AP Calculus BC

Name: SOLUTIONS

Unit 1 Review (Precalculus topics) – **NO CALCULATORS**

Date: ______ Per: ______

Please show work so we can see how you find your solutions.

Function Notation and Composition of Functions:

#1. Let $f(x) = x - 2$ and $g(x) = x^3$. Find $g(f(4))$.

$f(4) = (4) - 2 = 2$

$g(2) = (2)^3 = \boxed{8}$

#2. Express the function $F(x) = \tan^3(x)$ in the form $f(g(x))$.

$= (\tan x)^3$

$f(x) =$ x^3

$g(x) =$ $\tan x$

#3. If *If* $f(x) = 2x^2 - 5x$, *write* $\frac{f(x+h) - f(x)}{h}$ *in simplest form*.

$$\frac{[2(x+h)^2 - 5(x+h)] - [2(x)^2 - 5(x)]}{h}$$

$$\frac{2(x^2 + 2xh + h^2) - 5x - 5h - 2x^2 + 5x}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{5x} - 5h - \cancel{2x^2} + \cancel{5x}}{h} = \frac{4xh + 2h^2 - 5h}{h} = \frac{h(4x + 2h - 5)}{h(1)} = \boxed{4x + 2h - 5}$$

#4. Use the table to find the value of the function or composition at the given x.

x	-3	-1	0	3	5	7
$f(x)$	3	4	-1	-2	-1	5
$g(x)$	5	17	0	4	-3	-1

$g(-1) =$ 17

$f(g(5)) =$ 3
$f(-3)$

$f(f(7)) =$ -1
$f(5)$

$\checkmark$ stretched x3

left 2

$y = \ln x$

#5. **Multiple Choice:** Relative to the graph ~~$y = \cos x$~~, the graph of $y = 3\ln(x+2)$ is changed in what way?

(typo)

A) Shifted 2 units downward

B) Compressed horizontally by a factor of 3

C) Shifted 2 units to the right

D) Stretched vertically by a factor of 3

E) Shifted 2 units upward

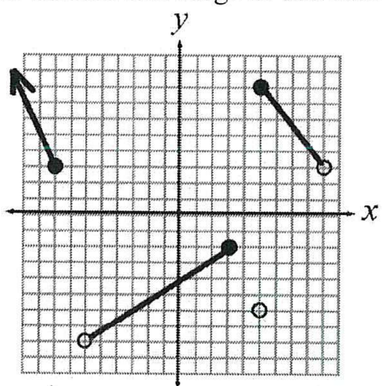
F) Shifted 2 units to the right and stretched 3 times vertically

$\boxed{\text{G)}}$ Shifted 2 units to the left and stretched 3 times vertically

H) Shifted 2 units to the left

Domain/Range and Interval notation:

#6. State the domain and range of this function:



∞

3

−2

−8

$\boxed{\text{range: } (-8,-2] \cup [3,\infty)}$

$-\infty$ −8 −6   3 5 9

$\boxed{\text{Domain: } (-\infty,-8] \cup (-6,3] \cup [5,9)}$

#7. **Multiple Choice:** The function $f(x) = \dfrac{x-2}{x-5} + \sqrt{(x-3)(x+2)}$ has as its domain all values of x such that

A) $(-\infty,2) \cup (2,3) \cup (3,5)$

B) $[3,5) \cup (5,\infty)$

C) $(-\infty,0) \cup [5,\infty)$

D) $(-\infty,0) \cup (3,5) \cup (5,\infty)$

E) $[3,5]$

F) $(3,5) \cup (5,\infty)$

G) $(-\infty,0) \cup (5,\infty)$

H) $(-\infty,0) \cup [3,5) \cup (5,\infty)$

$\div 0$ at $x = 5$

$\sqrt{0}$ at $x = 3, x = -2$

$\checkmark$ $\checkmark$ X $\checkmark$ $\checkmark$ X $\checkmark$

(−3) $\boxed{-2}$ (0) $\boxed{3}$ (4) $\boxed{5}$ (6)

$f(-3)$: $\frac{(-)}{(-)} + \sqrt{(-)(-)}$, $(+) + \sqrt{+}$ $\checkmark$

$f(-2)$: $\frac{(-)}{(-)} + \sqrt{(-)(0)}$, $(+) + \sqrt{0}$ $\checkmark$

$f(0)$: $\frac{(-)}{(-)} + \sqrt{(-)(+)}$, $(+) + \sqrt{-}$ X

$f(3)$: $\frac{(+)}{(-)} + \sqrt{(0)(+)}$, $(-) + \sqrt{0}$ $\checkmark$

$f(4)$: $\frac{(+)}{(-)} + \sqrt{(+)(+)}$, $(-) + \sqrt{+}$ $\checkmark$

$f(5)$: $\frac{(+)}{(0)} + \sqrt{(+)(+)}$, $\div 0$ X

$f(6)$: $\frac{(+)}{(+)} + \sqrt{(+)(+)}$, $(+) + \sqrt{+}$ $\checkmark$

$\boxed{\text{domain: } (-\infty,-2] \cup [3,5) \cup (5,\infty)}$ (none of the choices match this :( )

#8. **Multiple Choice:** Let $f(x)=\sqrt{x+2}$ and $g(x)=\sqrt{x^2-9}$. Find the domain of $(fg)(x)$. $= f(x)\cdot g(x)$

$\sqrt{x+2}\sqrt{x^2-9}$

$\sqrt{x+2}\sqrt{(x-3)(x+3)}$

$x=-2,\ x=3,\ x=-3$

A) $(-\infty,-3)\cup(3,\infty)$

B) $(2,\infty)$

C) $(-\infty,-2)$

D) $[3,\infty)$

E) $(-\infty,-3]\cup[3,\infty)$

F) $[2,\infty)$

G) $(-\infty,-2]$

H) $(3,\infty)$

$-\infty$ … $\boxed{-3}$ … $\boxed{-2}$ … $\boxed{3}$ … ∞

(−4)	$\boxed{-3}$	(−2.5)	$\boxed{-2}$	(0)	$\boxed{3}$	(4)
$\sqrt{(-)}\sqrt{+}$	$\sqrt{-}\sqrt{0}$	$\sqrt{-}\sqrt{-}$	$\sqrt{0}\sqrt{(-)(+)}$, $\sqrt{-}$	$\sqrt{(+)}\sqrt{(-)(+)}$, $\sqrt{-}$	$\sqrt{+}\sqrt{0}$	$\sqrt{+}\sqrt{(+)(+)}$
×	×	× ×	×	×	✓	✓

$\boxed{\text{domain: } [3,\infty)}$

Factoring and Completing the Square:

#9. Factor completely: x^2+x-42

M	A
−42	1
(7)(−6)	1 ✓
(−7)(6)	−1

$\boxed{(x+7)(x-6)}$

#10. Factor completely: $5x^2-13x+6$

M	A
30	−13
(10)(3)	
(−10)(−3)	✓

$\dfrac{(5x-10)}{5}\dfrac{(5x-3)}{1}$

$\boxed{(x-2)(5x-3)}$

#11. Factor completely: $3x^4-48x^2$ $=3x^2(x^2-16)$

$=\boxed{3x^2(x-4)(x+4)}$

#12. Complete the square to write the equation in the form of a circle: $x^2+y^2+4x-8y+19=0$

$(x^2+4x+4)+(y^2-8y+16)=-19+4+16$

$\boxed{(x+2)^2+(y-4)^2=1}$

Exponent Rules and Logarithms:

#13. Write in simplest form: $3a^2 4b^{-4} a^6 b^4 c^3$ $= 12a^8 \underbrace{b^0}_{(1)} c^3 = \boxed{12a^8c^3}$

#14. Write in simplest form without using a fraction or negative exponents: $\dfrac{2e^{-3x}}{5e^{-7x}}$ $= \dfrac{2e^{7x}}{5e^{3x}} = \boxed{\dfrac{2}{5}e^{4x}}$

Can't avoid this fraction

#15. Simplify the logarithmic expression into one logarithm: $4\log_3(x)+\log_3(y)-3\log_3(z)+\frac{1}{2}\log_3(x)$

$\log_3(x^4)+\log_3(y)-\log_3(z^3)+\log_3(x^{1/2})$

$\log_3\left(\dfrac{x^4 y x^{1/2}}{z^3}\right) = \boxed{\log_3\left(\dfrac{x^4 y\sqrt{x}}{z^3}\right)}$

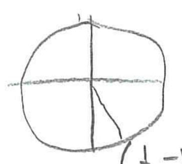
#16. Expand the logarithmic expression to the sum and/or difference of factors of logarithms with no exponents.

$\ln\left(\dfrac{5y^4}{x^6}\right) = \ln(5)+\ln(y^4)-\ln(x^6)$

$= \boxed{\ln(5)+4\ln(y)-6\ln(x)}$

Unit Circle Trigonometry and Basic Trig Identities:

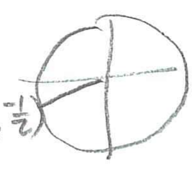
#17. Evaluate (answer in exact form, no decimals): $\cos\left(\dfrac{-\pi}{3}\right) = \boxed{\dfrac{1}{2}}$



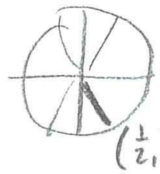
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

#18. Evaluate (answer in exact form, no decimals): $\cot\left(\dfrac{7\pi}{6}\right) = \dfrac{\cos\left(\frac{7\pi}{6}\right)}{\sin\left(\frac{7\pi}{6}\right)} = \dfrac{-\sqrt{3}/2}{-1/2} = \boxed{\sqrt{3}}$

$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$

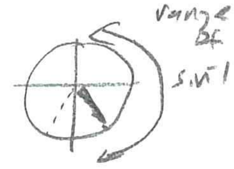


#19. Evaluate (answer in exact form, no decimals): $\csc\left(\dfrac{5\pi}{3}\right) = \dfrac{1}{\sin\left(\frac{5\pi}{3}\right)} = \dfrac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = \boxed{\dfrac{-2}{\sqrt{3}}}$



$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

angle where $y = -\frac{\sqrt{3}}{2}$

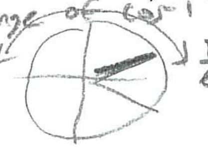


#20. Evaluate (answer in exact form, no decimals): $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \boxed{\frac{-\pi}{3}}$

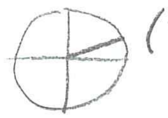
#21. Evaluate (answer in exact form, no decimals): $\sin\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \boxed{\frac{1}{2}}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

angle where $x = \frac{\sqrt{3}}{2}$



$\frac{\pi}{6}$, now $\sin\left(\frac{\pi}{6}\right)$



$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

#22. Simplify fully: $\frac{3\sin^3\theta}{\cos\theta} + 3\sin\theta\cos\theta = \frac{3\sin^3\theta}{\cos\theta} + \frac{3\sin\theta\cos^2\theta}{\cos\theta} = \frac{3\sin^3\theta + 3\sin\theta\cos^2\theta}{\cos\theta}$

$= \frac{3\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta} = \frac{3\sin\theta}{\cos\theta} = \boxed{3\tan\theta}$

$(\sin^2\theta + \cos^2\theta = 1)$ $\qquad \frac{\sin\theta}{\cos\theta} = \tan\theta$

Solving equations:

#23. Solve the equation (Exact answers only): $\frac{x^2-4}{3} = \frac{x}{4}$

$4(x^2-4) = 3x$

$4x^2 - 16 = 3x$

$4x^2 - 3x - 16 = 0$

not factorable, use quadratic formula:

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-16)}}{2(4)} = \frac{3 \pm \sqrt{9+256}}{8}$

$= \frac{3 \pm \sqrt{265}}{8}$ $\qquad \boxed{x = \frac{3+\sqrt{265}}{8},\ x = \frac{3-\sqrt{265}}{8}}$

$\begin{array}{r} {}^{3}16 \\ \times 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array}$

#24. Solve the equation (Exact answers only): $4x^2 + x - 3 = 0$

M	A
-12	1
$(-4)(3)$	-1
$(4)(-3)$	1 ✓

$\frac{(4x+4)}{4}\frac{(4x-3)}{1} = 0$

$(x+1)(4x-3) = 0$

$\boxed{x = -1}$ $\quad 4x - 3 = 0,\ 4x = 3,\ \boxed{x = \frac{3}{4}}$

#25. Solve the equation (Exact answers only): $e^{2x+6} = 4$

$\ln(e^{2x+6}) = \ln(4)$

$2x + 6 = \ln(4)$

$2x = \ln(4) - 6$

$\boxed{x = \frac{\ln(4) - 6}{2}}$

#26. Solve the equation (Exact answers only): $\ln(3x-1)=5$

$$e^{\ln(3x-1)}=e^5$$

$$3x-1=e^5$$

$$3x=e^5+1$$

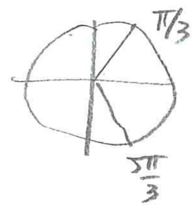
$$\boxed{x=\frac{e^5+1}{3}}$$

#27. Solve the equation (Exact answers only): $4\cos(\theta)-2=0 \quad (0\le\theta<2\pi)$

$$4\cos\theta=2$$

$$\cos\theta=\frac{1}{2}$$

$$\boxed{\theta=\frac{\pi}{3},\ \theta=\frac{5\pi}{3}}$$



#28. Solve the equation (Exact answers only): $2\sin^2(\theta)+3\sin(\theta)-2=0 \quad (0\le\theta<2\pi)$

$u=\sin\theta$

M	A
-4	3
$(4)(-1)$	3 ✓
$(-4)(1)$	-3

$$2u^2+3u-2=0$$

$$\frac{(2u+4)(2u-1)}{2 \quad\quad 1}=0$$

$$(u+2)(2u-1)=0$$

$$(\sin\theta+2)(2\sin\theta-1)=0$$

$\sin\theta+2=0$ | $2\sin\theta-1=0$
$\sin\theta=-2$ | $2\sin\theta=1$
N/A | $\sin\theta=\frac{1}{2}$

$$\boxed{\theta=\frac{\pi}{6},\ \theta=\frac{5\pi}{6}}$$