AP Calculus BC Semester 1 Final Exam Review **Name**______________________________________

Free-Response Part: You'll need to show work on the final using appropriate formats and explanations. The FRQ part of the final exam is NO CALCULATOR and the MCQ part REQUIRES A CALCULATOR, so do the FRQ problems in this review without a calculator.
Look at the posted solutions to see the required justifications and formatting for full credit.

#1. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -t(t-2)(t-3)$. At time $t = 0$, the particle is at position $x = 1$.

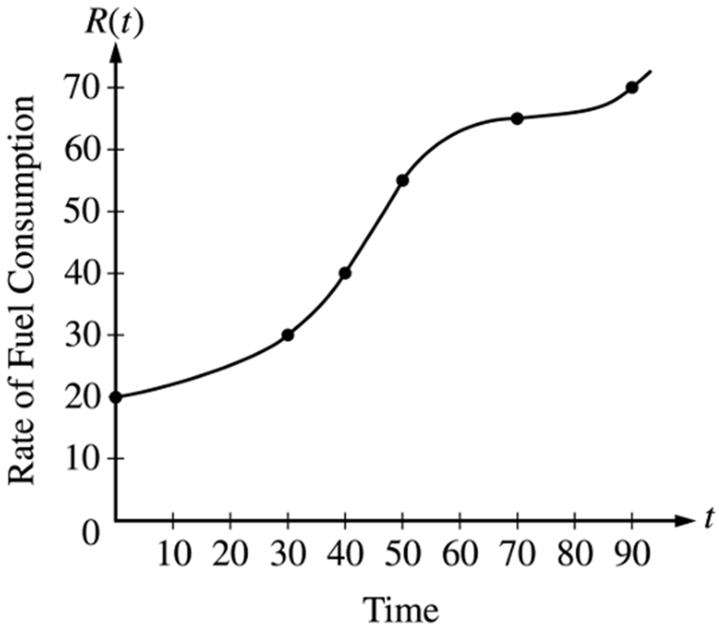
a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain your reasoning.

b) Find all time t in the open interval $0 < t < 4$ when the particle changes direction. Justify your answer.

c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

d) At what position is the particle at time $t = 3$?

#2. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes are shown.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

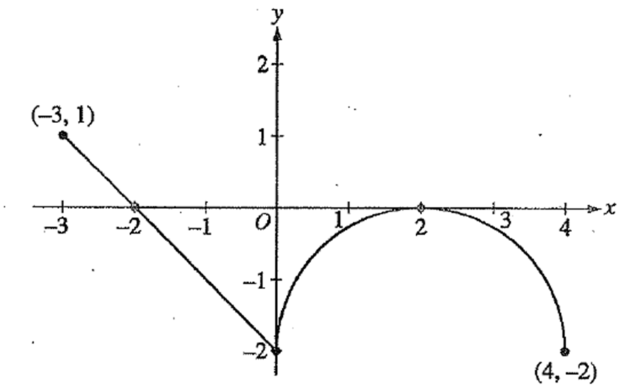
a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer and indicate units of measure.

b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table.

d) Using your approximation is part (c), how much total fuel was consumed during the entire 90 minute flight?

#3. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'(x)$, the derivative of $f(x)$, consists of one line segment and a semicircle, as shown in the figure.



Graph of f'

a) On what intervals, if any, is f increasing? Justify your answer.

b) Find the x-coordinates of each of the following: critical points, inflections points, and locations where the function may have an absolute maximum or absolute minimum over the interval $-3 \le x \le 4$.

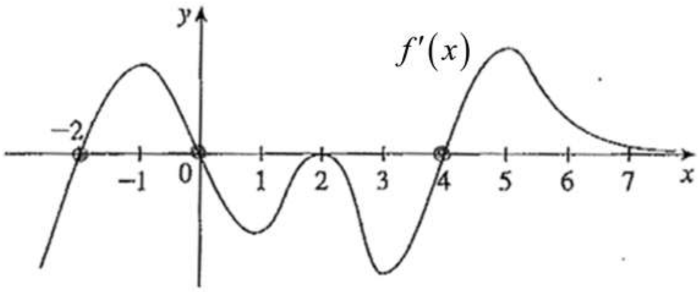
c) Find an equation for the line tangent to the graph of f at x = 0.

d) Find $f(-3)$ and $f(4)$.

#4. Find $\frac{dy}{dx}$ for $y - x^2 y^2 = 6$

#5. Find an equation of the tangent line to the curve $y^4 + xy = x^3 - x + 2$ at $(1,1)$.

#6. The figure shows the graph of $f'(x)$, the <u>derivative</u> of $f(x)$.



Find the following (approximate coordinates to the nearest 0.5):

a) All critical point x-values.
b) x-intervals where $f(x)$ is increasing/decreasing.
c) All inflection point x-values.
d) x-intervals where $f(x)$ is concave up/down.
e) All x-values where $f(x)$ has a local maximum or local minimum.

#7. Given

$$g(x) = \begin{cases} x^2 & x < 2 \\ -3 & x = 2 \\ 3x & x > 2 \end{cases}$$

a) Is *g(x)* continuous at $x = 2$?

b) Is *g(x)* continuous at $x = 3$?

#8. Sketch a slope field using at least 6 points in quadrant I for the differential equation $\frac{dy}{dx} = -x + 2y$

Solve the differential equation by separation of variables:

#9. $\frac{dy}{dx} = xy^2$ if $y(1) = 3$.

Solve the differential equation by separation of variables:

#10. $\frac{dy}{dx} = (2+x)y^2$ if $y(2) = 4$.

#11. $xyy' = 3 + x^2$ if $y(1) = 2$.

#12. For the function $g(t) = \frac{t}{t+2}$

a) Find the average rate of change over the interval $[1,4]$

b) Show that the Mean Value Theorem (or Rolle's Theorem) guarantees a time value within the interval $[1,4]$ where the instantaneous rate of change equals the average rate of change.

c) Find the time value where the instantaneous rate of change equals the average rate of change over the interval $[1,4]$.

#13. A particle moves along the x-axis such that its acceleration in m/s^2 at time t in seconds is given by $a(t)=t^3-3t$. If $v(2)=5$ and $x(1)=4$, find the velocity and position functions for the particle at any time, t.

#14. A particle moves along a line such that its position in meters at time t in seconds is given by

$$s(t) = t^3 - \frac{9}{2}t^2 + 6t$$

a) Find the velocity at time t.
b) What is the velocity after 1 second?
c) Find the acceleration at time t.
d) Find the acceleration of the particle at time $t = 4$ seconds.
e) When is the acceleration of the particle zero?
f) When is the particle at rest?
g) At what times in the interval $0 \le t \le 8$ is the particle changing direction?

Multiple-Choice Part: Although the posted solution for these MCQs show full solution work, on the actual final exam, you'll only need to choose an answer, so no work is required. However, this may mean that you need to simplify your work answer to obtain a listed answer. CALCULATOR REQUIRED for the MCQ part of the final exam.

#15. If $f(x)=\dfrac{e^{5x}}{x^3}$, find $f'(x)$

(A) $f'(x)=\dfrac{5e^{5x}}{3x^2}$ (B) $f'(x)=\dfrac{e^{5x}(3-5x)}{x^4}$ (C) $f'(x)=\dfrac{e^{5x}(x-3)}{x^4}$ (D) $f'(x)=\dfrac{e^{5x}(5x-3)}{x^4}$

#16. If $f(x)=\sin(4x)\ln(6x^2+3x)$, find $f'(x)$

(A) $f'(x)=\dfrac{4x+1}{x(x+1)}\sin(4x)+4\ln(6x^2+3x)\cos(4x)$

(B) $f'(x)=\dfrac{4(4x+1)\cos(4x)}{x(2x+1)}$

(C) $f'(x)=\dfrac{4\sin(4x)}{x}+4\ln(6x^2+3x)\cos(4x)$

(D) $f'(x)=\dfrac{\sin(4x)}{3x(2x+1)}+\ln(6x^2+3x)\cos(4x)$

#17. Let f be the function given by $f(x) = e^x \cos(2x)$. For what value of x in the interval $1 \le x \le 3$ is the slope of the tangent line to the graph of f equal to 1.2 ?

(A) 1.803 (B) 1.630 (C) 1.845 (D) 2.410

#18. Evaluate $\lim_{x \to 0} \dfrac{\ln(1-x) + x + \frac{1}{2}x^2}{x^3}$

(A) 0 (B) $-\dfrac{1}{3}$ (C) $\dfrac{1}{3}$ (D) *undefined*

#19. Evaluate $\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x}$

(A) 0 (B) -1 (C) 1 (D) *undefined*

#20. There are $40,000\ cm^2$ of material available to form into an open-topped square-bottom box. What is the maximum volume that can be enclosed within the box?

(A) $5,569.907\ cm^3$ (B) $115.470\ cm^3$ (C) $43.089\ cm^3$ (D) $769,800.359\ cm^3$

#21. An open-topped square-bottom box must enclose $50,000\ cm^3$. What is the minimum amount of material area that can be used to enclose this volume?

(A) $6,839.904\ cm^2$ (B) $46.416\ cm^2$ (C) $6,464.304\ cm^2$ (D) $129.099\ cm^2$

#22. A 25-foot long ladder is leaning against the wall of a house. The base of the ladder is pushed horizontally toward the wall at a rate of 2 feet per second. How fast is the top of the ladder moving up the wall when the bottom of the ladder is 15 feet from the wall?

(A) $2.667\ \frac{ft}{s}$ (B) $1.5\ \frac{ft}{s}$ (C) $0.667\ \frac{ft}{s}$ (D) $0.375\ \frac{ft}{s}$

#23. A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat. Suppose the winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat moving horizontally when there is 13 feet of rope out.

(A) $10.4\ \frac{ft}{s}$ (B) $0.096\ \frac{ft}{s}$ (C) $1.538\ \frac{ft}{s}$ (D) $0.65\ \frac{ft}{s}$

#24. Evaluate $\int \frac{x}{\sqrt{x^7}}\, dx$

(A) $\frac{2}{7}x^{\left(\frac{7}{2}\right)}+C$ (B) $\frac{2}{7}x^{\left(\frac{-7}{2}\right)}+C$ (C) $\frac{2}{5}x^{\left(\frac{-5}{2}\right)}+C$ (D) $-\frac{2}{3}x^{\left(\frac{-3}{2}\right)}+C$

#25. Evaluate $\int (4x+6)e^{(x^2+3x)}\, dx$

(A) $e^{(x^2+3x)}+C$ (B) $2e^{(x^2+3x)}+C$ (C) $(4x+2)e^{(x^2+3x)}+C$ (D) $(2x+3)e^{(x^2+3x)}+C$

#26. Evaluate $\int \frac{1}{x(\ln x)^2}\, dx$

(A) $\frac{-1}{\ln(x)}+C$ (B) $\frac{1}{2x^2}+C$ (C) $-\ln(x)+C$ (D) $\ln(x)+C$

#27. Evaluate $\int x^2 e^x\, dx$

(A) $\frac{1}{3}x^3e^x+C$

(B) $x^2e^x-2xe^x+C$

(C) $x^2e^x-2xe^x-e^x+C$

(D) $x^2e^x-2xe^x+2e^x+C$

#28. Evaluate $\int x\sin(2x)\,dx$

(A) $\frac{1}{2}x\cos(2x)-\frac{1}{4}\sin(2x)+C$

(B) $-x\cos(2x)+\sin(2x)+C$

(C) $-\frac{1}{2}x\cos(2x)+\frac{1}{4}\sin(2x)+C$

(D) $-\frac{1}{2}\cos(2x)+C$

#29. Evaluate $\int \frac{1}{x^2+x-20}\,dx$

(A) $\frac{2}{9}\arctan\left(\frac{2x+1}{9}\right)+C$

(B) $-\frac{1}{9}\ln|x-4|+\frac{1}{9}\ln|x+5|+C$

(C) $\frac{1}{9}\ln|x-4|-\frac{1}{9}\ln|x+5|+C$

(D) $\ln|x^2+x-20|+C$

#30. Evaluate $\int \frac{1}{x^2+4x+13}\,dx$

(A) $\ln|x^2+4x+13|+C$

(B) $-\frac{1}{8}\ln|x-2|+\frac{1}{8}\ln|x+6|+C$

(C) $\frac{1}{8}\ln|x-2|-\frac{1}{8}\ln|x+6|+C$

(D) $\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)+C$

#31. Evaluate $\int_0^5 \frac{1}{(x-1)^{1/5}}\,dx$

(A) 0
(B) 2.539
(C) 1.625
(D) *diverges*

#32. Evaluate $\int_1^3 \frac{2}{x-2}\,dx$

(A) 0
(B) 1
(C) 2
(D) *diverges*

#33. Evaluate $\int_4^\infty \frac{x}{x^{7/2}}\,dx$

(A) 0.083
(B) $2{,}304.000$
(C) 12.048
(D) *diverges*

#34. Find the <u>derivative</u> of the function $F(x)=\int_{2}^{e^{3x}} \cos(t^2)\,dt$ when $x = 0.2$

(A) -0.984
(B) -5.379
(C) -1.793
(D) 2.345

#35. Find the average value of the function $f(x)=-x^4+2x^2+4$ on the closed interval $[-2,1]$.

(A) 11.4
(B) 3.000
(C) 3.8
(D) 0.5

(For #36-#38) A particle moves along a line such that its position in meters at time t in seconds is given by $s(t) = 2t^3 - 5t^2 + 2t$

#36. When is the speed of the particle $1 \frac{m}{s}$?

(A) $t = 2.142\ s$

(B) $t = 0.107\ s, \quad t = 1.560\ s$

(C) $t = -0.281\ s, \quad t = 1.000\ s, \quad t = 1.781\ s, \quad t = 2.142\ s$

(D) $t = 0.107\ s, \quad t = 0.392\ s, \quad t = 1.274\ s, \quad t = 1.560\ s$

#37. How is the particle moving at time $t = 0.4$ seconds?

(A) *The particle is moving left*

(B) *The particle is moving right*

(C) *The particle is at rest*

(D) *Cannot be determined from the information given*

#38. Is the particle speeding up or slowing down at time $t = 0.4$ seconds?

(A) *The particle is slowing down*

(B) *The particle is speeding up*

(C) *The particle is at rest*

(D) *Cannot be determined from the information given*

For #39-40, find the area bounded by the given curves (use math9 to evaluate the integrals)

#39. $y = x^3, \quad y = x^2 - 4x + 4, \quad x = 2$

(A) 1.333
(B) 3.417
(C) 56.369
(D) 53.228

#40. $x - 2y + 7 = 0, \quad y^2 - 6y - x = 0$

(A) 144
(B) 36
(C) 32.667
(D) 588.106

For #41-43, use the disk method to find the volume generated by rotating the region bounded by the given curves about the specified axis (use math9 to evaluate the integrals).

#41. $y = x^2, \quad y = 4, \quad x = 0 \qquad \textit{about the } x-\textit{axis}$

(A) 80.425
(B) 25.600
(C) 53.617
(D) 442.336

#42. $y = e^{-2x}, \quad y = 0, \quad x = 1, \quad x = 0 \quad \textit{about the } x-\textit{axis}$

(A) 1.358
(B) 0.245
(C) 0.771
(D) 0.443

#43. $y = x^3, \quad y = 8, \quad x = 0, \quad \textit{about the } y-\textit{axis}$

(A) 57.446
(B) 20.106
(C) 18.286
(D) 60.319