Unit 10 Part 2 REVIEW – SOLUTIONS

(1) $g(x) = \frac{1}{x^5}$

from table: $\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$

$$\boxed{g(x) = \sum_{n=0}^{\infty} (-1)^n (x^5-1)^n}$$

(2) $g(x) = \sin(2-x^3)$

from table: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$g(x) = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (2-x^3)^{2n+1}}{(2n+1)!}}$$

(3) $g(x) = \frac{1}{3+x^2}$

from table: $\frac{1}{1+\underset{(u)}{x}} = \sum_{n=0}^{\infty} (-1)^n x^n$

$1 + u = 3 + x^2$

$u = 3 + x^2 - 1 = 2 + x^2$

so $g(x) = \boxed{\sum_{n=0}^{\infty} (-1)^n (2+x^2)^n}$

(4) $f(x) = \cos(3x^5)$

from table: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \ldots$

$f(x) = 1 - \frac{(3x^5)^2}{2!} + \frac{(3x^5)^4}{4!} - \frac{(3x^5)^6}{6!} + \frac{(3x^5)^8}{8!}$

$$\boxed{f(x) = 1 - \frac{9}{2}x^{10} + \frac{27}{8}x^{20} - \frac{81}{80}x^{30} + \frac{729}{4480}x^{40}}$$

(5) $f(x) = \sin(4x-3)$

from table: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$$\boxed{f(x) = (4x-3) + \frac{1}{6}(4x-3)^3 + \frac{1}{120}(4x-3)^5 - \frac{1}{5040}(4x-3)^7 + \frac{1}{9!}(4x-3)^9}$$

(6) $f(x) = \sin(3x)$ $n=5$ Maclaurin $(c=0)$

$f(x) = \sin(3x)$ $f(0) = 0$

$f'(x) = 3\cos(3x)$ $f'(0) = 3$

$f''(x) = -9\sin(3x)$ $f''(0) = 0$

$f'''(x) = -27\cos(3x)$ $f'''(0) = -27$

$f^{(4)}(x) = 81\sin(3x)$ $f^{(4)}(0) = 0$

$f^{(5)}(x) = 243\cos(3x)$ $f^{(5)}(0) = 243$

$P_5(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}}{5!}x^5$

$P_5(x) = 0 + 3x + \frac{0}{2}x^2 + \frac{(-27)}{6}x^3 + \frac{0}{24}x^4 + \frac{243}{120}x^5$

$$\boxed{P_5(x) = 3x - \frac{9}{2}x^3 + \frac{81}{40}x^5}$$

(7) $f(x) = \ln x$ $n=4$ Taylor, $c=2$

$f(x) = \ln x$ $f(2) = \ln 2$

$f'(x) = \frac{1}{x} = x^{-1}$ $f'(2) = \frac{1}{2}$

$f''(x) = -x^{-2}$ $f''(2) = -\frac{1}{4}$

$f'''(x) = 2x^{-3}$ $f'''(2) = \frac{1}{4}$

$f^{(4)}(x) = -6x^{-4}$ $f^{(4)}(2) = -\frac{3}{8}$

$P_4(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3 + \frac{f^{(4)}(2)}{4!}(x-2)^4$

$P_4(x) = \ln 2 + \frac{1}{2}(x-2) + \frac{(-1/4)}{2}(x-2)^2 + \frac{(1/4)}{6}(x-2)^3 + \frac{(-3/8)}{24}(x-2)^4$

$$\boxed{P_4(x) = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 - \frac{1}{64}(x-2)^4}$$

(8) $f(x)=xe^x$ $n=4$ Maclaurin

$f(x)=xe^x$ $f(0)=0$

$f'(x)=xe^x+e^x$ $f'(0)=1$

$f''(x)=xe^x+e^x+e^x$
$=xe^x+2e^x$ $f''(0)=2$

$f'''(x)=xe^x+e^x+2e^x$
$=xe^x+3e^x$ $f'''(0)=3$

$f^{(4)}(x)=xe^x+e^x+3e^x$
$=xe^x+4e^x$ $f^{(4)}(0)=4$

$$P_4(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4$$

$$P_4(x)=0+(1)x+\frac{2}{2}x^2+\frac{3}{6}x^3+\frac{4}{24}x^4$$

$$\boxed{P_4(x)=x+x^2+\tfrac{1}{2}x^3+\tfrac{1}{6}x^4}$$

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(9) $f(x)=\sqrt{x}$ $n=4$ Taylor $c=9$

$f(x)=x^{1/2}$ $f(9)=3$

$f'(x)=\frac{1}{2}x^{-1/2}$ $f'(9)=\frac{1}{6}$

$f''(x)=-\frac{1}{4}x^{-3/2}$ $f''(9)=-\frac{1}{108}$

$f'''(x)=\frac{3}{8}x^{-5/2}$ $f'''(9)=\frac{1}{648}$

$f^{(4)}(x)=\frac{-15}{16}x^{-7/2}$ $f^{(4)}(9)=-\frac{15}{6}\frac{1}{(\sqrt{9})^7}=\frac{-15}{13122}$

$$P_4(x)=f(9)+f'(9)(x-9)+\frac{f''(9)}{2!}(x-9)^2+\frac{f'''(9)}{3!}(x-9)^3+\frac{f^{(4)}(9)}{4!}(x-9)^4$$

$$P_4(x)=3+\frac{1}{6}(x-9)+\frac{(-1/108)}{2}(x-9)^2+\frac{(1/648)}{6}(x-9)^3+\frac{(-15/13122)}{24}(x-9)^4$$

$$\boxed{P_4(x)=3+\tfrac{1}{6}(x-9)-\tfrac{1}{216}(x-9)^2+\tfrac{1}{3888}(x-9)^3-\tfrac{15}{314928}(x-9)^4}$$

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(10) $f(x)=e^{3x}$ $n=5$ Maclaurin

$f(x)=e^{3x}$ $f(0)=1$

$f'(x)=3e^{3x}$ $f'(0)=3$

$f''(x)=9e^{3x}$ $f''(0)=9$

$f'''(x)=27e^{3x}$ $f'''(0)=27$

$f^{(4)}(x)=81e^{3x}$ $f^{(4)}(0)=81$

$f^{(5)}(x)=243e^{3x}$ $f^{(5)}(0)=243$

$$P_5(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+\frac{f^{(5)}(0)}{5!}x^5$$

$$\boxed{P_5(x)=1+3x+\frac{9}{2}x^2+\frac{27}{6}x^3+\frac{81}{24}x^4+\frac{243}{120}x^5}$$

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(11) $f(x)=\sin(x)$ $f(0.4)$ $c=0$ error $<.0002$

$$P(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}+\ldots$$

$$P(0.4)=0.4-\frac{(.4)^3}{3!}+\frac{(.4)^5}{5!}-\frac{(.4)^7}{7!}+\frac{(.4)^9}{9!}$$

0.4 .0107 | $8.5\cdot10^{-5}$ .000085 $<.0002$ error

$$P(x)=x-\frac{x^3}{3!}$$ ←

$\boxed{\text{3rd degree}}$

(2 terms)

(12) $f(x)=\ln(x)$ $c=1$ $f(1.4)$ error $<.0002$

$$\ln(x)=(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\frac{(x-1)^5}{5}-\frac{(x-1)^6}{6}+\frac{(x-1)^7}{7}+\Big|\frac{(x-1)^8}{8}+$$

$$\ln(1.4)=(1.4-1)-\frac{(1.4-1)^2}{2}+\frac{(1.4-1)^3}{3}-\frac{(1.4-1)^4}{4}+\frac{(1.4-1)^5}{5}-\frac{(1.4-1)^6}{6}+\frac{(1.4-1)^7}{7}-\Big|\frac{(1.4-1)^8}{8}$$

0.4 .08 .0213 .0064 .002048 | $6.8\cdot10^{-4}$ .00068 $2.34\cdot10^{-4}$ .000234 | $8.19\cdot10^{-5}$ .00008 $<.0$

$$P(x)=(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\frac{(x-1)^5}{5}-\frac{(x-1)^6}{6}+\frac{(x-1)^7}{7}$$

$\boxed{\text{7th degree}}$

(7 term)

(13) $f(x)=e^x$ $c=0$ $f(0.7)$ error $< .0004$

$P(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots$

$P(0.7) = 1 + 0.7 + \frac{(0.7)^2}{2!} + \frac{(0.7)^3}{3!} + \frac{(0.7)^4}{4!} + \frac{(0.7)^5}{5!} + \frac{(0.7)^6}{6!} + \frac{(0.7)^7}{7!} + \cdots$

1   0.7   .245   .057   .01   .0014 | 1.61 $\cdot$ 10⁻⁴ .00016 $<$ .0004

$P(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$ $\boxed{5^{th} \text{ degree}}$

(6 terms)

(14) $\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}$

ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{x^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{x^{2n}} \right|$

$= \lim_{n\to\infty} \left| \frac{x^2 x^{2n} (2n)!}{(2n+2)(2n+1)(2n)!\, x^{2n}} \right|$

$= \underbrace{\lim_{n\to\infty} \frac{1}{(2n+2)(2n+1)}}_{\frac{1}{\infty}} |x|$

$0\,|x| < 1$

true for all x, so

interval of convergence is $\boxed{(-\infty,\infty)}$

(15) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-4)^n}{n 9^n}$

ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x-4)^{n+1}}{(n+1)9^{n+1}} \cdot \frac{n 9^n}{(x-4)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-4)(x-4)^n\, n\, 9^n}{(n+1)\, 9 \cdot 9^n\, (x-4)^n} \right|$

$= \lim_{n\to\infty} \frac{n}{n+1} \left| \frac{x-4}{9} \right|$

L'Hop

$= \lim_{n\to\infty} \frac{1}{1} \left| \frac{x-4}{9} \right|$

$\left| \frac{x-4}{9} \right| < 1$

$|x-4| < 9$

$-9 < x-4 < 9$
$+4 \quad +4 \quad +4$

$-5 < x < 13$

now check endpoints

$x = -5$:

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-5-4)^n}{n 9^n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-9)^n}{n 9^n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \underbrace{\left(\frac{-9}{9}\right)^n}_{(-1)^n}$

$= \sum_{n=1}^{\infty} (-1)(-1)^n(-1)^n \frac{1}{n}$

$= -\sum_{n=1}^{\infty} \frac{1}{n}$ p-series w/ $p=1$ diverges

$x = 13$:

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(13-4)^n}{n 9^n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(9)^n}{n 9^n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \underbrace{\left(\frac{9}{9}\right)^n}_{(1)^n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

alternating series test

$a_{n+1} < a_n$? $\frac{1}{n+1} < \frac{1}{n}$ ✓

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

converges

$\boxed{\text{So interval of convergence is } (-5, 13]}$

(16) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^{n+1}}{n+1}$

ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(x-1)^{n+2}}{(n+2)} \frac{(n+1)}{(x-1)^{n+1}}\right|$

$= \lim_{n\to\infty} \frac{n+1}{n+2} \left|\frac{(x-1)(x-1)^{n+1}}{(x-1)^{n+1}}\right|$

L'Hop

$= \lim_{n\to\infty} \frac{1}{1} |x-1|$

$|x-1| < 1$

$-1 < x-1 < 1$

$+1 \quad +1 \quad +1$

$0 < x < 2$

now check endpoints

$x=0$:

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{((0)-1)^{n+1}}{n+1}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-1)^{n+1}}{(n+1)}$

$= \sum_{n=1}^{\infty} (1)^{n+1} \frac{1}{n+1}$

$= \sum_{n=1}^{\infty} \frac{1}{n+1}$

Limit comparison w/ $\frac{1}{n} = b_n$

$0 < a_n \le b_n$?

$\frac{1}{n+1} < \frac{1}{n}$ ✓

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{n}{n+1}$

L'Hop

$= \lim_{n\to\infty} \frac{1}{1} = 1 > 0$

now check:

$\sum_{n=1}^{\infty} \frac{1}{n}$ p-series w/ $p=1$ diverges

diverges for $x=0$

(could also use integral test)

$x=2$:

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{((2)-1)^{n+1}}{(n+1)}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(1)^{n+1}}{n+1}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+1}$

alternating series test

$a_{n+1} < a_n$?

$\frac{1}{n+2} < \frac{1}{n+1}$ ✓

$\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

converges

$\boxed{\text{so interval of convergence is } (0, 2]}$

(17) $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{2n+1}$

(a) $f(x)$

ratio ~~test~~

$$\lim_{n\to\infty} \left| \frac{x^{2n+2}}{2(n+1)+1} \frac{2n+1}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \frac{2n+1}{2n+3} \left| \frac{x^2 x^{2n}}{x^{2n}} \right|$$

L'Hop

$$\lim_{n\to\infty} \frac{2}{2} \left| x^2 \right|$$

$$|x^2| < 1$$

$$x^2 < 1$$

$$-1 < x < 1$$

now check endpoints:

$x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2n+1}$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{2n+1}$$

$$= \sum_{n=1}^{\infty} \frac{(1)^n}{2n+1} = \sum_{n=1}^{\infty} \frac{1}{2n+1}$$

Limit comparison w/ $\frac{1}{2n} = b_n$

$0 < a_n \le b_n$?

$$\frac{1}{2n+1} < \frac{1}{2n} \checkmark$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{2n+1}}{\frac{1}{2n}}$$

$$= \lim_{n\to\infty} \frac{2n}{2n+1}$$

L'Hop

$$= \lim_{n\to\infty} \frac{2}{2} = 1 > 0$$

now: $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$

p-series w/ $p=1$ diverges

so $x = -1$ diverges

$x = 1$: $\sum_{n=1}^{\infty} \frac{(1)^{2n}}{2n+1}$

$$= \sum_{n=1}^{\infty} \frac{1}{2n+1}$$

Same as this so diverges

so $f(x)$ interval of convergence is

$$\boxed{(-1, 1)}$$

17b $f(x)=\sum_{n=1}^{\infty}\frac{x^{2n}}{2n+1}$

$f'(x)=\sum_{n=1}^{\infty}\frac{2n\,x^{2n-1}}{2n+1}$

ratio test

$\lim_{n\to\infty}\left|\frac{2(n+1)x^{2(n+1)-1}}{2(n+1)+1}\cdot\frac{2n+1}{2n\,x^{2n-1}}\right|$

$=\lim_{n\to\infty}\frac{2(n+1)(2n+1)}{(2n+3)2n}\left|\frac{x^{2n+1}}{x^{2n-1}}\right|$

$=\lim_{n\to\infty}\frac{2(2n^2+2n+n+1)}{4n^2+6n}\left|\frac{x^2x^{2n-1}}{x^{2n-1}}\right|$

$=\lim_{n\to\infty}\frac{4n^2+6n+2}{4n^2+6n}|x^2|$

L'Hop

$=\lim_{n\to\infty}\frac{8n+6}{8n+6}$

$=\lim_{n\to\infty}\frac{8}{8}=1\,|x^2|<1$

$-1<x<1$

now check endpoints

$x=-1$:

$\sum_{n=1}^{\infty}\frac{2n(-1)^{2n-1}}{2n+1}$

$=\sum_{n=1}^{\infty}(-1)^{2n}(-1)^{-1}\frac{2n}{2n+1}$

$=\sum_{n=1}^{\infty}(-1)^n(-1)^n\frac{1}{-1}\frac{2n}{2n+1}$

$=\sum_{n=1}^{\infty}(1)^n(-1)\frac{2n}{2n+1}$

$=-\sum_{n=1}^{\infty}\frac{2n}{2n+1}$

nth term

$\lim_{n\to\infty}\frac{2n}{2n+1}$

L'Hop

$=\lim_{n\to\infty}\frac{2}{2}=1\neq0$

diverges

$x=1$:

$\sum_{n=1}^{\infty}\frac{2n(1)^{2n-1}}{2n+1}$

$=\sum_{n=1}^{\infty}\frac{2n}{2n+1}$

nth term

$\lim_{n\to\infty}\frac{2n}{2n+1}$

L'Hop

$=\lim_{n\to\infty}\frac{2}{2}=1\neq0$

diverges

so for $f'(x)$,
interval of convergence is $\boxed{(-1,1)}$

7c $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{2n+1}$

$\int f(x)dx = \sum_{n=1}^{\infty} \frac{x^{2n+1}}{(2n+1)(2n+1)}$

now check endpoint

ratio test

$\lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)(2(n+1)+1)} \cdot \frac{(2n+1)(2n+1)}{x^{2n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{x^{2n+3}}{(2n+3)(2n+3)} \frac{(2n+1)(2n+1)}{x^{2n+1}} \right|$

$= \lim_{n\to\infty} \frac{(2n+1)(2n+1)}{(2n+3)(2n+3)} \left| \frac{x^2 x^{2n+1}}{x^{2n+1}} \right|$

$= \lim_{n\to\infty} \frac{4n^2+4n+1}{4n^2+12n+9} |x^2|$

L'Hop (x2)

$= \lim_{n\to\infty} \frac{4}{4} = 1 \quad |x^2| < 1$

$-1 < x < 1$

$x = -1$:

$\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{(2n+1)^2}$

$= \sum_{n=1}^{\infty} \frac{(-1)(-1)^n(-1)^n}{(2n+1)^2}$

$= -\sum_{n=1}^{\infty} \frac{(1)^n}{(2n+1)^2}$

$= -\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$

Integral test

terms positive? yes ✓

decreasing?

$g(x) = \frac{1}{(2x+1)^2} = (2x+1)^{-2}$

$g'(x) = -2(2x+1)^{-3}(2)$

$= \frac{-4}{(2x+1)^3} \quad \frac{(-)}{(+)} = -$

− decreasing ✓

$\int_1^{\infty} \frac{1}{(2x+1)^2} dx \quad u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{1}{2}\int_3^{\infty} u^{-2} du$

$\lim_{b\to\infty} \frac{1}{2}\int_3^b u^{-2} du$

$\lim_{b\to\infty} \left[-\frac{1}{2}\frac{1}{u}\right]_3^{\infty}$

$-\frac{1}{2}\frac{1}{(\infty)} - \left[-\frac{1}{2}\frac{1}{3}\right]$

$0 \quad + \frac{1}{6}$

converges at $x = -1$

$x = 1$

$\sum_{n=1}^{\infty} \frac{(1)^{2n+1}}{(2n+1)^2}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$

same but not negative so also converges for $x = 1$

So interval of convergence for $\int f(x)dx$ is $\boxed{[-1, 1]}$

Calculus 2 - Unit 10 Part 2 REVIEW

Find the Maclaurin series for the given function (use the elementary forms table):

1) $g(x)=\dfrac{1}{x^5}$

2) $g(x)=\sin\left(2-x^3\right)$

3) $g(x)=\dfrac{1}{3+x^2}$

Writing first 5 terms of a power expansion for the given function (use the elementary forms table):

4) $f(x)=\cos\left(3x^5\right)$

5) $f(x)=\sin(4x-3)$

Find the specified Maclaurin or Taylor polynomial (for these you must use the definitions):

6) Find the $n=5$ Maclaurin polynomial for the function $f(x)=\sin(3x)$

7) Find the $n=4$ Taylor polynomial centered at c = 2 for the function $f(x)=\ln(x)$

8) Find the $n=4$ Maclaurin polynomial for the function $f(x)=xe^x$

9) Find the $n=4$ Taylor polynomial centered at c = 9 for the function $f(x)=\sqrt{x}$

10) Find the $n=5$ Maclaurin polynomial for the function $f(x)=e^{3x}$

11) Determine the degree of the Maclaurin polynomial centered at 0 required to approximate $f(0.4)$ for the function $f(x)=\sin(x)$ for the error to be less than 0.0002.

12) Determine the degree of the ~~Maclaurin~~ Taylor polynomial centered at 1 required to approximate $f(1.4)$ for the function $f(x)=\ln(x)$ for the error to be less than 0.0002.

13) Determine the degree of the Maclaurin polynomial centered at 0 required to approximate $f(0.7)$ for the function $f(x)=e^x$ for the error to be less than 0.0004.

14) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{x^{2n}}{(2n)!}$ (consider the endpoints).

15) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-4)^n}{n9^n}$ (consider the endpoints).

16) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^{n+1}}{n+1}$ (consider the endpoints).

17) If $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{x^{2n}}{2n+1}$ find the interval of convergence for (a) $f(x)$ (b) $f'(x)$ (c) $\int f(x)\,dx$