AP Calculus BC

Name: answers/solutions

10.1 Worksheet

Per: ____________

Write the first 5 terms of the sequence.

1. $a_n = 4n - 3$

$\boxed{1, 5, 9, 13, 17}$

2. $a_n = \sin\left(\frac{n\pi}{2}\right)$

$\boxed{1, 0, -1, 0, 1}$

(can use a calculator table :))
enter a_n as Y1

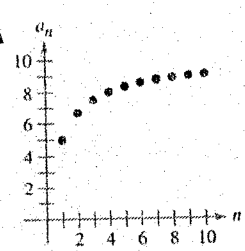
3. $a_n = (-1)^{n+1}\left(\frac{2}{n}\right)$

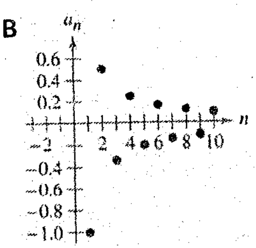
$\boxed{2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}}$

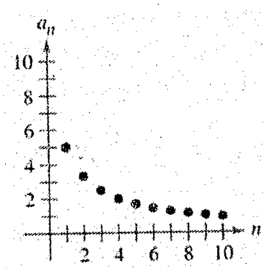
4. $a_n = 2 + \frac{2}{n} - \frac{1}{n^2}$

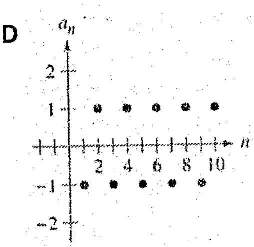
$\boxed{3, \frac{11}{4}, \frac{23}{9}, \frac{39}{16}, \frac{59}{25}}$

Match the sequence with the given nth term with its graph.

A


B


C


D


5. $a_n = \frac{10}{n+1}$

C

6. $a_n = \frac{10n}{n+1}$

A

7. $a_n = (-1)^n$

D

8. $a_n = \frac{(-1)^n}{n}$

B

Write the next two apparent terms of the sequence. Describe the pattern you used to find these terms.

9. $2, 5, 8, 11, \ldots$ $\boxed{14, 17}$

10. $5, 10, 20, 40, \ldots$ $\boxed{80, 160}$

$\times 2 \quad \times 2 \quad \times 2$

Simplify the ratio of the factorials.

11. $\frac{(n+1)!}{n!}$ $= \boxed{n+1}$

12. $\frac{(2n-1)!}{(2n+1)!}$

$$\frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$$

$$\boxed{\frac{1}{(2n+1)(2n)}}$$

Find the limit (if possible) of the sequence with the given nth term.

13. $a_n = \frac{n+1}{n}$

$$\lim_{n\to\infty} \frac{n+1}{n} = \boxed{1}$$

14. $a_n = \frac{2n}{\sqrt{n^2+1}}$

$$\lim_{n\to\infty} \frac{2n}{\sqrt{n^2+1}} \quad \frac{\infty}{\infty}$$

(L'Hop)
$$\lim_{n\to\infty} \frac{2}{\frac{1}{2}(n^2+1)^{-1/2}(2n)}$$

$$\lim_{n\to\infty} \frac{2\sqrt{n^2+1}}{n}$$ not getting simpler

instead: $\lim_{n\to\infty} \frac{2n\left(\frac{1}{n}\right)}{\sqrt{n^2+1}\left(\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{2}{\sqrt{n^2+1}\sqrt{\frac{1}{n^2}}}$$

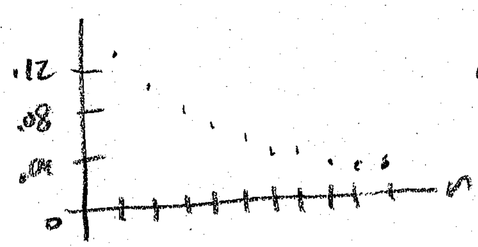
$$\lim_{n\to\infty} \frac{2}{\sqrt{\frac{n^2+1}{n^2}}} = \lim_{n\to\infty} \frac{2}{\sqrt{1+\frac{1}{n^2}}} = \frac{2}{\sqrt{1+0}} = \boxed{2}$$

Use your graphing calculator to graph the first 10 terms of the sequence with the given nth term. Use the graph to make an inference about the convergence or divergence of the sequence. Verify your inference analytically and, if the sequence converges, find its limit.

15. $a_n = \frac{4n+1}{n}$

$\lim_{n\to\infty} \frac{4n+1}{n} = \boxed{4}$ $\boxed{\text{converges}}$

16. $a_n = \frac{1}{n^{3/2}}$



converges

$\lim_{n\to\infty} \frac{1}{n^{3/2}}$

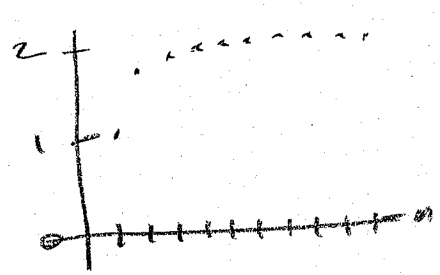
$\frac{1}{\infty}$

$\boxed{0}$ $\boxed{\text{converges}}$

17. $a_n = \sin\left(\frac{n\pi}{2}\right)$

$\boxed{\text{diverges}}$ terms oscillate between $1, 0, -1$

18. $a_n = 2 - \frac{1}{4^n}$



converges

$\lim_{n\to\infty} \left(2 - \frac{1}{4^n}\right)$

$2 - 0$

$\boxed{2}$ $\boxed{\text{converges}}$

Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

19. $a_n = \frac{5}{n+2}$

$\lim_{n\to\infty} \frac{5}{n+2} = \boxed{0}$ $\boxed{\text{converges}}$

20. $a_n = 8 + \frac{5}{n}$

$\lim_{n\to\infty} \left(8 + \frac{5}{n}\right)$

$8 + 0$

$\boxed{8}$ $\boxed{\text{converges}}$

21. $a_n = (-1)^n \left(\frac{n}{n+1}\right)$

$\boxed{\text{diverges}}$ (terms eventually oscillate between 1 and -1)

22. $a_n = \frac{1+(-1)^n}{n^2}$

$\lim_{n\to\infty} \frac{1+(-1)^n}{n^2}$

$= \lim_{n\to\infty} \frac{1}{n^2} + \lim_{n\to\infty} (-1)^n \frac{1}{n^2}$

$0 \quad + \quad 0$ (alternating sign)

$\boxed{0}$ $\boxed{\text{converges}}$

23. $a_n = \frac{(n+1)!}{n!}$ $\boxed{\text{diverges}}$

24. $\frac{(n-2)!}{n!}$

$$\lim_{n\to\infty} \frac{(n-2)!}{n!}$$

$$\lim_{n\to\infty} \frac{(n-2)!}{n(n-1)(n-2)!}$$

$$\lim_{n\to\infty} \frac{1}{n(n-1)} \quad \frac{1}{\infty} = \boxed{0} \ \boxed{\text{converges}}$$

25. $a_n = \frac{\sin(n)}{n}$ $\boxed{0}$ $\boxed{\text{converges}}$

26. $a_n = \cos\left(\frac{\pi n}{n^2}\right)$ $\lim_{n\to\infty} \cos\left(\frac{\pi n}{n^2}\right)$

argument: $\lim_{n\to\infty} \frac{\pi n}{n^2} \quad \frac{\infty}{\infty}$

(L'Hôp)

$$= \lim_{n\to\infty} \frac{\pi}{2n} \quad \frac{\pi}{\infty} = 0$$

so terms approach $\cos(0) = \boxed{1}$ $\boxed{\text{converges}}$

Write an expression for the nth term of the sequence.

27. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$

$\boxed{a_n = (-1)^{n-1}\frac{1}{n^2}}$

28. $-2, 1, 6, 13, 22, \ldots$

first, add 2:

first, add 2:	0	3	8	15	24
n:	1	2	3	4	5
n^2:	1	4	9	16	25
n^2-1:	0	3	8	15	24

so $a_n = (n^2-1)-2$

$\boxed{a_n = n^2-3}$

check: $n=1$ -2
$n=2$ 1
$n=3$ 6
$n=4$ 13
$n=5$ 22 ✓

29. $2, 1+\frac{1}{2}, 1+\frac{1}{3}, 1+\frac{1}{4}, 1+\frac{1}{5}, \ldots$

$\boxed{a_n = 1+\frac{1}{n}}$

30. $\frac{1}{2\cdot 3}, \frac{2}{3\cdot 4}, \frac{3}{4\cdot 5}, \frac{4}{5\cdot 6}, \ldots$

n: 1 2 3 4

$\boxed{a_n = \frac{n}{(n+1)(n+2)}}$

Determine whether the sequence with the given nth term is monotonic and whether it is bounded. Use your graphing calculator to confirm your results.

31. $a_n = 4 - \frac{1}{n}$

$\boxed{\text{monotonic, increasing}}$

$\boxed{\text{bounded } (3 \text{ to } 4)}$

32. $a_n = \frac{3n}{n+2}$

$f(n) = \frac{3n}{n+2}$

$f'(n) = \frac{(n+2)(3) - (3n)(1)}{(n+2)^2}$

$= \frac{3n+6-3n}{(n+2)^2} = \frac{6}{(n+2)^2} \frac{(+)}{(+)}$

$f'(n) > 0$ $\boxed{\text{monotonic, increasing}}$

lower bound at $n=1$

$\frac{3(1)}{(1)+2} = 1$

upper bound?

$\lim_{n\to\infty} \frac{3n}{n+2}$ $\frac{\infty}{\infty}$

(L'Hop) $\lim_{n\to\infty} \frac{3}{1} = 3$

$\boxed{\text{bounded } (1 \text{ to } 3)}$

33. $a_n = \left(\frac{2}{3}\right)^n$

$\boxed{\text{monotonic, decreasing}}$ $\boxed{\text{bounded } (0 \text{ to } 2/3)}$

34. $a_n = \left(\frac{3}{2}\right)^n$

exponential function

$f(n) = \left(\frac{3}{2}\right)^n$

$f'(n) = \left(\frac{3}{2}\right)^n \ln\left(\frac{3}{2}\right)$

(+) (+)

$f'(n) > 0$

$\boxed{\text{monotonic, increasing}}$

lower bound at $n=1$

$\left(\frac{3}{2}\right)^1 = \frac{3}{2}$

upper bound?

$\lim_{n\to\infty} \left(\frac{3}{2}\right)^n$

$\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\cdots$

$(1.5)(1.5)(1.5)\cdots$

gets larger as $n \to \infty$

$\boxed{\text{diverges}}$

10.2 Worksheet

Find the sequence of partial sums S_1, S_2, S_3, S_4, and S_5.

1. $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots$

$$\boxed{\begin{aligned} S_1 &= 1 \\ S_2 &= \tfrac{5}{4} = 1.25 \\ S_3 &= \tfrac{49}{36} = 1.361 \\ S_4 &= \tfrac{205}{144} = 1.4236 \\ S_5 &= \tfrac{5269}{3600} = 1.46361 \end{aligned}}$$

2. $\frac{1}{2*3}+\frac{2}{3*4}+\frac{3}{4*5}+\frac{4}{5*6}+\frac{5}{6*7}+\cdots$

$S_1 = \frac{1}{2\cdot3} = \boxed{\frac{1}{6} = 0.1666\overline{6}}$

$S_2 = \frac{1}{2\cdot3}+\frac{2}{3\cdot4} = \boxed{\frac{1}{3} = 0.33\overline{3}}$

$S_3 = \frac{1}{2\cdot3}+\frac{2}{3\cdot4}+\frac{3}{4\cdot5} = \boxed{\frac{29}{60} = 0.4833\overline{3}}$

$S_4 = \frac{1}{2\cdot3}+\frac{2}{3\cdot4}+\frac{3}{4\cdot5}+\frac{4}{5\cdot6} = \boxed{\frac{37}{60} = 0.61666\overline{6}}$

$S_5 = \frac{1}{2\cdot3}+\frac{2}{3\cdot4}+\frac{3}{4\cdot5}+\frac{4}{5\cdot6}+\frac{5}{6\cdot7} = \boxed{\frac{103}{140} = 0.7357143}$

3. $3-\frac{9}{2}+\frac{27}{4}-\frac{81}{8}+\frac{243}{16}+\cdots$

$$\boxed{\begin{aligned} S_1 &= 3 \\ S_2 &= -\tfrac{3}{2} = -1.5 \\ S_3 &= \tfrac{21}{4} = 5.25 \\ S_4 &= \tfrac{-39}{8} = -4.875 \\ S_5 &= \tfrac{165}{16} = 10.3125 \end{aligned}}$$

4. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\cdots$

$S_1 = \boxed{1}$

$S_2 = 1+\frac{1}{2} = \boxed{\frac{3}{2} = 1.5}$

$S_3 = 1+\frac{1}{2}+\frac{1}{4} = \boxed{\frac{7}{4} = 1.75}$

$S_4 = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{6} = \boxed{\frac{23}{12} = 1.91666\overline{6}}$

$S_5 = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8} = \boxed{\frac{49}{24} = 2.04166\overline{6}}$

Verify that the infinite series diverges.

5. $\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^n$

(geometric, diverges)

6. $\sum_{n=0}^{\infty} 4(-1.05)^n$ geometric w/ $|r| = 1.05 > 1$, $\boxed{\text{diverges}}$

7. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

(use $\lim_{n\to\infty} \frac{n^2}{n^2+1}$, show limit $\neq 0$, diverges)

(nth term test)

8. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

nth term test

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \;\; \frac{\infty}{\infty} \rightarrow$ (L'Hopital's) $= \lim_{n\to\infty} \frac{1}{\frac{1}{2}(n^2+1)^{-1/2}2n} = \lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n}$ (no progress)

instead, try dividing numerator and denominator by n:

$= \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}\sqrt{\frac{1}{n^2}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n^2+1}{n^2}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \frac{1}{\sqrt{1+0}} = 1$

$1 \neq 0$, so $\boxed{\text{diverges}}$

Verify that the infinite series converges.

9. $\sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n$ (geometric, converges)

10. $\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n$ geometric

w/ $|r| = \frac{1}{2} < 1$, $\boxed{\text{converges}}$

11. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (use nth term test, converges)

12. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ nth term test

$\lim_{n\to\infty} \frac{1}{n(n+2)} = 0$, $\boxed{\text{converges}}$

13. Find the sum of the series, use a graphing utility to find the indicated partial sum S_n and complete the table.

$\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$

n	5	10	20	50	100
S_n	2.7976	3.164336	3.39356	3.55125	3.60784

$\boxed{S = \frac{11}{3} = 3.6667}$

for table partial sums, use MATH > summation Σ(

set this value → $\sum_{n=1}^{20} \frac{6}{n(n+3)}$

- use partial fraction expansion to separate into 2 terms
- result is a telescoping series middle terms cancel leaving 3 terms on each end
- take $\lim_{n\to\infty}$ of end terms & add to 1st terms

Find the sum of the convergent series.

14. $\sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)^n$ geometric w/ $|r| = \frac{1}{5} < 1$, converges

to sum $S = \frac{a}{1-r} = \frac{1}{1-(-\frac{1}{5})} = \frac{1}{(\frac{6}{5})} = \boxed{\frac{5}{6}}$

15. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ geometric, converges to sum $\boxed{S = 15}$

$$\sum_{n=1}^{\infty}\left(-\frac{1}{3}\frac{1}{3n+2}+\frac{1}{3}\frac{1}{3n-1}\right)$$

16. $\sum_{n=1}^{\infty}\frac{1}{9n^2+3n-2} = \sum_{n=1}^{\infty}\frac{1}{(3n+2)(3n-1)} = \frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)$ telescoping series

$9n^2+3n-2$ M: -18, A: 3; $(6)(-3)$

$\frac{(9n+6)}{3}\frac{(9n-3)}{3}$

$(3n+2)(3n-1)$

partial fractions:

$= \frac{A}{3n+2}+\frac{B}{3n-1}$

$A(3n-1)+B(3n+2)=1$

$(3A+3B)n+(-A+2B)=(0)n+(1)$

$\begin{cases}3A+3B=0\\ -A+2B=1\end{cases}$

$\left[\begin{array}{cc|c}3&3&0\\-1&2&1\end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c}1&0&-1/3\\0&1&1/3\end{array}\right]$ $-1/3 = A$, $1/3 = B$

$= \frac{1}{3}\left[\left(\frac{1}{2}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{8}}\right)+\left(\cancel{\frac{1}{8}}-\frac{1}{11}\right)+\dots\left(\cancel{\frac{1}{3n-1}}-\frac{1}{3n+2}\right)\right]$

$= \frac{1}{3}\frac{1}{2} - \frac{1}{3}\lim_{n\to\infty}\frac{1}{3n+2}$

$= \frac{1}{6} - 0 = \boxed{\frac{1}{6} = 0.16667}$

Can check w/calculator summation)

17. $\sum_{n=1}^{\infty}(\sin(1))^n$

(geometric) $\boxed{S = \frac{\sin(1)}{1-\sin(1)} = 5.308}$

Write the repeating decimal as a geometric series and write the sum of the series as a ratio of two integers.

18. $0.\overline{36}$ $= .363636\ldots = .36 + .0036 + .000036 \ldots$

$\boxed{\sum_{n=0}^{\infty} 0.36\left(\frac{1}{100}\right)^n}$ geometric w/ $r = \frac{1}{100} < 1$, converges to $S = \frac{a}{1-r}$

$S = \frac{0.36}{1-\left(\frac{1}{100}\right)} = \frac{0.36}{\left(\frac{99}{100}\right)} = \boxed{\frac{36}{99}}$

19. $0.\overline{81}$ $= \boxed{\sum_{n=0}^{\infty} 0.81\left(\frac{1}{100}\right)^n = \frac{81}{99}}$

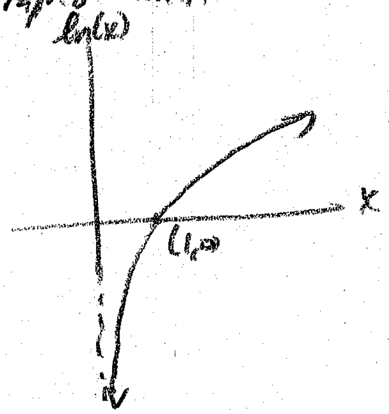
Determine if the series is convergent or divergent.

20. $\sum_{n=0}^{\infty} \frac{3^n}{1000}$ $= \sum_{n=0}^{\infty} \frac{1}{1000}(3)^n$ geometric w/ $r = 3 > 1$, $\boxed{\text{diverges}}$

21. $\sum_{n=0}^{\infty} (1.075)^n$ geometric, $\boxed{\text{diverges}}$

22. $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$ not geometric, use nth term test: $\lim_{n\to\infty} \ln\left(\frac{1}{n}\right)$

graph of $\ln(x)$



$\lim_{n\to\infty} \frac{1}{n} = 0$

so as n increases:

$\lim_{n\to\infty} \ln\left(\frac{1}{n}\right) = \lim_{x\to 0^+} \ln(x) \to -\infty$

so $\lim_{n\to\infty} \ln\left(\frac{1}{n}\right) \to -\infty \neq 0$

$\therefore$ $\boxed{\text{Series diverges}}$

23. $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$ nth term test, $\boxed{\text{diverges}}$

24. $\sum_{n=1}^{\infty} e^{-n}$ nth term test —or— geometric

$\lim_{n\to\infty} e^{-n}$

$\lim_{n\to\infty} \frac{1}{e^n} = 0$

$\boxed{\text{Converges}}$

$\sum_{n=1}^{\infty} (e^{-1})^n = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$

$r = \frac{1}{e} = 0.3678 < 1$

$\boxed{\text{Converges}}$

25. $\sum_{n=1}^{\infty} \left(1 + \frac{k}{n}\right)^n$ nth term test, $\boxed{\text{diverges}}$

10.3 Worksheet

Confirm that the integral test can be applied to the series, then use it to determine if the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{1}{n+3}$ integral test applies, $\boxed{\text{Series diverges}}$

2. $\sum_{n=1}^{\infty} \frac{2}{3n+5}$ $f(x) = \frac{2}{3x+5}$

✓ • $f(x) > 0$, continuous for $x \geq 1$

✓ • $f'(x) = \frac{(3x+5)(0) - 2(3)}{(3x+5)^2}$

$= \frac{-6}{(3x+5)^2}$ $\frac{(-)}{(+)}$

$f'(x) < 0$, so decreasing

integral test applies

$\int_1^{\infty} \frac{2}{3x+5} dx = \lim_{b\to\infty} \int_1^b \frac{2}{3x+5} dx$

$= \lim_{b\to\infty} \left[\frac{2}{3} \ln|3b+5|\right] - \frac{2}{3} \ln|3(2)+5|$

$\infty - \frac{2}{3} \ln|11|$

diverges, so

$\boxed{\text{Series diverges}}$

$\int \frac{2}{3x+5} dx$ u sub $u = 3x+5$ $du = 3dx$ $dx = \frac{1}{3} du$

$\frac{2}{3} \int \frac{1}{u} du$

$\frac{2}{3} \ln|u|$

$\frac{2}{3} \ln|3x+5|$

3. $\sum_{n=1}^{\infty} e^{-n}$

integral test applies, $\boxed{\text{Series converges}}$

4. $\sum_{n=1}^{\infty} ne^{-n/2}$ $f(x) = xe^{-1/2x}$

✓ • $f(x) > 0$, continuous for $x \geq 1$

✓ • $f'(x) = x(-\frac{1}{2}e^{-1/2x}) + e^{-1/2x}(1)$

$= e^{-1/2x}(-\frac{1}{2}x+1)$ $\left(\begin{matrix}-\frac{1}{2}x+1=0 \\ x=2\end{matrix}\right)$

$(+)$ $(-)$ for $x \geq 2$

decreasing for $n \geq 2$

So integral test applies at $n \geq 2$

$\sum_{n=1}^{\infty} ne^{-1/2n} = e^{-1/2} + \sum_{n=2}^{\infty} ne^{-1/2n}$

$\int_2^{\infty} xe^{-1/2x} dx = \lim_{b\to\infty} \int_2^b xe^{-1/2x} dx$

$\int xe^{-1/2x} dx$ by parts: $u = x$ $dv = e^{-1/2x} dx$ $du = dx$ $v = -2e^{-1/2x}$

$uv - \int v du$

$-2xe^{-1/2x} + 2\int e^{-1/2x} dx$

$\lim_{b\to\infty} \left[-2xe^{-1/2x} - 4e^{-1/2x}\right]_2^b$

$e^{-1/2} + \lim_{b\to\infty} \left[-2\underline{b}e^{-1/2b} - 4e^{-1/2b}\right] - \left(-2(2)e^{-1/2(2)} - 4e^{-1/2(2)}\right)$

$e^{-1/2} - \infty + 4e^{-1} + 4e^{-1}$ diverges

So the $\boxed{\text{series diverges}}$

5. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ integral test applies, $\boxed{\text{Series converges}}$ (use integration by parts)

6. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ $f(x) = n^{-1}(\ln(n))^{-1/2}$

✓ • $f(x) > 0$, continuous $x \geq 1$

✓ • $f'(x) = n^{-1}\left(-\frac{1}{2}(\ln(n))^{-3/2}\frac{1}{n}\right) + (\ln(n))^{-1/2}(-n^{-2})$

$= \frac{1}{2n^2(\ln(n))^{3/2}} - \frac{1}{n^2(\ln(n))^{1/2}}$

$= \frac{1}{2n^2(\ln(n))^{3/2}} - \frac{2\ln(n)}{2n^2(\ln(n))^{3/2}}$ $\frac{(-)}{(+)}$ for $n > 2$

$\underline{\text{decreasing}}$ so integral test applies

$\int_1^{\infty} \frac{1}{x}(\ln(x))^{-1/2}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}(\ln(x))^{-1/2}dx$

$\underline{\text{usub}}$ $u = \ln(x)$, $du = \frac{1}{x}dx$

$\int u^{-1/2}du = 2u^{1/2} = 2\sqrt{\ln(x)}$

$\lim_{b\to\infty} [2\sqrt{\ln(b)}] - 2\sqrt{\ln(1)}$

$\infty - 0$

diverges

so $\boxed{\text{series diverges}}$

7. $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$

integral test applies, $\boxed{\text{series diverges}}$ (use u-sub)

8. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}}$ $f(x) = (x+2)^{-1/2}$

✓ • $f(x) > 0$, continuous for $x \geq 1$

✓ • $f'(x) = -\frac{1}{2}(x+2)^{-3/2}(1) = \frac{-1}{(x+2)^{3/2}}$ $\frac{(-)}{(+)}$

$f'(x) < 0$ $\underline{\text{decreasing}}$

integral test applies

$\int_1^{\infty} (x+2)^{-1/2}dx = \lim_{b\to\infty} \int_1^b (x+2)^{-1/2}dx$

$\underline{\text{usub}}$ $u = x+2$, $du = dx$

$\int u^{-1/2}du$

$2u^{1/2} = 2\sqrt{x+2}$

$= \lim_{b\to\infty} [2\sqrt{b+2}] - 2\sqrt{1+2}$

$\infty - 2\sqrt{3}$

diverges

so $\boxed{\text{series diverges}}$

Explain why the integral test does not apply to the series.

9. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$\boxed{f(x) \text{ is not } > 0 \text{ for all } n}$

10. $\sum_{n=1}^{\infty} e^{-n} \cos(n)$

$\cos(n)$ changes sign

e^{-n} always positive

So $\boxed{a_n \text{ is not } > 0 \text{ for all } n}$

11. $\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n}$

$\boxed{\text{not decreasing, } f'(x) \text{ is not } < 0 \text{ for all } n}$

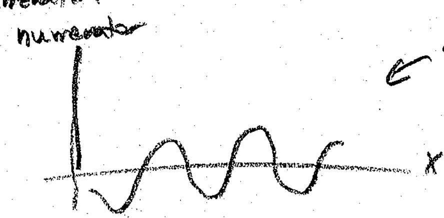
12. $\sum_{n=1}^{\infty} \left(\frac{\sin(n)}{n}\right)^2$

· $f(x) > 0$, continuous $x \geq 1$? <u>yes</u>

· $f'(x) = 2\left(\frac{\sin x}{x}\right)^1\left(\frac{x\cos x - \sin x}{x^2}\right) = \frac{2(x\sin x\cos x - \sin^2 x)}{x^3}$ ← ??

← (+)

graph the numerator:



← so sign of $f'(x)$ is changing

$\boxed{\text{not decreasing}}$

Use the integral test to determine the convergence or divergence of the p-series.

13. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

$\boxed{\text{Converges}}$

14. $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ ✓ $f(x) > 0$, continuous for $x \geq 1$

$f(x) = x^{-1/2}$ ✓ $f'(x) = -\frac{1}{2}x^{-3/2} = \frac{-1}{2x^{3/2}}$ $\frac{(-)}{(+)}$

$f'(x) < 0$ decreasing

integral test applies

$\int_1^{\infty} x^{-1/2}\,dx = \lim_{b\to\infty} \int_1^b x^{-1/2}\,dx$

$\left[\frac{x^{1/2}}{(1/2)} \right]_1^b$

$\lim_{b\to\infty} \left[2\sqrt{b} \right) - 2\sqrt{1}$

$\infty - 2$ $\boxed{\text{diverges}}$

15. $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$

$\boxed{\text{diverges}}$

16. $\sum_{n=1}^{\infty} \frac{1}{n^5}$ ✓ $f(x) > 0$, continuous for $x \geq 1$

$f(x) = x^{-5}$ ✓ $f'(x) = -5x^{-6} = \frac{-5}{x^6}$ $\frac{(-)}{(+)}$

$f'(x) < 0$ decreasing

integral test applies

$\int_1^{\infty} x^{-5}\,dx = \lim_{b\to\infty} \int_1^b x^{-5}\,dx$

$\left[\frac{x^{-4}}{-4} \right]_1^b$

$\lim_{b\to\infty} \left[-\frac{1}{4(b)^4} \right] - \left(\frac{-1}{4(1)^4} \right)$

$0 + \frac{1}{4}$ $\boxed{\text{converges}}$

Use theorem 8.11 to determine the convergence or divergence of the p-series.

17. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

p-series w/ $p = 3/2 > 1$

$\boxed{\text{Converges}}$

18. $\sum_{n=1}^{\infty} \frac{1}{n^{1.03}}$

p-series w/ $p = 1.03 > 1$

$\boxed{\text{Converges}}$

10.4 Worksheet

Use the Direct Comparison Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

$\boxed{\text{diverges}}$

2. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

compare to $\sum_{n=1}^{\infty} \frac{1}{3n^2} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n^2}$ p-series w/ $p=2$, converges

$\frac{1}{3n^2+2} < \frac{1}{n^2}$

So $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ also $\boxed{\text{converges}}$

3. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

$\boxed{\text{diverges}}$

4. $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$

compare to $\sum_{n=0}^{\infty} \frac{4^n}{5^n} = \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$ geometric w/ $r=\frac{4}{5}$, converges

$\frac{4^n}{5^n+3} < \frac{4^n}{5^n}$

So $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$ also $\boxed{\text{converges}}$

5. $\sum_{n=0}^{\infty} \frac{1}{n!}$ $\boxed{\text{converges}}$ (compare to $\frac{1}{n^2}$)

6. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ compare to $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}} = \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ p-series w/ $p=1/3$, diverges

$\frac{1}{4\sqrt[3]{n}} < \frac{1}{4\sqrt[3]{n}-1}$

so $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ also $\boxed{\text{diverges}}$

Use the Limit Comparison Test to determine the convergence or divergence of the series.

7. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $\boxed{\text{diverges}}$

8. $\sum_{n=1}^{\infty} \frac{5}{4^n+1}$ compare to $\sum_{n=1}^{\infty} \frac{5}{4^n} = 5\sum_{n=1}^{\infty} \frac{1}{4^n} = 5\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ geometric w/ $r=1/4$, converges

(nth term test inconclusive)

$$\lim_{n\to\infty} \frac{\left(\frac{5}{4^n+1}\right)}{\left(\frac{5}{4^n}\right)} = \lim_{n\to\infty} \frac{4^n}{4^n+1} \left(\frac{\infty}{\infty}\right)$$

(L'Hôpital)

$$= \lim_{n\to\infty} \frac{4^n \ln 4}{4^n \ln 4} = 1 \text{ (finite, positive)}$$

So series are "linked", $\therefore \sum_{n=1}^{\infty} \frac{5}{4^n+1}$ also $\boxed{\text{converges}}$

9. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$

$\boxed{\text{diverges}}$

10. $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$ compare to $\sum_{n=1}^{\infty} \frac{2^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$ geometric w/ $r = \frac{2}{5}$, converges

$$\lim_{n\to\infty} \frac{\left(\frac{2^n+1}{5^n+1}\right)}{\left(\frac{2^n}{5^n}\right)} = \lim_{n\to\infty} \frac{5^n(2^n+1)}{2^n(5^n+1)} = \lim_{n\to\infty} \frac{10^n+5^n}{10^n+2^n} \approx \lim_{n\to\infty} \frac{10^n}{10^n} = 1$$ (finite, positive)

(L'Hôpital's won't simplify)

but 10^n much $> 5^n$ or 2^n

so series are "linked"

$\therefore \sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$ also $\boxed{\text{converges}}$

11. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

$\boxed{\text{converges}}$

12. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$ compare to $\sum_{n=1}^{\infty} \frac{n}{n2^n} = \sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ geometric w/ $r = \frac{1}{2}$, converges

(nth term test inconclusive)

$$\lim_{n\to\infty} \frac{\left(\frac{n}{(n+1)2^{n-1}}\right)}{\left(\frac{1}{2^n}\right)} = \lim_{n\to\infty} \frac{2^n n}{(n+1)2^{n-1}} = \lim_{n\to\infty} \frac{2 \cdot 2^{n-1} n}{(n+1)2^{n-1}} = \lim_{n\to\infty} \frac{2n}{n+1} \quad \left(\frac{\infty}{\infty}\right)$$

(L'Hôp) $= \lim_{n\to\infty} \frac{2}{1} = 2$ (finite, positive)

so series are "linked"

$\therefore \sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$ also $\boxed{\text{converges}}$

13. Which test (nth-Term Test, Geometric Series Test, p-Series Test, Telescoping Series Test, Integral Test, Direct Comparison Test, Limit Comparison Test) would you use to determine if the series converges or diverges? All must be used at least once.

a. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$ $\boxed{\text{p-series}}$

b. $\sum_{n=0}^{\infty} 5\left(-\frac{4}{3}\right)^{n}$ $\boxed{\text{geometric}}$

c. $\sum_{n=1}^{\infty} \frac{1}{5^n+1}$ $\boxed{\text{direct comparison}}$ w/ $\sum_{n=1}^{\infty} \frac{1}{5^n}$ $\boxed{\text{Geometric}}$ (converges)

d. $\sum_{n=2}^{\infty} \frac{1}{n^3-8}$ $\boxed{\text{limit comparison}}$ w/ $\sum_{n=2}^{\infty} \frac{1}{n^3}$ $\boxed{\text{p-series}}$

e. $\sum_{n=1}^{\infty} \frac{2n}{3n-2}$ $\boxed{\text{nth term test}}$ (will diverge, so conclusive)

f. $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$ $\boxed{\text{telescoping series}}$

g. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$ $\boxed{\text{Integral test}}$

h. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ $\boxed{\text{direct comparison}}$ w/ $\sum_{n=1}^{\infty} \frac{3}{n^2}$ $\boxed{\text{p-series}}$ (converges)

10.5 Worksheet

Determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$ $\boxed{\text{Converges}}$

2. $\sum_{n=1}^{\infty} \frac{n*(-1)^{n+1}}{3n+2}$ alternating series $\lim_{n\to\infty} \frac{n}{3n+2} \left(\frac{\infty}{\infty}\right)$

$\overset{(L'Hop)}{=} \lim_{n\to\infty} \frac{1}{3} \neq 0$ $\boxed{\text{diverges}}$

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ $\boxed{\text{Converges}}$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$ alternating series $\lim_{n\to\infty} \frac{1}{e^n} = 0$ ✓

$a_{n+1} \overset{?}{\leq} a_n$

$\frac{1}{e^{n+1}} < \frac{1}{e^n}$ $\boxed{\text{converges}}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^n(5n-1)}{4n+1}$

$\boxed{\text{diverges}}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} * n}{n^2+5}$

alternating series

$\lim_{n\to\infty} \frac{n}{n^2+5} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{1}{2n} = 0$ ✓

$a_{n+1} \overset{?}{\leq} a_n$

$\frac{n+1}{(n+1)^2+5} \overset{?}{\leq} \frac{n}{n^2+5}$ (can't tell, use derivative)

$f'(n) = \frac{(n^2+5)(1) - n(2n)}{(n^2+5)^2} = \frac{-n^2+5}{(n^2+5)^2} \quad \frac{(-)}{(+)}$

$f'(n) < 0$, so $a_{n+1} < a_n$ ✓

$\boxed{\text{converges}}$

7. $\sum_{n=1}^{\infty} \frac{(-1)^n * n}{\ln(n+1)}$

$\boxed{\text{diverges}}$

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} * \ln(n)}{n}$

alternating series

$\lim_{n\to\infty} \frac{\ln(n)}{n} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$a_{n+1} \overset{?}{\leq} a_n$

$\frac{\ln(n+1)}{n+1} \overset{?}{\leq} \frac{\ln(n)}{n}$ (can't tell, use derivative)

$f'(n) = \frac{n\left(\frac{1}{n}\right) - \ln(n)(1)}{n^2} = \frac{1-\ln(n)}{n^2} \quad \frac{(-)}{(+)}$

$f'(n) < 0$, so $a_{n+1} < a_n$ ✓

$\boxed{\text{converges}}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$\boxed{\text{Converges}}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} * n^2}{n^2+4}$ alternating series $\lim_{n\to\infty} \frac{n^2}{n^2+4} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{2n}{2n} = 1 \neq 0$ $\boxed{\text{diverges}}$

11. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{\ln(n+1)}$

$\boxed{\text{diverges}}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln(n+1)}{n+1}$ alternating series $\lim_{n\to\infty} \frac{\ln(n+1)}{n+1} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $\lim_{n\to\infty} \frac{\left(\frac{1}{n+1}\right)}{1} = \lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

$a_{n+1} \overset{>}{\underset{=}{<}} a_n$

$\frac{\ln(n+2)}{n+2} \overset{>}{\underset{=}{<}} \frac{\ln(n+1)}{n+1}$ (hard to tell, switch to derivative)

$f'(n) = \frac{(n+1)\frac{1}{n+1} - \ln(n+1)(1)}{(n+1)^2} = \frac{1-\ln(n+1)}{(n+1)^2}$ $\frac{(-)}{(+)}$

$f'(n) < 0$, so $a_{n+1} < a_n$ ✓

$\boxed{\text{Converges}}$

13. $\sum_{n=1}^{\infty} \sin\left(\frac{(2n+1)\pi}{2}\right)$

$\boxed{\text{diverges}}$

14. $\sum_{n=1}^{\infty} \frac{1}{n}\cos(n\pi)$

nth term test $\lim_{n\to\infty} \frac{\cos(n\pi)}{n} \leftarrow \frac{(-1\text{ to }1)}{\leftarrow \infty} = 0$ ✓

$\boxed{\text{converges}}$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

$\boxed{\text{converges}}$ (alternating series test)

16. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

alternating series

$\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$ ✓

$a_{n+1} \overset{?}{\leq} a_n$

$\frac{1}{(2(n+1)+1)!} < \frac{1}{(2n+1)!}$ ✓

$\boxed{\text{converges}}$

17. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$

$\boxed{\text{Converges}}$ alternating series test, for decreasing use derivative and graph numerator, decreases for $n>3$.

18. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$ alternating Series $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt[3]{n}} = \lim_{n\to\infty} \frac{n^{1/2}}{n^{1/3}} = \lim_{n\to\infty} n^{1/6} \to \infty$ $\boxed{\text{diverges}}$

Approximate the sum of the series by using the first 6 terms.

19. $\sum_{n=1}^{\infty} \frac{5*(-1)^{n}}{n!}$ $\boxed{-\frac{455}{144}}$

20. $\sum_{n=1}^{\infty} \frac{4*(-1)^{n+1}}{\ln(n+1)}$ $= \frac{4}{\ln(2)} - \frac{4}{\ln(3)} + \frac{4}{\ln(4)} - \frac{4}{\ln(5)} + \frac{4}{\ln(6)} - \frac{4}{\ln(7)} \boxed{\approx 2.7067}$

21. $\sum_{n=1}^{\infty} \frac{2*(-1)^{n+1}}{n^3}$ $\boxed{\approx 1.76867}$

22. $\sum_{n=1}^{\infty} \frac{n*(-1)^{n+1}}{3^n}$ $\approx \frac{1}{3} - \frac{2}{9} + \frac{3}{27} - \frac{4}{81} + \frac{5}{243} - \frac{6}{729} \approx \boxed{\frac{5}{27}}$

How many terms are required to approximate the series with an error of less than 0.001?

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

$\boxed{N = 10 \text{ terms}}$

24. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3-1}$ $|R_N| \le a_{N+1} = \frac{1}{2(N+1)^3-1} < 0.001$

$$2(N+1)^3 - 1 > \frac{1}{0.001}$$

$$2(N+1)^3 - 1 > 1000$$

$$2(N+1)^3 > 1001$$

$$(N+1)^3 > \frac{1001}{2}$$

$$N+1 > \sqrt[3]{\frac{1001}{2}}$$

$$N > \sqrt[3]{\frac{1001}{2}} - 1 = 6.939$$

$$\boxed{N = 7 \text{ terms}}$$

Determine whether the series converges absolutely, conditionally, or diverges.

25. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ $\boxed{\text{Converges absolutely}}$

26. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n!}\right| = \sum_{n=1}^{\infty} \frac{1}{n!}$ compare w/ $\sum_{n=1}^{\infty} \frac{1}{n^2}$

p-series w/ $p=2$
converges

n:	1	2	3	4	5
$n!$:	1	2	6	24	120
n^2:	1	4	9	16	25

$\hookrightarrow$ for $n>3$, $n^2 < n!$

for $n>3$, $n^2 < n!$, so $\frac{1}{n!} < \frac{1}{n^2}$ so $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n!}\right|$ converges

Since $\sum_{n=1}^{\infty} |a_n|$ converges, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ $\boxed{\text{Converges absolutely}}$

27. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ $\boxed{\text{converges conditionally}}$

28. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} * n^2}{(n+1)^2}$ $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1} n^2}{(n+1)^2}\right| = \sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}$ hmm.. integral test? or comparison w/ $\sum \frac{n^2}{n^2+2n}$?
Seems difficult, so let's check alternating series test first...

alternating series test, $\lim_{n\to\infty} \frac{n^2}{(n+1)^2}$ $\frac{\infty}{\infty}$

(L'Hop) $= \lim_{n\to\infty} \frac{2n}{2(n+1)(1)}$ $\frac{\infty}{\infty}$

$= \lim_{n\to\infty} \frac{2}{2} = 1 \neq 0$ $\boxed{\text{diverges}}$

29. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n*\ln(n)}$ $\boxed{\text{converges conditionally}}$ (use integral test for $\sum |a_n|$)

30. $\sum_{n=2}^{\infty} \frac{(-1)^n * n}{n^3-5}$ $\sum_{n=2}^{\infty} \left| \frac{(-1)^n n}{n^3-5} \right| = \sum_{n=2}^{\infty} \frac{n}{n^3-5}$ compare w/ $\sum_{n=2}^{\infty} \frac{1}{n^2}$ (p-series w/ p=2, converges)

Limit Comparison $\lim_{n\to\infty} \frac{\left(\frac{n}{n^3-5}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{n^3}{n^3-5} \left(\frac{\infty}{\infty}\right) \overset{\text{L'Hop}}{=} \lim_{n\to\infty} \frac{3n^2}{3n^2} = 1$ (finite, positive) so series are "linked"

$\therefore \sum_{n=2}^{\infty} \left| \frac{(-1)^n n}{n^3-5} \right|$ also converges

since $\sum |a_n|$ converges, $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-5}$ $\boxed{\text{converges absolutely}}$

31. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}$

$\boxed{\text{converges absolutely}}$ (for $\sum |a_n|$, use direct comparison w/ $\sum \frac{1}{n^2}$)

32. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$ $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\sqrt{n+4}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ p-series w/ p=1/2, diverges

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$ alternating series test

- $\lim_{n\to\infty} \frac{1}{\sqrt{n+4}} = 0$ ✓
- $a_{n+1} \leq a_n$

$\frac{1}{\sqrt{n+5}} < \frac{1}{\sqrt{n+4}}$ ✓

converges

Since $\sum |a_n|$ diverges, but $\sum a_n$ converges, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$ $\boxed{\text{converges conditionally}}$

AP Calculus BC

10.6 Worksheet

Name: ____________________

Period: ________

Verify the formula

1. $\frac{n*5^{n+2}}{5^{n+1}} = 5n$ $\boxed{\text{verified}}$

2. $\frac{(n+1)*4^n}{n*4^{n+1}} = \frac{n+1}{4n}$

$$\frac{(n+1)\cdot \cancel{4^n}}{n\cdot 4\cdot \cancel{4^n}} = \frac{n+1}{4n} \quad \boxed{\text{verified}}$$

3. $\frac{(n+1)!}{(n-2)!} = (n+1)(n)(n-1)$

$\boxed{\text{verified}}$

4. $\frac{(2k-2)!}{(2k)!} = \frac{1}{(2k)(2k-1)}$

$$\frac{\cancel{(2k-2)!}}{(2k)(2k-1)\cancel{(2k-2)!}} = \frac{1}{(2k)(2k-1)} \quad \boxed{\text{verified}}$$

Use the Ratio Test to determine the convergence or divergence of the series.

5. $\sum_{n=1}^{\infty} \frac{1}{5n}$

$\boxed{\text{Converges}}$

6. $\sum_{n=1}^{\infty} \frac{1}{n!}$ ratio test $\lim_{n\to\infty} \left| \frac{\left(\frac{1}{(n+1)!}\right)}{\left(\frac{1}{n!}\right)} \right| = \lim_{n\to\infty} \left| \frac{n!}{(n+1)!} \right|$

$= \lim_{n\to\infty} \left| \frac{n!}{(n+1)n!} \right|$

$= \lim_{n\to\infty} \left| \frac{1}{n+1} \right| = 0 < 1$ $\boxed{\text{converges}}$

7. $\sum_{n=0}^{\infty} \frac{n!}{3^n}$

$\boxed{\text{diverges}}$

8. $\sum_{n=1}^{\infty} \frac{6^n}{n!}$ ratio test $\lim_{n\to\infty} \left| \frac{\left(\frac{6^{n+1}}{(n+1)!}\right)}{\left(\frac{6^n}{n!}\right)} \right| = \lim_{n\to\infty} \left| \frac{6^{n+1}}{(n+1)!} \cdot \frac{n!}{6^n} \right|$

$= \lim_{n\to\infty} \left| \frac{6 \cdot 6^n n!}{(n+1)n! \, 6^n} \right|$

$= \lim_{n\to\infty} \left| \frac{6}{n+1} \right| = 0 < 1$ $\boxed{\text{converges}}$

9. $\sum_{n=1}^{\infty} \frac{9^n}{n^5}$

$\boxed{\text{diverges}}$

10. $\sum_{n=1}^{\infty} \frac{n}{4^n}$ ratio test $\lim_{n\to\infty} \left| \frac{n+1}{4^{n+1}} \cdot \frac{4^n}{n} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)4^n}{4 \cdot 4^n n} \right| = \lim_{n\to\infty} \left| \frac{n+1}{4n} \right| \quad \frac{\infty}{\infty}$

(L'Hop) $= \lim_{n\to\infty} \left| \frac{1}{4} \right| = \frac{1}{4} < 1$ $\boxed{\text{converges}}$

11. $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$ (this problem is challenging)

should get to $\lim_{n\to\infty} \left| \frac{6}{(n+2)} \cdot \frac{(n+1)^n}{(n+2)^n} \right|$ separate limits:

$\lim_{n\to\infty} \left| \frac{6}{n+2} \right| \cdot \underbrace{\lim_{n\to\infty} \left| \left(\frac{n+1}{n+2}\right)^n \right|}_{\text{for this, use } \ln y = \ln\left[\lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)^n\right]}$

to move down to n

12. $\sum_{n=0}^{\infty} \frac{5^n}{2^n+1}$ ratio test $\lim_{n\to\infty} \left| \frac{5^{n+1}}{2^{n+1}+1} \cdot \frac{2^n+1}{5^n} \right|$

$= \lim_{n\to\infty} \left| \frac{5 \cdot 5^n}{2 \cdot 2^n + 1} \cdot \frac{(2^n+1)}{5^n} \right| = \lim_{n\to\infty} \left| \frac{5(5^n(2^n+1))}{2(2^n+\frac{1}{2})5^n} \right|$

$= \frac{5}{2} \lim_{n\to\infty} \left| \frac{2^n+1}{2^n+\frac{1}{2}} \right| \left(\frac{\infty}{\infty}\right)$ L'Hop this doesn't help, but for $n\to\infty$ $2^n+1 \approx 2^n$ & $2^n+\frac{1}{2} \approx 2^n$

$= \frac{5}{2} \frac{2^n}{2^n} = \frac{5}{2} > 1$ $\boxed{\text{diverges}}$

13. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

$\boxed{\text{diverges}}$

14. $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$ ratio test $\lim_{n\to\infty} \left| \frac{((n+1)!)^2}{(3n+3)!} \cdot \frac{(3n)!}{(n!)^2} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)^2(n!)^2}{(3n+3)(3n+2)(3n+1)(3n)!} \cdot \frac{(3n)!}{(n!)^2} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)} \right|$

$= \lim_{n\to\infty} \left| \frac{n^2+2n+1}{(3n+3)(9n^2+9n+2)} \right| = \lim_{n\to\infty} \left| \frac{n^2+2n+1}{27n^3+54n^2+33n+6} \right| = 0 < 1$ $\boxed{\text{converges}}$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

$\boxed{\text{converges absolutely}}$ (use ratio test to check $\Sigma |a_n|$)

16. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n2^n}$ ratio test $\lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{3^n} \right|$

$= \lim_{n\to\infty} \left| \frac{3 \cdot 3^n \cdot 2^n}{(n+1)2 \cdot 2^n \cdot 3^n} \right| = \lim_{n\to\infty} \left| \frac{3n}{2n+2} \right| \left(\frac{\infty}{\infty}\right) \overset{(\text{L'H})}{=} \lim_{n\to\infty} \frac{3}{2} = \frac{3}{2} > 1$ $\Sigma |a_n|$ diverges

w/ ratio test, if using on alternating series, no need to separately check Σa_n, it either diverges or converges absolutely

Use the Root Test to determine the convergence or divergence of the series.

17. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

$\boxed{\text{converges}}$

18. $\sum_{n=1}^{\infty}\left(\frac{2n}{n+1}\right)^n$ root test $\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{2n}{n+1}\right)^n\right|} = \lim_{n\to\infty} \frac{2n}{n+1} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{2}{1} = 2 > 1$ $\boxed{\text{diverges}}$

19. $\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln(n))^n}$

$\boxed{\text{Converges}}$

20. $\sum_{n=1}^{\infty}\left(-\frac{3n}{2n+1}\right)^n$ root test $\lim_{n\to\infty} \sqrt[n]{\left|\left(-\frac{3n}{2n+1}\right)^n\right|} = \lim_{n\to\infty} \frac{3n}{2n+1} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{3}{2} = \frac{3}{2} > 1$ $\boxed{\text{diverges}}$

21. $\sum_{n=1}^{\infty}(2\sqrt[n]{n}+1)^n$

$\boxed{\text{diverges}}$

22. $\sum_{n=0}^{\infty} e^{-3n}$ root test $\lim_{n\to\infty} \sqrt[n]{|(e^{-3})^n|} = \lim_{n\to\infty} e^{-3} = e^{-3} = .04979 < 1$

$\boxed{\text{converges}}$

23. $\sum_{n=1}^{\infty} \frac{n}{3^n}$ $\boxed{\text{converges}}$

24. $\sum_{n=1}^{\infty} \left(\frac{n}{500}\right)^n$ root test $\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n}{500}\right)^n\right|} = \lim_{n\to\infty} \frac{n}{500} \to \infty$, $\boxed{\text{diverges}}$

25. $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^n}$

$\boxed{\text{converges}}$

26. $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$ $\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n!}{n^2}\right)^n\right|} = \lim_{n\to\infty} \frac{n!}{n^2} \left(\frac{\infty}{\infty}\right)$

~~root test~~

$\to \infty$

$\boxed{\text{diverges}}$

n:	1	2	3	4	5	6
$n!$:	1	2	6	24	120	720
n^2:	1	4	9	16	25	36
$\frac{n!}{n^2}$:	1	0.5	.667	1.5	4.8	20 $\to \infty$

Use ~~either the Root Test or Ratio Test~~ any test in the chapter to determine the convergence or divergence of the series.

27. $\sum_{n=1}^{\infty} \frac{5*(-1)^{n+1}}{n}$ $\boxed{\text{converges conditionally}}$

$\sum |a_n|$ p-series

$\sum a_n$ alt. series test

28. $\sum_{n=1}^{\infty} \frac{100}{n} = 100 \sum_{n=1}^{\infty} \frac{1}{n}$ p-series, w/ $p=1$, $\boxed{\text{diverges}}$

29. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$

$\boxed{\text{Converges}}$ (p-series)

30. $\sum_{n=1}^{\infty} \left(\frac{2\pi}{3}\right)^n$ root test $\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{2\pi}{3}\right)^n\right|} = \lim_{n\to\infty} \frac{2\pi}{3} = \frac{2\pi}{3} = 2.094 > 1$

$\boxed{\text{diverges}}$

31. $\sum_{n=1}^{\infty} \frac{5n}{2n-1}$ $\boxed{\text{diverges}}$ (n^{th} term test)

32. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ compare to $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$ p-series w/ p=1, diverges

$\frac{n}{2n^2+1} < \frac{n}{2n^2}$ wrong side for direct comparison, try limit comparison

limit comparison $\lim_{n\to\infty} \frac{\left(\frac{n}{2n^2+1}\right)}{\left(\frac{1}{2n}\right)} = \lim_{n\to\infty} \frac{2n^2}{2n^2+1} \left(\frac{\infty}{\infty}\right)$

(L'Hop) $= \lim_{n\to\infty} \frac{4n}{4n} = 1$ (finite, positive) so series are "linked"

$\therefore \sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ also $\boxed{\text{diverges}}$

Calculus 2 - Unit 10 Part 1 REVIEW
Determine whether the series diverges or converges.
<u>You must show a valid test and work to justify your answer.</u>

#1) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n^3+1}{n^3-1}\right)^n$

#2) $\displaystyle\sum_{n=1}^{\infty}\frac{8^n}{5^n}$

#3) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{4^n-1}$

#4) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\ln(n)}$

#5) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{3^n}$

#6) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n+1}$

#7) $\displaystyle\sum_{n=1}^{\infty}\frac{4n}{2n^2+1}$

#8) $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n^2}$

#9) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^3}{n^3+4}$

#10) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{5n-1}{4n+1}$

#11) $\displaystyle\sum_{n=0}^{\infty}5\frac{2^n}{3^n}$

#12) $\displaystyle\sum_{n=1}^{\infty}\sin\left(\frac{(2n-1)\pi}{2}\right)$

#13) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n}$

#14) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n-1}$

#15) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(n)}{n^2}$

#16) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+1}$

#17) $\displaystyle\sum_{n=1}^{\infty}(-2)^n$

#18) $\displaystyle\sum_{n=1}^{\infty}\frac{5n^4}{n^4+n^2+7}$

#19) $\displaystyle\sum_{n=1}^{\infty}\frac{9^n}{n^5}$

#20) $\displaystyle\sum_{n=1}^{\infty}\frac{5}{n^{0.4}}$

#21) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

#22) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{3^n}$

Determine whether the series converges absolutely, conditionally, or diverges:

#23) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$

#24) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

#25) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{(n+1)^2}$

#26) Determine the minimum number of terms required to approximate the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n^3 - 1}$ with an error of less than 0.0005.

Unit 10 Part 1 REVIEW — SOLUTIONS

① $\sum_{n=1}^{\infty} \left(\frac{2n^3+1}{n^3-1}\right)^n$

root test

(b/c $\sim^n$)

$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$?

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n^3+1}{n^3-1}\right)^n}$

$\lim_{n\to\infty} \frac{2n^3+1}{n^3-1} \quad \frac{\infty}{\infty}$

L'Hop.

$\lim_{n\to\infty} \frac{6n^2}{3n^2} = \lim_{n\to\infty} \frac{12n}{6n}$

$= \lim_{n\to\infty} \frac{12}{6} = 2 > 1$

$\boxed{\text{diverges}}$

② $\sum_{n=1}^{\infty} \frac{8^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{8}{5}\right)^n$

geometric $r = \frac{8}{5}$

$\left|\frac{8}{5}\right| > 1$ $\boxed{\text{diverges}}$

③ $\sum_{n=1}^{\infty} \frac{3^n}{4^n-1}$

limit comparison

compare to $\frac{3^n}{4^n} = b_n$

$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$?

$\lim_{n\to\infty} \frac{\left(\frac{3^n}{4^n-1}\right)}{\left(\frac{3^n}{4^n}\right)} = \lim_{n\to\infty} \frac{4^n}{4^n-1}$

L'Hop doesn't help, but for large n, $4^n - 1 \approx 4^n$

so $\lim_{n\to\infty} \frac{4^n}{4^n-1} = \lim_{n\to\infty} \frac{4^n}{4^n} = 1 > 0$

now check $\sum_{n=1}^{\infty} \frac{3^n}{4^n} = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

geometric w/ $r = \frac{3}{4}$

$\left|\frac{3}{4}\right| < 1$ converges

so $\sum_{n=1}^{\infty} \frac{3^n}{4^n-1}$ $\boxed{\text{converges}}$

note: direct compare doesn't work because

$\frac{3^n}{4^n} < \frac{3^n}{4^n-1}$

so the fact the this converges doesn't force the RHS to converge (because it is beneath it)

④ $\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$

try nth term

(not matching other forms)

$\lim_{n\to\infty} a_n = 0$?

$\lim_{n\to\infty} \frac{n}{\ln(n)} \quad \frac{\infty}{\infty}$

L'Hop

$\lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n = \infty$

$\neq 0$ so

$\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$ $\boxed{\text{diverges}}$

⑤ $\sum_{n=1}^{\infty} \frac{n!}{3^n}$ ratio test (w/ $n!$, try ratio 1st)

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$?

$\lim_{n\to\infty} \left|\frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}\right|$

$\lim_{n\to\infty} \left|\frac{(n+1)n! \, 3^n}{3 \cdot 3^n \, n!}\right|$

$\lim_{n\to\infty} \left|\frac{n+1}{3}\right| = \infty$ $\boxed{\text{diverges}}$

(6) $\sum_{n=1}^{\infty} \frac{1}{5^n+1}$

direct comparison w/ $\frac{1}{5^n} = b_n$

(denom ± 1)

$0 < a_n < b_n$?

$\frac{1}{5^n+1} < \frac{1}{5^n}$ ✓

check $\sum_{n=1}^{\infty} \frac{1}{5^n} = \sum_{n=1}^{\infty} \frac{1^n}{5^n}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

geometric w/ $r = \frac{1}{5}$

converges

so $\sum_{n=1}^{\infty} \frac{1}{5^n+1}$ also $\boxed{\text{converges}}$

(terms are $< \frac{1}{5^n}$)

(7) $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$

nth term = 0, not geometric, not p-series, not good for root, not alternating, not easy to compare

try integral test

- terms positive? yes ✓
- decreasing? check derivative

$f(x) = \frac{4x}{2x^2+1}$ $\quad f'(x) = \frac{(2x^2+1)(4) - (4x)(4x)}{(2x^2+1)^2}$

$= \frac{8x^2+4-16x^2}{(2x^2+1)^2}$

$= \frac{-8x^2+4}{(2x^2+1)^2}$ $\quad \frac{(-)}{(+)}$ −

decreasing ✓

now $\int_1^{\infty} \frac{4x}{2x^2+1} dx$

$u = 2x^2+1$

$\frac{du}{dx} = 4x$, $du = 4x\,dx$

$\int_3^{\infty} \frac{1}{u} du = \lim_{b \to \infty} \int_3^b \frac{1}{u} du$

$= \lim_{b \to \infty} [\ln|u|]_3^b$

$= \lim_{b \to \infty} \ln|b| - \ln|3|$

$\infty - 3 = \infty$ diverges

so $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$ also $\boxed{\text{diverges}}$

(8) $\sum_{n=1}^{\infty} \frac{2}{n^2} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$

p-series w/ $p = 2 > 1$ $\boxed{\text{converges}}$

(11) $\sum_{n=0}^{\infty} 5 \frac{2^n}{3^n} = \sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

geometric w/ $r = \frac{2}{3}$

$\left|\frac{2}{3}\right| < 1$

$\boxed{\text{converges}}$

(9) $\sum_{n=1}^{\infty} \frac{2n^3}{n^3+4}$

nth term

$\lim_{n \to \infty} \frac{2n^3}{n^3+4} \;\frac{\infty}{\infty}$

L'Hop

$= \lim_{n \to \infty} \frac{6n^2}{3n^2} = \lim_{n \to \infty} \frac{12n}{6n}$

$= \lim_{n \to \infty} \frac{12}{6} = 2 \neq 0$

$\boxed{\text{diverges}}$

(10) $\sum_{n=1}^{\infty} (-1)^n \frac{5n-1}{4n+1}$

alternating series test

$a_{n+1} < a_n$?

$\frac{5(n+1)-1}{4(n+1)+1} < \frac{5n-1}{4n+1}$

$\frac{5n+4}{4n+5} < \frac{5n-1}{4n+1}$

not obvious, so let's use derivative:

$f(x) = \frac{5x-1}{4x+1}$

$f'(x) = \frac{(4x+1)(5) - (5x-1)(4)}{(4x+1)^2}$

$= \frac{20x+5-20x+4}{(4x+1)^2} = \frac{9}{(4x+1)^2}$ $\quad \frac{+}{+}$ +

not true that $a_{n+1} < a_n$

also... $\lim_{n \to \infty} \frac{5n-1}{4n+1} \;\frac{\infty}{\infty}$

L'Hop $\lim_{n \to \infty} \frac{5}{4} \neq 0$

so $\boxed{\text{diverges}}$

(12) $\sum_{n=1}^{\infty} \sin\left(\frac{(2n-1)\pi}{2}\right)$

doesn't ~~fit~~ patterns, try writing out some terms...

$\sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{2}\right) + \sin\left(\frac{5\pi}{2}\right) + \ldots$

$1 + (-1) + (1) + (-1)$

oscillating between 1 & -1

$\boxed{\text{diverges}}$

(13) $\sum_{n=1}^{\infty} \frac{1}{5^n} = \sum_{n=1}^{\infty} \frac{1^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

geometric w/ $r = \frac{1}{5}$ $\left|\frac{1}{5}\right| < 1$ $\boxed{\text{converges}}$

could also do ratio test:

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$?

$\lim_{n\to\infty} \left|\frac{1}{5^{n+1}} \cdot \frac{5^n}{1}\right| = \lim_{n\to\infty} \left|\frac{5^n}{5 \cdot 5^n}\right| = \lim_{n\to\infty} \left|\frac{1}{5}\right|$

$= \frac{1}{5} < 1$ (converges)

(14) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

direct comparison w/ $\frac{1}{2n}$

but use $a_n = \frac{1}{2n}$, $b_n = \frac{1}{2n-1}$

$0 < a_n \leq b_n$ ✓

now check $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$

p-series w/ $p = 1$ diverges

and since this is under $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

$\sum_{n=1}^{\infty} \frac{1}{2n-1}$ also $\boxed{\text{diverges}}$

could also do limit comparison w/ $\frac{1}{2n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$? $\left(\text{w/ } a_n = \frac{1}{2n-1}\right)$

$\lim_{n\to\infty} \frac{\frac{1}{2n-1}}{\frac{1}{2n}} = \lim_{n\to\infty} \frac{2n}{2n-1}$ $\frac{\infty}{\infty}$

L'Hop $= \lim_{n\to\infty} \frac{2}{2} = 1 > 0$

now check $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$

p-series w/ $p=1$

diverges

so $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ also (diverges)

(15) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ (ln ... usually integral test)

- terms positive? yes ✓
- decreasing? $f(x) = \frac{\ln x}{x^2}$ $f'(x) = \frac{x^2\left(\frac{1}{x}\right) - \ln x(2x)}{x^4}$

$= \frac{x - 2x\ln x}{x^4}$

$= \frac{x(1 - 2\ln x)}{x^4}$ (for $x>$) $\frac{(-)}{(+)}$

decreasing ✓

now $\int_1^{\infty} \frac{\ln x}{x^2}\,dx$

by parts: $u = \ln x$ $dv = x^{-2}dx$

$\frac{du}{dx} = \frac{1}{x}$ $\int dv = \int x^{-2}dx$

$du = \frac{1}{x}dx$ $v = -x^{-1} = \frac{-1}{x}$

$uv - \int v\,du$

$(\ln x)\left(\frac{-1}{x}\right) - \int \left(-\frac{1}{x}\right)\frac{1}{x}dx$

$-\frac{1}{x}\ln x + \int x^{-2}dx$

$\lim_{b\to\infty} \left[-\frac{1}{x}\ln x - \frac{1}{x}\right]_1^b$

$-\lim_{b\to\infty} \frac{1}{b}\ln b - \lim_{b\to\infty}\frac{1}{b} - \left[-\frac{1}{1}\ln 1^{0} - \frac{1}{1}\right]$

$\frac{\infty}{\infty}$

L'Hop

$-\lim_{b\to\infty} \frac{\frac{1}{b}}{1} - \frac{1}{\infty}$

$\frac{\frac{1}{\infty}}{1}$

$0 - 0 + 0 + 1 = 1$

integral converges

so series $\boxed{\text{converges}}$

(16) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

limit comparison w/ $\frac{n}{n^2}, \frac{1}{n} = b_n$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{n}{n^2+1}\right)}{\frac{1}{n}}$

$\lim_{n\to\infty} \frac{n^2}{n^2+1} \quad \frac{\infty}{\infty}$

L'Hop

$\lim_{n\to\infty} \frac{2n}{2n} = \lim_{n\to\infty} \frac{2}{2} = 1 > 0$

now check $\sum_{n=1}^{\infty} \frac{1}{n}$

p-series w/ p=1

diverges

so $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ also $\boxed{\text{diverges}}$

(17) $\sum_{n=1}^{\infty} (-2)^n = -2 + (-2) + (-2)^{\cdots} = -\infty$

$\boxed{\text{diverges}}$

(18) $\sum_{n=1}^{\infty} \frac{5n^4}{n^4+n^2+7}$

nth term $\lim_{n\to\infty} \frac{5n^4}{n^4+n^2+7} \quad \frac{\infty}{\infty}$

L'Hop: $\lim_{n\to\infty} \frac{20n^3}{4n^3+2n} = \lim_{n\to\infty} \frac{60n^2}{12n^2+2}$

$= \lim_{n\to\infty} \frac{120n}{24n} = \lim_{n\to\infty} \frac{120}{2} = 60 \neq 0$

$\boxed{\text{diverges}}$

(19) $\sum_{n=1}^{\infty} \frac{9^n}{n^5}$

ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1 ?$

$\lim_{n\to\infty} \left|\frac{9^{n+1}}{(n+1)^5} \frac{n^5}{9^n}\right|$

$\lim_{n\to\infty} \left|\frac{9 \cdot 9^n}{(n+1)^5} \frac{n^5}{9^n}\right|$

$\lim_{n\to\infty} \left|\frac{9n^5}{(n+1)^5}\right| \quad \frac{\infty}{\infty}$

L'Hop $\lim_{n\to\infty} \frac{45n^4}{5(n+1)^4} \quad \frac{\infty}{\infty}$

$= \lim_{n\to\infty} \frac{180n^3}{20(n+1)^3} = \lim_{n\to\infty} \frac{540n^2}{60(n+1)^2}$

$= \lim_{n\to\infty} \frac{1080n}{120(n+1)} = \lim_{n\to\infty} \frac{1080}{120} = 9 > 1$

$\boxed{\text{diverges}}$

(20) $\sum_{n=1}^{\infty} \frac{5}{n^{0.4}} = 5 \sum_{n=1}^{\infty} \frac{1}{n^{0.4}}$

p-series w/ p=0.4

$\boxed{\text{diverges}}$

(21) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$

root test

$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1 ?$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n}$

$\lim_{n\to\infty} \frac{n}{2n+1} \quad \frac{\infty}{\infty}$

L'Hop

$\lim_{n\to\infty} \frac{1}{2} < 1$

$\boxed{\text{converges}}$

(22) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{3^n}$

alternating series test

$a_{n+1} < a_n ?$

$\frac{1}{3^{n+1}} < \frac{1}{3^n}$ ✓

$\lim_{n\to\infty} \frac{1}{3^n} = 0$ ✓

$\boxed{\text{converges}}$

(23) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$, $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n!}$, $\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{n!}\right| = \sum_{n=1}^{\infty} \frac{1}{n!}$

alternating series test

$a_{n+1} < a_n$?

$\frac{1}{(n+1)!} < \frac{1}{n!}$ ✓

$\lim_{n\to\infty} \frac{1}{n!} = 0$ ✓

converges

ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$?

$\lim_{n\to\infty} \left|\frac{1}{(n+1)!} \cdot \frac{n!}{1}\right|$

$\lim_{n\to\infty} \left|\frac{1}{(n+1)n!} \cdot \frac{n!}{1}\right| = \lim_{n\to\infty} \frac{1}{n+1} = 0$

converges

Because $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n!}$ and $\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{n!}\right|$ both converge,

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ $\boxed{\text{converges absolutely}}$

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(24) $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$, $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$, $\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

alternating series test

$a_{n+1} < a_n$?

$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$ ✓

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

converges

p-series w/ $p = \frac{1}{2}$

diverges

Because $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$ converges but $\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{\sqrt{n}}\right|$ diverges,

$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$ $\boxed{\text{converges conditionally}}$

(25) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{(n+1)^2}$, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{(n+1)^2}$

alternating series test

$a_{n+1} < a_n$?

use derivative: $f(x) = \frac{x^2}{(x+1)^2}$ $f'(x) = \frac{(x+1)^2(2x) - (x^2)(2(x+1)'(1))}{(x+1)^4}$

$= \frac{2x(x+1)^2 - 2x^2(x+1)}{(x+1)^4}$

$= \frac{2x(x+1)[x+1-x]}{(x+1)^4} = \frac{2x(x+1)}{(x+1)^4} \frac{+}{+} +$

not decreasing

also, $\lim_{n\to\infty} \frac{n^2}{(n+1)^2}$ $\frac{\infty}{\infty}$

L'Hop $= \lim_{n\to\infty} \frac{2n}{2(n+1)} = \lim_{n\to\infty} \frac{2}{2} = 1 \neq 0$

diverges

so $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{(n+1)^2}$ $\boxed{\text{diverges}}$

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(26) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n^3-1}$ approx w/ error < 0.0005

$|R_N| < a_{N+1}$

$\frac{1}{2(N+1)^3 - 1} < 0.0005$

$2(N+1)^3 - 1 > \frac{1}{0.0005} = 2000$

$2(N+1)^3 = 2001$

$(N+1)^3 = \frac{2001}{2}$

$N+1 = \sqrt[3]{\frac{2001}{2}}$

$N = \sqrt[3]{\frac{2001}{2}} - 1 = 9.0016 \nearrow$

need $\boxed{10 \text{ terms}}$