AP Calculus BC

Name: answers / Solutions

2.1 Homework

Find the average rate of change of the function over the closed interval.

1. $f(x) = 5x + 1$; over [1, 5]

$\boxed{5}$

2. $f(x) = -2x + 7$; over [-5, 2]

$f(2) = -2(2)+7 = 3$

$f(-5) = -2(-5)+7 =$

$\frac{f(2)-f(-5)}{2-(-5)} = \frac{3-17}{2+5} = \frac{-14}{7} = \boxed{-2}$

3. $f(x) = x^2 + 5x - 2$; over [2, 7]

$\boxed{14}$

4. $f(x) = 3x - 2$; over [2, 7]

$f(2) = 3(2) - 2 = 4$

$f(7) = 3(7) - 2 = 19$

$\frac{f(7)-f(2)}{7-2} = \frac{19-4}{7-2} = \frac{15}{5} = \boxed{3}$

5. Use the position function to answer the following questions. Position $x(t) = -t^2 + 18t - 65$, where t is time measured in hours and x(t) is position of an object in the x-direction from an arbitrary reference point and is measured in kilometers.

a. What is the average rate of change (velocity) of the object as it moves away from the reference point from time...

i) t = 5 to t = 9? 4 km/hr

ii) t = 9 to t = 13? −4 km/hr

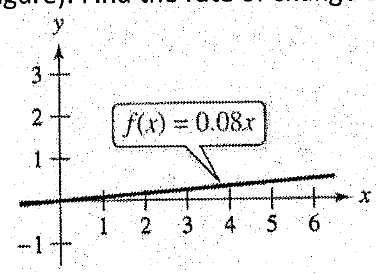
iii) t = 5 to t = 13? 0 km/hr

b. Describe what is happening to the position of the object from time t = 5 to time t = 13.

(explanation)

For 6 & 7: Decide whether the problem can be solved using precalculus or whether calculus is required. If the problem can be solved using precalculus, then solve it. If the problem seems to require calculus, explain your reasoning and use a graphical or numeric approach to estimate the solution.

6. A bicyclist is riding on a path modeled by the function $f(x) = 0.08x$, where x and f(x) are measured in miles (see figure). Find the rate of change of elevation at x = 2.

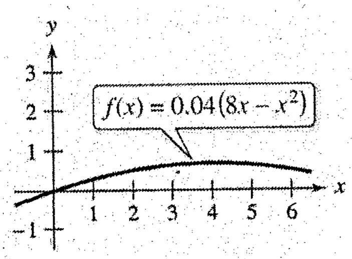


Rate of change = instantaneous rate of change (requires calculus)

... but since this is a line, slope = 0.08

So rate of change = $\boxed{0.08 \frac{\text{miles height}}{\text{miles horizontally}}}$

7. A bicyclist is riding on a path modeled by the function $f(x) = 0.04(8x - x^2)$, where x and f(x) are measured in miles (see figure). Find the rate of change of elevation at x = 2.

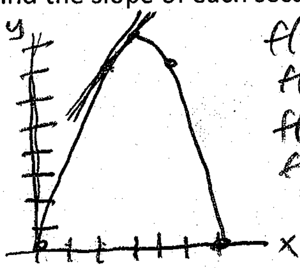


$\boxed{\frac{1}{5} \frac{\text{miles height}}{\text{miles horizontally}}}$

8. Consider the function $f(x) = 6x - x^2$ and the point P at (2, 8) on the graph of f.

a. Graph f and the secant lines passing through point P at (2, 8) and point Q at (x, f(x)) for x-values of 3, 2.5 and 1.5.

b. Find the slope of each secant line.



$f(2)=8$
$f(3)=9$
$f(2.5)=8.75$
$f(1.5)=6.75$

$(2,8)-(3,9)$: $m = \frac{9-8}{3-2} = \frac{1}{1} = \boxed{1}$

$(2,8)-(2.5,8.75)$: $m = \frac{8.75-8}{2.5-2} = \frac{0.75}{0.5} = \boxed{1.5}$

$(2,8)-(1.5,6.75)$: $m = \frac{6.75-8}{1.5-2} = \frac{-1.25}{-0.5} = \boxed{2.5}$

c. Use the results of part b to estimate the slope of the tangent line to the graph of f at point P (2, 8). Describe how to improve your approximation of the slope.

m is between 1.5 and 2.5 $\approx \boxed{2}$

To improve the approximation, we can choose point (x, f(x)) closer to (2, 8)
for example: (2.01, f(2.01))

9. Consider the function $f(x) = \sqrt{x}$ and the point P at (4, 2) on the graph of f.

a. Graph f and the secant lines passing through point P at (4, 2) and point Q at (x, f(x)) for x-values of 1, 3 and 5.

b. Find the slope of each secant line.

(graph) $\boxed{m = 0.333,\ 0.268,\ 0.236}$

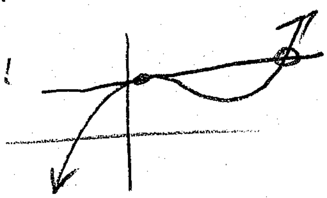
c. Use the results of part b to estimate the slope of the tangent line to the graph of f at point P (4, 2). Describe how to improve your approximation of the slope.

$\boxed{m \approx 0.252}$

(explanation)

10. Can a tangent line to a graph intersect the graph at more than one point? Explain your reasoning.

yes, for example!



... if the function has more than 2 regions where f changes between increasing and decreasing

11. How would you describe the instantaneous rate of change of an automobile's position on a highway?

(explanation)

AP Calculus BC Name: ______________________

2.2 Homework

Complete the table. Use the result to estimate the limit. Use your calculator to graph the function to confirm your results.

1. $\lim_{x \to 4} \frac{x-4}{x^2-5x+4} = \boxed{\frac{1}{3}}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)				?			

2. $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \boxed{\frac{1}{2}}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0,51317	0,50126	0,50013	?	0,49988	0,49876	0,48809

3. $\lim_{x \to 0} \frac{\sin(x)}{x} = \boxed{1}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)				?			

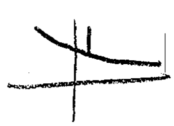
4. $\lim_{x \to 0} \frac{\cos(x)-1}{x} = \boxed{0}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0,04996	0,005	$5,10^{-4}$	?	$-5,10^{-4}$	-0,005	-0,05

5. $\lim_{x \to 0} \frac{e^x-1}{x} = \boxed{1}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)				?			

6. $\lim_{x \to 0} \frac{\ln(x+1)}{x} = \boxed{1}$



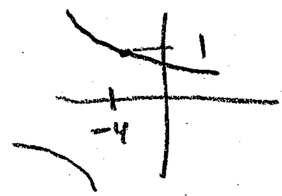
x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	1,0536	1,005	1,0005	?	0,9995	0,99503	0,9531

Create a table of values for the function and use the result to estimate the limit. Use your calculator to graph the function to confirm your result.

7. $\lim_{x\to 1} \frac{x-2}{x^2+x-6}$ $= 0{,}25 = \boxed{\frac{1}{4}}$

8. $\lim_{x\to -4} \frac{x+4}{x^2+9x+20}$ $= \boxed{1}$

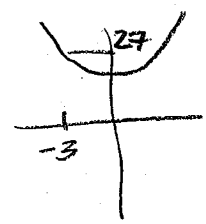
x	$\frac{x+4}{x^2+9x+20}$
$-4{,}1$	$1{,}1111$
$-4{,}01$	$1{,}0101$
$-3{,}99$	$0{,}9901$
$-3{,}9$	$0{,}90909$



9. $\lim_{x\to 1} \frac{x^4-1}{x^6-1}$ $= \boxed{\frac{2}{3}}$

10. $\lim_{x\to -3} \frac{x^3+27}{x+3}$ $= \boxed{27}$

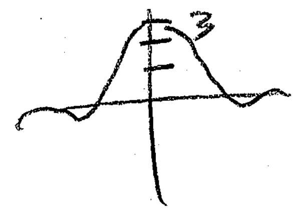
x	$\frac{x^3+27}{x+3}$
$-3{,}1$	$27{,}91$
$-3{,}01$	$27{,}09$
$-2{,}99$	$26{,}91$
$-2{,}9$	$26{,}11$



11. $\lim_{x\to 0} \frac{\sin(2x)}{x}$ $= \boxed{2}$

12. $\lim_{x\to 0} \frac{\sin(3x)}{x}$ $= \boxed{3}$

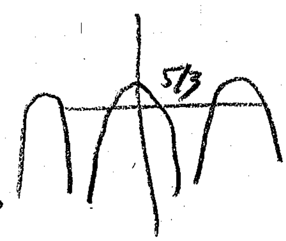
x	$\frac{\sin(3x)}{x}$
$-0{,}1$	$2{,}9552$
$-0{,}01$	$2{,}9996$
$0{,}01$	$2{,}9996$
$0{,}1$	$2{,}9552$



13. $\lim_{x\to 0} \frac{\tan(x)}{\tan(2x)}$ $= \boxed{\frac{1}{2}}$

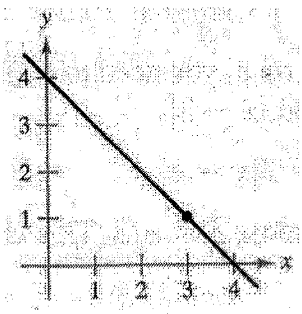
14. $\lim_{x\to 0} \frac{5\tan(x)}{\tan(3x)}$ $= 1{,}666 = \boxed{\frac{5}{3}}$

x	$\frac{5\tan x}{\tan(3x)}$
$-0{,}1$	$1{,}6218$
$-0{,}01$	$1{,}6662$
$0{,}01$	$1{,}6662$
$0{,}1$	$1{,}6218$

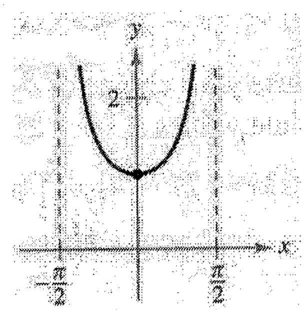


Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

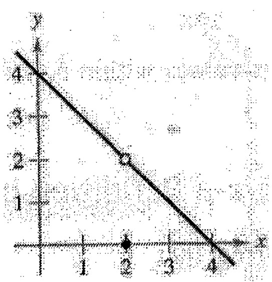
15. $\lim_{x\to 3}(4-x)$ $= \boxed{1}$



16. $\lim_{x\to 0}\sec(x)$ $= \boxed{1}$

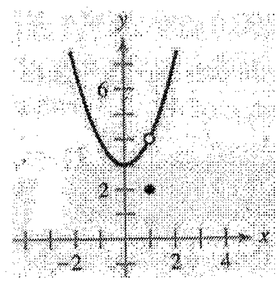


17. $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} 4-x & x \neq 2 \\ 0 & x = 2 \end{cases}$



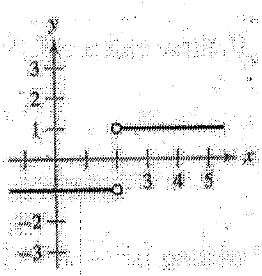
$= \boxed{2}$

18. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x^2+3 & x \neq 1 \\ 2 & x = 1 \end{cases}$

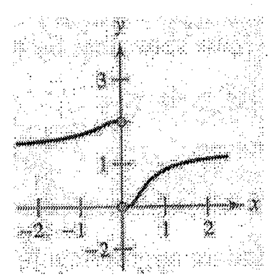


$= \boxed{4}$

19. $\lim_{x\to 2}\frac{|x-2|}{x+3}$ $\boxed{\text{DNE}}$



20. $\lim_{x\to 0}\frac{4}{2+e^{1/x}}$ $\boxed{\text{DNE}}$

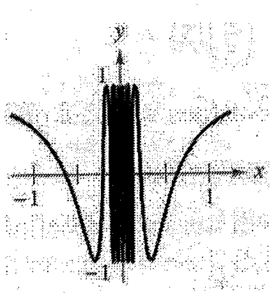


$\lim_{x\to 0^-}\frac{4}{2+e^{1/x}} = 2$

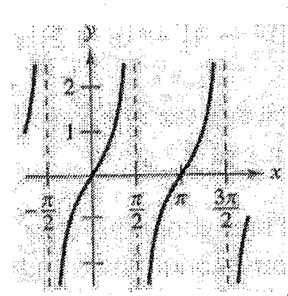
$\lim_{x\to 0^+}\frac{4}{2+e^{1/x}} = 0$

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

21. $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ $\boxed{\text{DNE}}$



22. $\lim_{x\to \pi/2}\tan(x)$ $\boxed{\text{DNE}}$



$\lim_{x\to \pi/2^-}\tan x = \infty$

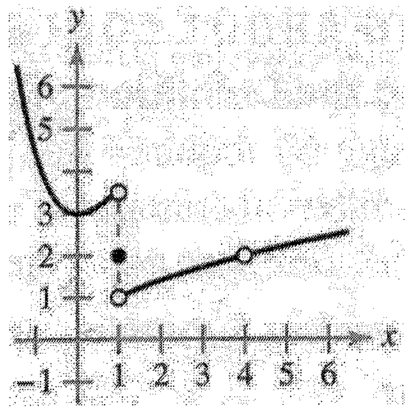
$\lim_{x\to \pi/2^+}\tan x = -\infty$

$\lim_{x\to \pi/2^-} f(x) \neq \lim_{x\to \pi/2^+} f(x)$

and you can't equate infinities because they aren't numbers

Use the graph of the function f to decide whether the value of the quantity exists. If it does, find it. If not, explain why.

23.



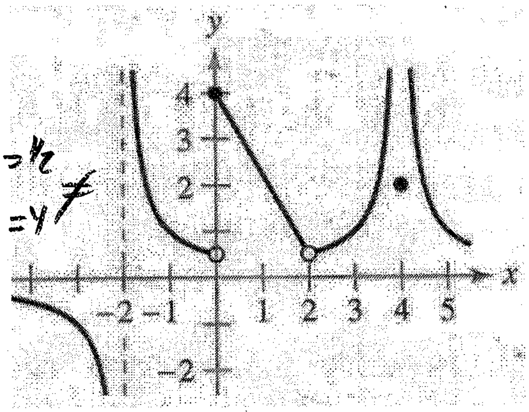
a. $f(1)$ $= \boxed{2}$

b. $\lim_{x\to 1} f(x)$ $\boxed{\text{DNE}}$

c. $f(0)$ $= \boxed{3}$

d. $\lim_{x\to 4} f(x)$ $= \boxed{2}$

24.



a. $f(-2)$ $\boxed{\text{DNE}}$

b. $\lim_{x\to 1} f(x)$ $= \boxed{2}$

c. $f(0)$ $= \boxed{4}$

d. $\lim_{x\to 0} f(x)$ $\boxed{\text{DNE}}$ $\lim_{x\to 0^-} f(x) = 1/2 \neq \lim_{x\to 0^+} f(x) = 4$

e. $f(2)$ $\boxed{\text{DNE}}$

f. $\lim_{x\to 2} f(x)$ $= \boxed{\frac{1}{2}}$

g. $f(4)$ $= \boxed{2}$

h. $\lim_{x\to 4} f(x)$ $\boxed{\text{DNE}}$ (can't equate infinities)

Sketch the graph of f. Then identify the values of c for which $\lim_{x\to c} f(x)$ exists.

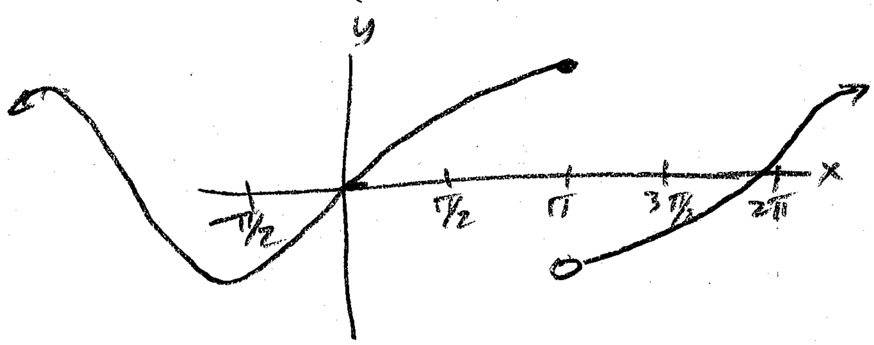
25. $f(x) = \begin{cases} x^2 & x \le 2 \\ 8 - 2x & 2 < x < 4 \\ 4 & x \ge 4 \end{cases}$

$\lim_{x\to c} f(x)$ exists (graph)

for all x

$\boxed{\text{except } x = 4}$

26. $f(x) = \begin{cases} \sin(x) & x < 0 \\ 1 - \cos(x) & 0 \le x \le \pi \\ \cos(x) & x > \pi \end{cases}$



$\lim_{x\to c} f(x)$ exists for all x

except $\boxed{x = \pi}$

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2.3 Homework

Find the limit. Use a graphing calculator to verify.

1. $\lim_{x\to -3} x^2 + 3x$ $= \boxed{0}$

2. $\lim_{x\to 2} -x^3 + 1$ $= -(2)^3+1 = \boxed{-7}$

3. $\lim_{x\to 3} \sqrt{x+1}$ $= \boxed{2}$

4. $\lim_{x\to 2} \sqrt[3]{12x+3}$ $= \sqrt[3]{12(2)+3} = \sqrt[3]{27} = \boxed{3}$

5. $\lim_{x\to 0} \sec(2x)$ $= \boxed{1}$

6. $\lim_{x\to \pi} \cos(3x)$ $= \cos(3\pi) = \boxed{-1}$

7. $\lim_{x\to 3} \tan\left(\frac{\pi x}{4}\right)$ $= \boxed{-1}$

8. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$ $= \frac{1}{\cos\left(\frac{7\pi}{6}\right)} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = \boxed{\frac{-2}{\sqrt{3}}}$

9. $\lim_{x\to 1} \ln(3x) + e^x$ $= \boxed{\ln(3)+e}$

10. $\lim_{x\to 1} \ln\left(\frac{x}{e^x}\right)$ $= \ln\left(\frac{1}{e^1}\right) = \ln(e^{-1}) = \boxed{-1}$

Write a simpler function that agrees with the given function at all but one point. Then find the limit of the function.

11. $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} -x^3 - 4 & x \neq -2 \\ -2 & x = 2 \end{cases}$ $\boxed{f(x) = -x^2 - 4}$ $\lim_{x\to 2} f(x) = \boxed{-12}$

12. $\lim_{x\to 3} f(x)$, where $f(x) = \begin{cases} 3x^2 - x + 1 & x \neq 3 \\ 3 & x = 3 \end{cases}$ $\lim_{x\to 3} f(x) = 3(3)^2 - (3) + 1 = \boxed{25}$

$\boxed{f(x) = 3x^2 - x + 1}$

Find the limit.

13. $\lim_{x\to 0} \frac{x}{x^2 - x}$ $= \boxed{-1}$

14. $\lim_{x\to 0} \frac{2x}{x^2+4x}$ $= \lim_{x\to 0} \frac{x(2)}{x(x+4)} = \lim_{x\to 0} \frac{2}{x+4}$

$= \frac{2}{(0)+4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

15. $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-16}$ $= \boxed{\frac{3}{8}}$

16. $\lim_{x\to 2} \frac{x^2+2x-8}{x^2-x-2}$

$= \lim_{x\to 2} \frac{(x+4)(x-2)}{(x+1)(x-2)}$

$= \lim_{x\to 2} \frac{x+4}{x+1} = \frac{(2)+4}{(2)+1} = \frac{6}{3} = \boxed{2}$

Find the limit.

17. $\lim_{x\to4}\frac{\sqrt{x+5}-3}{x-4}$ $= \boxed{\frac{1}{6}}$

18. $\lim_{x\to3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$

$= \lim_{x\to3}\frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to3}\frac{(x-3)(1)}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to3}\frac{1}{\sqrt{x+1}+2}$

$= \frac{1}{\sqrt{(3)+1}+2} = \boxed{\frac{1}{4}}$

Find the limit of the transcendental function.

19. $\lim_{x\to0}\frac{\sin(x)}{5x}$ $= \boxed{\frac{1}{5}}$

20. $\lim_{x\to0}\frac{3(1-\cos(x))}{x}$

$= 3\lim_{x\to0}\frac{1-\cos x}{x} = 3(0) = \boxed{0}$

21. $\lim_{x\to0}\frac{(\sin(x))(1-\cos(x))}{x^2}$ $= \boxed{0}$

22. $\lim_{x\to0}\frac{(\cos(x))(\tan(x))}{x}$

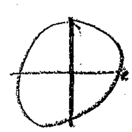
$= \lim_{x\to0}\frac{\cancel{\cos x}}{1}\cdot\frac{1}{x}\cdot\frac{\sin x}{1}\cdot\frac{1}{\cancel{\cos x}}$

$= \lim_{x\to0}\frac{\sin x}{x} = \boxed{1}$

23. $\lim_{x\to0}\frac{\sin^2(x)}{x}$ $= \boxed{0}$

24. $\lim_{x\to0}\frac{\tan^2(x)}{x}$

$= \lim_{x\to0}\frac{\sin x}{x}\cdot\frac{\sin x}{1}\cdot\frac{1}{(\cos x)^2}$

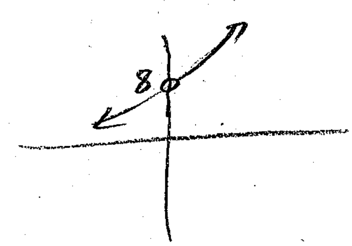

(1,0)

$= (1)\left(\frac{0}{1}\right)\left(\frac{1}{1^2}\right) = \boxed{0}$

25. $\lim_{x\to0}\frac{1-e^{-x}}{e^{x}-1}$ $= \boxed{1}$

26. $\lim_{x\to0}\frac{4(e^{2x}-1)}{e^{x}-1}$ $= \boxed{8}$

from calculator graph!



Find the limit of the transcendental function.

27. $\lim_{t\to 0}\frac{\sin(3t)}{2t}$ $= \boxed{\frac{3}{2}}$

28. $\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)}$ $= \boxed{\frac{2}{3}}$

$\lim_{x\to 0} \frac{\sin 2x}{1} \cdot \frac{1}{\sin 3x}$

$= \lim_{x\to 0} \frac{\sin 2x}{(2x)} \cdot \frac{(3x)}{\sin 3x} \cdot \frac{(2x)}{(3x)}$

$(1)(1)\left(\frac{2}{3}\right) = \boxed{\frac{2}{3}}$

Use a graphing calculator to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.

29. $\lim_{t\to 0}\frac{\sin(3t)}{t}$ $= \boxed{3}$

30. $\lim_{x\to 0}\frac{\cos(x)-1}{2x^2}$ $= \boxed{-\frac{1}{4}}$

x	$\frac{\cos x - 1}{2x^2}$
-0.1	-0.2498
-0.01	-0.25
0.01	-0.25
0.1	-0.2498

$\lim_{x\to 0} \frac{(\cos x - 1)}{2x^2}\frac{(\cos x + 1)}{(\cos x + 1)}$

$\lim_{x\to 0} \frac{\cos^2 x - 1}{2x^2(\cos x + 1)}$

$\lim_{x\to 0} \frac{\sin^2 x}{2x^2(\cos x + 1)}$

$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)\frac{1}{2}\frac{1}{(\cos x + 1)}$

$(1)(1)\left(\frac{1}{2}\right)\left(-\frac{1}{1+1}\right) = \boxed{-\frac{1}{4}}$

Find $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$.

31. $f(x) = 3x - 2$ $= \boxed{3}$

32. $f(x) = x^2 - 4x$

$\lim_{\Delta x\to 0} \frac{[(x+\Delta x)^2 - 4(x+\Delta x)] - [x^2 - 4x]}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 4x - 4\Delta x - x^2 + 4x}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{2x\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x} = \lim_{\Delta x\to 0} \frac{\Delta x(2x + \Delta x - 4)}{\Delta x(1)}$

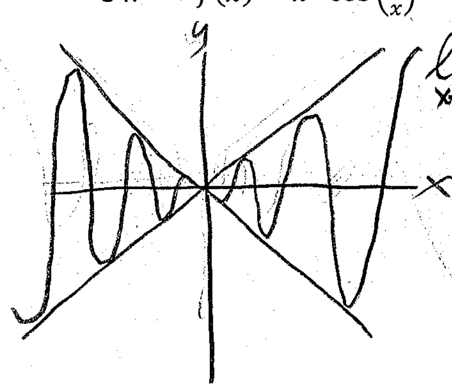
$= \lim_{\Delta x\to 0} (2x + \Delta x - 4) = 2x + (0) - 4 = \boxed{2x - 4}$

Use a graphing calculator to graph the given function and the equations $y = |x|$ and $y = -|x|$ in the same viewing window. Using the graphs to observe the Squeeze Theorem visually, find $\lim_{x\to 0} f(x)$.

33. $f(x) = x \cdot \sin\left(\frac{1}{x}\right)$

$\lim_{x\to 0} f(x) = \boxed{0}$

34. $f(x) = x \cdot \cos\left(\frac{1}{x}\right)$



$-|x| \le f(x) \le |x|$

$\lim_{x\to 0} -|x| \le \lim_{x\to 0} f(x) \le \lim_{x\to 0} |x|$

$0 \le \lim_{x\to 0} f(x) \le 0$

so $\lim_{x\to 0} f(x) = \boxed{0}$

AP Calculus BC

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2.4 Homework (odds and 30)

Find the limit (if it exists). If it does not exist, explain why.

1. $\lim\limits_{x\to 5^+} \frac{x-5}{x^2-25}$ $= \boxed{\frac{1}{10}}$

2. $\lim\limits_{x\to 4^+} \frac{4-x}{x^2-16}$ $= \lim\limits_{x\to 4^+} \frac{-(x-4)}{(x-4)(x+4)}$

$= \lim\limits_{x\to 4^+} \frac{-1}{x+4} = \boxed{\frac{-1}{8}}$

3. $\lim\limits_{x\to -3^-} \frac{x}{\sqrt{x^2-9}}$ $\boxed{\text{DNE}}$

4. $\lim\limits_{x\to 4^-} \frac{\sqrt{x}-2}{x-4}$ $\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4^-} \frac{x-4}{(x-4)(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4^-} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

5. $\lim\limits_{x\to 0^-} \frac{|x|}{x}$ $= \boxed{-1}$

6. $\lim\limits_{x\to 10^+} \frac{|x-10|}{x-10}$ for $x > 10$, $|x-10| = x-10$

$= \lim\limits_{x\to 10^+} \frac{x-10}{x-10} = \boxed{1}$

7. $\lim\limits_{\Delta x\to 0^-} \frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}$ $= \boxed{\frac{-1}{x^2}}$

8. $\lim\limits_{\Delta x\to 0^+} \frac{(x+\Delta x)^2+x+\Delta x-(x^2+x)}{\Delta x}$

$= \lim\limits_{\Delta x\to 0^+} \frac{x^2+2x\Delta x+(\Delta x)^2+x+\Delta x-x^2-x}{\Delta x}$

$= \lim\limits_{\Delta x\to 0^+} \frac{2x\Delta x+(\Delta x)^2+\Delta x}{\Delta x}$

$= \lim\limits_{\Delta x\to 0^+} \frac{\Delta x(2x+\Delta x+1)}{\Delta x}$

$= \lim\limits_{\Delta x\to 0^+} (2x+\Delta x+1) = 2x+(0)+1$

$= \boxed{2x+1}$

9. $\lim_{x\to 3^-} f(x)$, and $f(x) = \begin{cases} \frac{x+2}{2} & x \le 3 \\ \frac{12-2x}{3} & x > 3 \end{cases}$

$\boxed{\frac{5}{2}}$

10. $\lim_{x\to 3^-} f(x)$, and $f(x) = \begin{cases} x^2 - 4x + 6 & x < 3 \\ -x^2 + 4x - 2 & x \ge 3 \end{cases}$

$\lim_{x\to 3^-} (x^2 - 4x + 6) = (3)^2 - 4(3) + 6$

$= 9 - 12 + 6$

$= \boxed{3}$

11. $\lim_{x\to 4^-} 5[\![x]\!] - 7 = \boxed{8}$

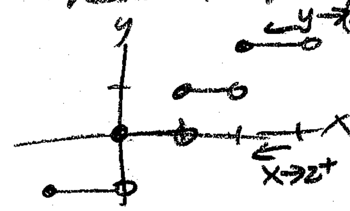
12. $\lim_{x\to 2^+} 2x - [\![x]\!]$ ← greatest integer function (the 'round down to nearest integer func)

$= 2(2) - \lim_{x\to 2^+} [\![x]\!]$

$= 2(2) - 2$

$= 4 - 2$

$= \boxed{2}$



13. $\lim_{x\to 3} 2 - [\![-x]\!]$ $\boxed{\text{DNE}}$

14. $\lim_{x\to 1} 1 - [\![-\frac{x}{2}]\!] = \boxed{2}$

$\lim_{x\to 1^-} 1 - [\![-\frac{x}{2}]\!]$

$1 - [\![-\frac{0.99}{2}]\!]$

$1 - [\![-.495]\!]$ round down

$1 - (-1)$

$\boxed{2}$

$\lim_{x\to 1^+} 1 - [\![-\frac{x}{2}]\!]$

$1 - [\![-0.55]\!]$ round down

$1 - (-1)$

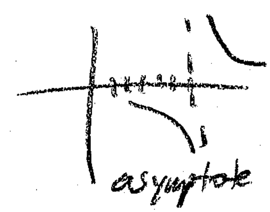
$\boxed{2}$

equal, so $\lim_{x\to 1} 1 - [\![-\frac{x}{2}]\!] = \boxed{2}$

Find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

15. $f(x) = \frac{6}{x}$ $\boxed{x=0 \text{ not removable}}$

16. $f(x) = \frac{4}{x-6}$ $\boxed{x=6 \text{ not removable}}$



17. $f(x) = 3x - \cos(x)$

$\boxed{\text{Continuous everywhere}}$

18. $f(x) = x^2 - 4x + 4$

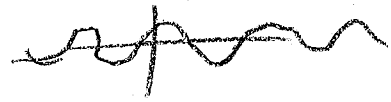
$\boxed{\text{continuous everywhere}}$

(polynomial)

Continued...

19. $f(x) = \frac{1}{4-x^2}$ $\boxed{x=2,\ x=-2 \text{ not removable}}$

20. $f(x) = \cos\left(\frac{\pi x}{2}\right)$ $\boxed{\text{Continuous everywhere}}$



21. $f(x) = \frac{x}{x^2-x}$ $\boxed{x=0,\text{ removable};\ x=1,\text{ not removable}}$

22. $f(x) = \frac{x}{x^2-4}$ $\Rightarrow \frac{x}{(x-2)(x+2)}$

$\boxed{x=2,\ x=-2 \text{ not removable}}$ (vertical asymptotes)

Discuss the continuity of the composite function $h(x) = f(g(x))$.

23. $f(x) = x^2, g(x) = x - 1$

$\boxed{\text{Continuous everywhere}}$

24. $f(x) = \frac{1}{\sqrt{x}}, g(x) = x - 1$

$f(x-1) = \frac{1}{\sqrt{x-1}}$ $-\infty \quad \infty$

$\boxed{\text{continuous over } (1, \infty)}$

25. $f(x) = \frac{1}{x-6}, g(x) = x^2 + 5$

$\boxed{\text{continuous over } (-\infty,-7)\cup(-7,1)\cup(1,\infty)}$

26. $f(x) = \sin(x), g(x) = x^2$

$f(x^2) = \sin(x^2)$

$\boxed{\text{continuous everywhere}}$

Use your calculator to find the zeros of the function. Approximate your answers to 4 decimal places.

27. $f(x) = x^3 + x - 1$

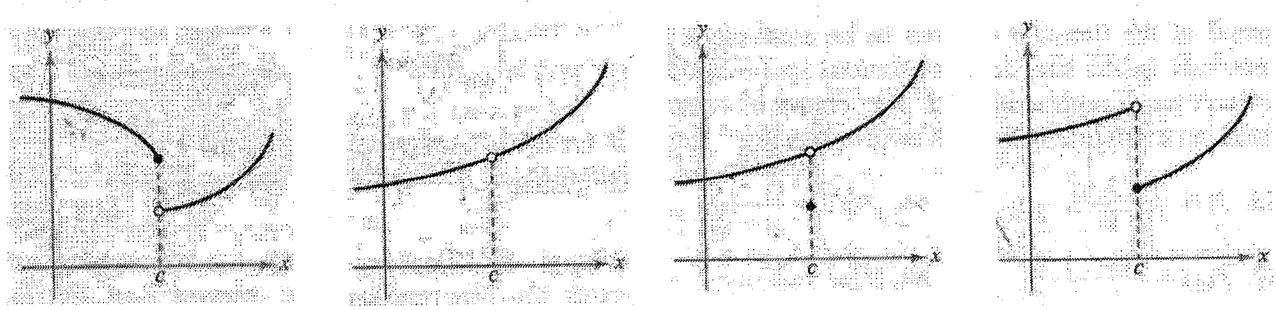
$x = \boxed{0.6823}$

28. $f(x) = x^4 - x^2 + 3x - 1$

$\boxed{x = -1.7455 \text{ and } x = 0.3733}$

Why is continuity ruined for the graphs at x = c?

29.



explain each) your choices: 1) $f(c)$ DNE

2) $\lim_{x\to c} f(x)$ DNE b/c $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$

3) $\lim_{x\to c} f(x) \neq f(c)$

30. Explain why each function is continuous or discontinuous.

(a) The temperature at a specific location as a function of time

(b) The temperature at a specific time as a function of the distance due west from New York City

(c) The altitude above sea level as a function of the distance due west from New York City

(d) The cost of a taxi ride as a function of the distance traveled

(e) The current in the circuit for the lights in a room as a function of time

(a) continuous (temperature always changes continuously)

(b) probably continuous (altitude doesn't change abruptly - unless you have a completely sheer cliff)

(c) probably continuous (unless there is a sheer cliff)

(d) discontinuous (money is usually rounded to nearest cent, so changes abruptly)

(e) discontinuous (light is either on or off)

31. Given that

$$\lim_{x \to 2} f(x) = 4 \qquad \lim_{x \to 2} g(x) = -2 \qquad \lim_{x \to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

$\boxed{-6} =$ (a) $\lim_{x \to 2} [f(x) + 5g(x)]$

(b) $\lim_{x \to 2} [g(x)]^3 = \boxed{-8}$

$\boxed{2} =$ (c) $\lim_{x \to 2} \sqrt{f(x)}$

(d) $\lim_{x \to 2} \frac{3f(x)}{g(x)} = \boxed{-6}$

$\boxed{\text{DNE}}$ (e) $\lim_{x \to 2} \frac{g(x)}{h(x)}$

(f) $\lim_{x \to 2} \frac{g(x)h(x)}{f(x)} = \boxed{0}$

AP Calculus BC

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2.5 Homework

Find each limit, if possible.

1a. $\lim_{x\to\infty} \frac{x^2+2}{x^3-1}$ $= \boxed{0}$

b. $\lim_{x\to\infty} \frac{x^2+2}{x^2-1}$ $= \boxed{1}$

c. $\lim_{x\to\infty} \frac{x^2+2}{x-1}$ $= \boxed{\infty}$

2a. $\lim_{x\to\infty} \frac{3-2x}{3x^3-1}$

$= \lim_{x\to\infty} \frac{\left(\frac{3}{x^3} - \frac{2}{x^2}\right)}{\left(3-\frac{1}{x^3}\right)}$

$= \frac{0}{3} = \boxed{0}$

b. $\lim_{x\to\infty} \frac{3-2x}{3x-1}$

$= \lim_{x\to\infty} \frac{\left(\frac{3}{x} - 2\right)}{\left(3-\frac{1}{x}\right)}$

$= \boxed{\frac{-2}{3}}$

c. $\lim_{x\to\infty} \frac{3-2x^2}{3x-1}$

$= \lim_{x\to\infty} \frac{\left(\frac{3}{x} - 2x\right)}{\left(3-\frac{1}{x}\right)}$

$= \lim_{x\to\infty} \frac{-2x}{3} = \boxed{-\infty}$

3a. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^2-4}$ $= \boxed{0}$

b. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4}$ $= \boxed{\frac{-2}{3}}$

c. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x-4}$ $= \boxed{-\infty}$

4a. $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^2+1}$

$= \lim_{x\to\infty} \frac{\left(\frac{5}{x^{1/2}}\right)}{\left(4+\frac{1}{x^2}\right)} = \frac{0}{4} = \boxed{0}$

b. $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^{3/2}+1}$

$= \lim_{x\to\infty} \frac{5}{4+\frac{1}{x^{3/2}}} = \boxed{\frac{5}{4}}$

c. $\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1}$

$= \lim_{x\to\infty} \frac{\left(\frac{5x^{3/2}}{x^{1/2}}\right)}{4+\frac{1}{x^{1/2}}}$

$= \lim_{x\to\infty} \frac{(5x)}{4+\frac{1}{x^{1/2}}} \Rightarrow \frac{\infty}{4} = \boxed{\infty}$

Find the limit.

5. $\lim_{x\to\infty} \frac{2x-1}{3x+2}$ $= \boxed{\frac{2}{3}}$

6. $\lim_{x\to\infty} \frac{4x^2+5}{x^2+3}$

$= \lim_{x\to\infty} \frac{\left(4+\frac{5}{x^2}\right)}{\left(1+\frac{3}{x^2}\right)} = \frac{4}{1} = \boxed{4}$

7. $\lim_{x\to\infty} \frac{x}{x^2-1}$ $= \boxed{0}$

8. $\lim_{x\to\infty} \frac{5x^3+1}{10x^3-3x^2+7}$

$= \lim_{x\to\infty} \frac{\left(5+\frac{1}{x^3}\right)}{\left(10-\frac{3}{x}+\frac{7}{x^3}\right)} = \frac{5}{10} = \boxed{\frac{1}{2}}$

Continued…

9. $\lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2-x}}$ $= \boxed{1}$

10. $\lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ for $x>0$, $x=\sqrt{x^2}$

$= \lim\limits_{x\to\infty} \frac{\left(\frac{x}{x}\right)}{\left(\frac{\sqrt{x^2+1}}{x}\right)} = \lim\limits_{x\to\infty} \frac{\left(\frac{x}{x}\right)}{\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)} = \lim\limits_{x\to\infty} \frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}$

$= \lim\limits_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+0}} = \boxed{1}$

11. $\lim\limits_{x\to\infty} \frac{2x+1}{\sqrt{x^2-x}}$ $= \boxed{2}$

12. $\lim\limits_{x\to\infty} \frac{5x^2+2}{\sqrt{x^2+3}}$ for $x>0$, $x=\sqrt{x^2}$

$= \lim\limits_{x\to\infty} \frac{\left(\frac{5x^2+2}{x}\right)}{\left(\frac{\sqrt{x^2+3}}{x}\right)} = \lim\limits_{x\to\infty} \frac{\left(5x+\frac{2}{x}\right)}{\left(\frac{\sqrt{x^2+3}}{\sqrt{x^2}}\right)} = \lim\limits_{x\to\infty} \frac{\left(5x+\frac{2}{x}\right)}{\sqrt{\frac{x^2}{x^2}+\frac{3}{x^2}}}$

$= \lim\limits_{x\to\infty} \frac{\left(5x+\frac{2}{x}\right)}{\sqrt{1+\frac{3}{x^2}}} = \frac{5(\infty)+0}{\sqrt{1+0}} = \boxed{\infty}$

13. $\lim\limits_{x\to\infty} \frac{\sin(2x)}{x}$ $= \boxed{0}$

14. $\lim\limits_{x\to\infty} \frac{x-\cos(x)}{x}$

$= \lim\limits_{x\to\infty} \frac{x}{x} - \lim\limits_{x\to\infty} \frac{\cos x}{x}$ ← oscillates between −1 and 1; ← goes to ∞

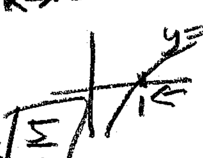
$= 1 - \frac{\#}{\infty}$

$= 1 - 0 = \boxed{1}$

15. $\lim\limits_{x\to\infty} \log(1+10^{-x})$ $= \boxed{0}$

16. $\lim\limits_{x\to\infty} \left(\frac{5}{2} + \ln\left(\frac{x^2+1}{x^2}\right)\right)$ $\lim\limits_{x\to\infty} \frac{1}{x^2} \to 0$ from right

$= \lim\limits_{x\to\infty} \frac{5}{2} + \lim\limits_{x\to\infty} \ln\left(1+\frac{1}{x^2}\right)$ so $1+\frac{1}{x^2} \to 1$ from right



as $1+\frac{1}{x^2} \to 1$, $\ln\left(1+\frac{1}{x^2}\right) \to 0$

$= \frac{5}{2} + 0 = \boxed{\frac{5}{2}}$

On your graphing calculator, graph f and g. Verify algebraically that f and g represent the same function. Zoom out so far that the graphs appear to be a straight line. What equation does this line appear to be?

17. $f(x) = \frac{9x^3-3x^2+2}{x(x-3)}$

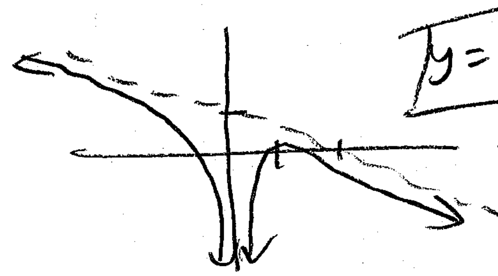
typo → $g(x) = 9x + \frac{2}{x(x-3)}$

$\boxed{y = 9x}$

18. $f(x) = -\frac{x^3-2x^2+2}{2x^2}$

$g(x) = -\frac{1}{2}x + 1 - \frac{1}{x^2}$

$\boxed{y = -\frac{1}{2}x + 1}$



AP Calculus BC Name: ______________________________

2.6 Homework (odds, 18, 20)

Find the slope of the tangent line to the graph of the function at the given point.

1. $f(x) = 3 - 5x$ at (-1, 8)

$\boxed{-5}$

2. $g(x) = \frac{3}{2}x + 1$ at (-2, -2)

line, so slope is $\boxed{\frac{3}{2}}$ (for all x)

3. $f(x) = 2x^2 - 3$ at (2, 5)

$\boxed{8}$

4. $g(x) = 5 - x^2$ at (3, -4)

$m = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3} \frac{[5-(x)^2] - [5-(3)^2]}{x-3}$

$= \lim_{x \to 3} \frac{-x^2+9}{x-3} = \lim_{x \to 3} \frac{-(x^2-9)}{x-3} = \lim_{x \to 3} \frac{-(x-3)(x+3)}{(x-3)}$

$= \lim_{x \to 3} -(x+3) = -((3)+3) = \boxed{-6}$

Find the derivative of the function by the limit process.

5. $f(x) = x^2 + x - 3$

$\boxed{f'(x) = 2x+1}$

6. $f(x) = x^2 - 5$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{[(x+h)^2 - 5] - [(x)^2 - 5]}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h(1)}$

$= \lim_{h \to 0} (2x+h) = 2x + (0) = \boxed{2x}$

7. $f(x) = -x^2 + 3x + 7$

$\boxed{f'(x) = -2x+3}$

8. $f(x) = -x^2 - 4x$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{[-(x+h)^2 - 4(x+h)] - [-(x^2) - 4(x)]}{h}$

$= \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 - 4x - 4h + x^2 + 4x}{h}$

$= \lim_{h \to 0} \frac{-2xh - h^2 - 4h}{h} = \lim_{h \to 0} \frac{h(-2x - h - 4)}{h(1)}$

$= \lim_{h \to 0} (-2x - h - 4) = -2x - (0) - 4$

$= \boxed{-2x - 4}$

Find the slope of the tangent line and the equation of the tangent line to the graph of f at the given point. Then use a graphing calculator to graph both the line and f to confirm your results.

9. $f(x) = x^3$ at (2, 8)

$\boxed{(y-8) = 12(x-2)}$

10. $f(x) = x^2 + 5x - 6$ at (3, 18) at $x=3$

$f'(x) = \lim_{h\to 0} \frac{[(x+h)^2 + 5(x+h) - 6] - [(x)^2 + 5x - 6]}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - 6 - x^2 - 5x + 6}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 + 5h}{h} = \lim_{h\to 0} \frac{h(2x+h+5)}{h(1)}$

$= \lim_{h\to 0} (2x+h+5) = 2x + (0) + 5 = 2x+5$

so $f'(3) = 2(3)+5 = 11$

tan line: $(y-y_0) = m(x-x_0)$

$\boxed{(y-18) = 11(x-3)}$

11. $f(x) = \sqrt{x}$ at (1, 1)

$\boxed{(y-1) = \frac{1}{2}(x-1)}$

12. $f(x) = \sqrt{x-1}$ at (5, 2) at $x=5$ (conjugate)

$f'(x) = \lim_{h\to 0} \frac{[\sqrt{(x+h)-1}] - [\sqrt{x-1}]}{h} \cdot \frac{(\sqrt{x+h-1} + \sqrt{x-1})}{(\sqrt{x+h-1} + \sqrt{x-1})}$

$= \lim_{h\to 0} \frac{(x+h-1)-(x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \lim_{h\to 0} \frac{h(1)}{h(\sqrt{x+h-1} - \sqrt{x-1})}$

$= \frac{1}{\sqrt{x+0-1} - \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$

so $f'(5) = \frac{1}{2\sqrt{5-1}} = \frac{1}{4}$

tan line: $(y-y_0) = m(x-x_0)$

$\boxed{(y-2) = \frac{1}{4}(x-5)}$

13. $f(x) = x + \frac{4}{x}$ at (-4, -5)

$\boxed{(y+5) = \frac{3}{4}(x+4)}$

14. $f(x) = \frac{6}{x+2}$ at (0, 3)

$f'(x) = \lim_{h\to 0} \frac{\left[\frac{6}{(x+h)+2}\right] - \left[\frac{6}{x+2}\right]}{h}$

$= \lim_{h\to 0} \frac{\left(\frac{6}{x+h+2} - \frac{6}{x+2}\right)}{h} \cdot \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}$

$= \lim_{h\to 0} \frac{6(x+2) - 6(x+h+2)}{h(x+h+2)(x+2)}$ ---continued---

Find an equation of the line that is tangent to the graph of f AND parallel to the given line.

15. $f(x) = -\frac{1}{4}x^2$ parallel to $x + y = 0$

$\boxed{(y+1) = -(x-2)}$

16. $f(x) = 2x^2$ parallel to $4x + y + 3 = 0$

$f'(x) = \lim_{h\to 0} \frac{[2(x+h)^2] - [2x^2]}{h}$

$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} = \lim_{h\to 0} \frac{h(4x+2h)}{h(1)}$

$= \lim_{h\to 0} (4x+2h) = 4x + 2(0) = 4x$

line slope: $4x + y + 3 = 0$

$y = -4x - 3$ $m = -4$

so $f'(x) = 4x = -4$, happens when $x = -1$

point: $f(-1) = 2(-1)^2 = 2$ $(-1, 2)$

tan line: $\boxed{(y-2) = -4(x+1)}$

M continued...

$$= \lim_{h \to 0} \frac{6x+12-6x-6h-12}{h(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{h(-6)}{h(x+h+2)(x+2)}$$

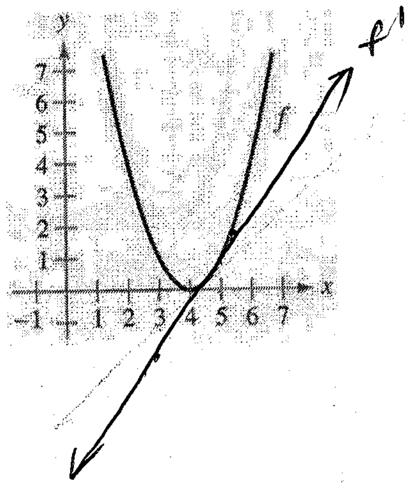
$$f'(x) = \frac{-6}{(x+0+2)(x+2)} = \frac{-6}{(x+2)^2}$$

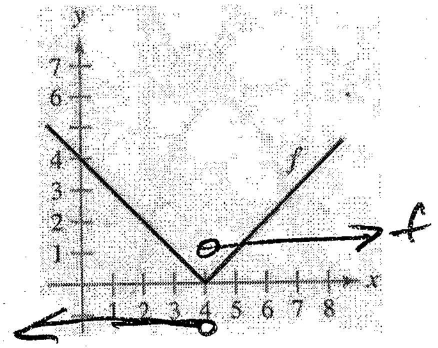
So $f'(0) = \frac{-6}{(0+2)^2} = \frac{-6}{4} = \frac{-3}{2}$

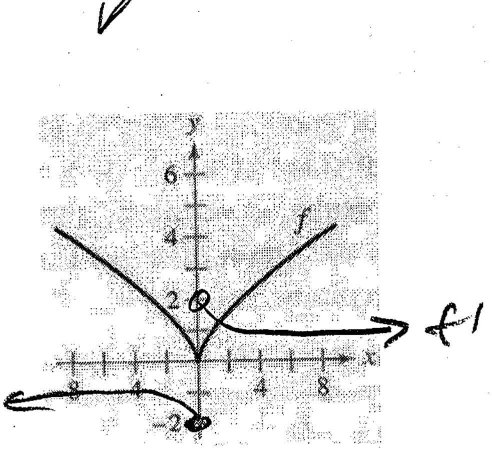
tan line: $(y-y_0) = m(x-x_0)$

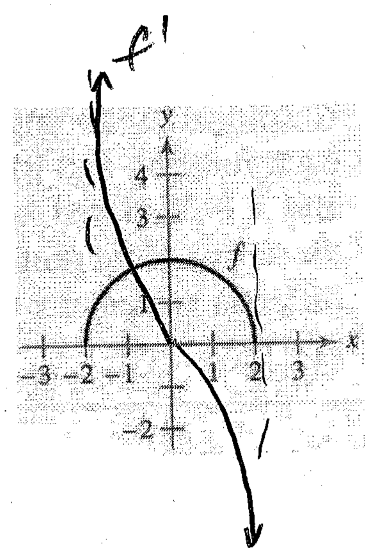
$$\boxed{(y-3) = \frac{-3}{2}(x-0)}$$

17. Sketch an appropriate graph of f' over the graph of f.



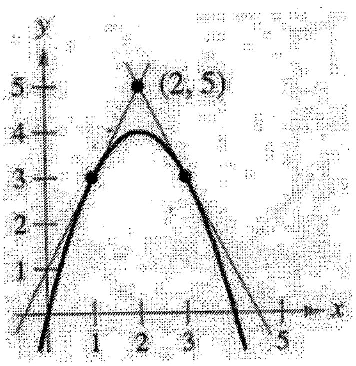






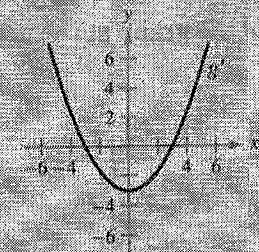
18. Find equations of the two tangent lines to the graph of f that pass through the indicated point.

$f(x) = 4x - x^2$



$$\boxed{\begin{array}{l}(y-5)=2(x-2)\\(y-5)=-2(x-2)\end{array}}$$

19. **HOW DO YOU SEE IT?** The figure shows the graph of g'.



(a) $g'(0) =$

(b) $g'(3) =$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $g(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

(a) $g'(0) = -3$

(b) $g'(3) = 0$

(d–f) (explanations)

remember: g' gives information about whether g is increasing or decreasing
g would give information about y-coordinates

20. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

(a) Find the velocity of the rock after one second.

(b) Find the velocity of the rock when $t = a$.

(c) When will the rock hit the surface?

(d) With what velocity will the rock hit the surface?

(a) $v(1) = \boxed{6.28 \text{ m/s}}$

(b) $v(a) = \boxed{10 - 3.72a}$

(c) $\boxed{5.376 \text{ sec}}$

(d) $= \boxed{-10 \text{ m/s}}$

21. The displacement (in meters) of a particle moving in a straight line is given by $s = t^2 - 8t + 18$, where t is measured in seconds.

(a) Find the average velocity over each time interval:

(i) [3, 4] (ii) [3.5, 4]
(iii) [4, 5] (iv) [4, 4.5]

(b) Find the instantaneous velocity when $t = 4$.

(c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part (a) and the tangent line whose slope is the instantaneous velocity in part (b).

(a) (i) $\boxed{-1}$

(ii) $\boxed{-0.5}$

(iii) $\boxed{1}$

(iv) $\boxed{0.5}$

(b) $\boxed{0}$

(c) (graph)