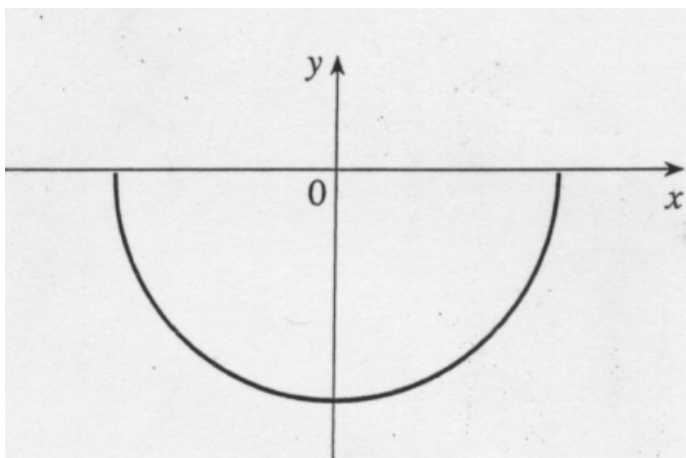   (c) Illustrate part (b) by graphing the curve and the tangent line on the same axes.

#14) Given the graph of f, sketch a graph of its derivative.
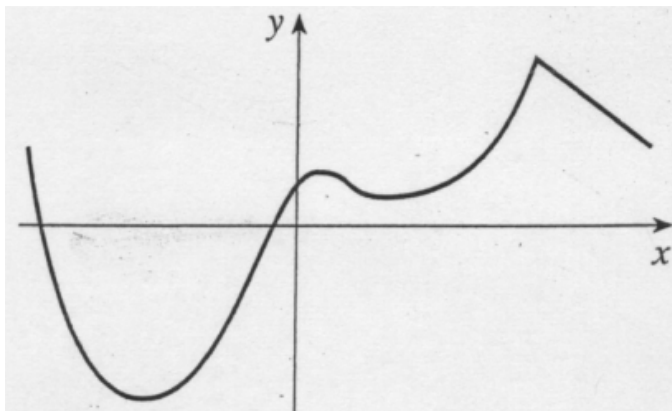
#15) Sketch the graph of f, then sketch the derivative of f.

$$f(x) = \begin{cases} 0, & x \le 0 \\ x & x > 0 \end{cases}$$

#16) Given the graph of *f*, sketch a graph of its derivative.

#17) Given the graph of *f*, sketch a graph of its derivative.