

Free-Response Part: You'll need to show work on the final using appropriate formats and explanations. The FRQ part of the final exam is **NO CALCULATOR** and the MCQ part **REQUIRES A CALCULATOR**, so do the FRQ problems in this review without a calculator.

Look at the posted solutions to see the required justifications and formatting for full credit.

#1. A particle moves along the x -axis so that its velocity at time t is given by $v(t) = -t(t-2)(t-3)$.

At time $t = 0$, the particle is at position $x = 1$.

a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain your reasoning.

b) Find all time t in the open interval $0 < t < 4$ when the particle changes direction. Justify your answer.

c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

d) At what position is the particle at time $t = 3$?

$$a) \quad v(t) = -t(t^2 - 5t + 6) = -t^3 + 5t^2 - 6t$$

$$a(t) = -3t^2 + 10t - 6$$

$$\boxed{a(2) = -3(2)^2 + 10(2) - 6} = -3(4) + 20 - 6 = 2$$

Because $a(2) > 0$, the speed of the particle is increasing at $t = 2$.

$$b) \quad v(t) = 0$$

$$-t(t-2)(t-3) = 0 \quad \text{at } t=0, t=2, t=3$$

$$(c) \quad \begin{array}{c} | \qquad \qquad \qquad | \qquad \qquad \qquad | \qquad \qquad \qquad | \qquad \qquad \qquad | \\ \hline \qquad \qquad \qquad 2 \qquad \qquad \qquad \qquad \qquad \qquad 3 \qquad \qquad \qquad \qquad \qquad \qquad (4) \end{array}$$

$$v(t) = (-t)(t-2)(t-3)$$

$$\begin{array}{ccc} v(1) & v(2.5) & v(3.5) \\ (-)(-)(-) & (-)(+)(-) & (-)(+)(+) \\ v(1) < 0 & v(2.5) > 0 & v(3.5) < 0 \end{array}$$

The particle changes direction when the sign of $v(t)$ changes, which occurs at $t=2$ and $t=3$ for $0 < t < 4$.

#1 continued...

c) From part (b), $v(t) < 0$ for $0 < t < 2$ and $v(t) > 0$ for $2 < t < 3$.

∴ particle total distance traveled:

$$= (-1) \int_0^2 v(t) dt + \int_2^3 v(t) dt$$

$$= (-1) \int_0^2 (-t^3 + 5t^2 - 6t) dt + \int_2^3 (-t^3 + 5t^2 - 6t) dt$$

$$= (-1) \left[-\frac{1}{4}t^4 + \frac{5}{3}t^3 - 3t^2 \right]_0^2 + \left[-\frac{1}{4}t^4 + \frac{5}{3}t^3 - 3t^2 \right]_2^3$$

$$= \boxed{(-1) \left[\left(-\frac{1}{4}(2)^4 + \frac{5}{3}(2)^3 - 3(2)^2 \right) - (0) \right] + \left[\left(-\frac{1}{4}(3)^4 + \frac{5}{3}(3)^3 - 3(3)^2 \right) - \left(-\frac{1}{4}(2)^4 + \frac{5}{3}(2)^3 - 3(2)^2 \right) \right]}$$

$$= \left(= \frac{27}{12} = 3.08\overline{33} \right)$$

$$d) x(3) = x(0) + \int_0^3 v(t) dt$$

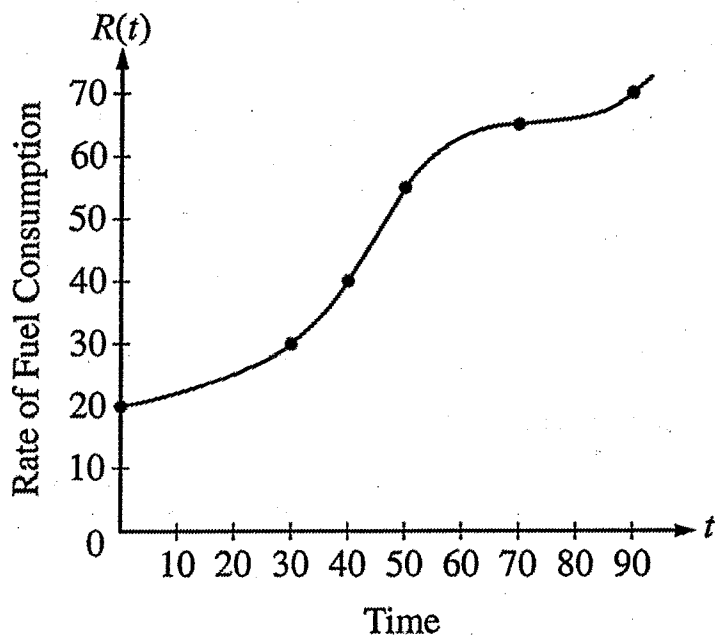
$$x(3) = 1 + \int_0^3 (-t^3 + 5t^2 - 6t) dt$$

$$= 1 + \left[-\frac{1}{4}t^4 + \frac{5}{3}t^3 - 3t^2 \right]_0^3$$

$$= \boxed{1 + \left[\left(-\frac{1}{4}(3)^4 + \frac{5}{3}(3)^3 - 3(3)^2 \right) - (0) \right]}$$

$$= (-1.25)$$

- #2. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes are shown.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer and indicate units of measure.
- b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table.
- d) Using your approximation in part (c), how much total fuel was consumed during the entire 90 minute flight?

#2

a) $R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10} = \frac{15}{10} \text{ gallons per minute}$

b) Since $R'(t)$ is increasing fastest at $t = 45$ min, $R'(t)$ must have a maximum at $t = 45$ min,

$\therefore R''(45) = 0$

c)

Interval	t_i	$R(t_i)$	Δt	ΔR
$[0, 30]$	0	20	30	= 600
$[30, 40]$	30	30	10	= 300
$[40, 50]$	40	40	10	= 400
$[50, 70]$	50	55	20	= 1100
$[70, 90]$	70	65	20	= 1300

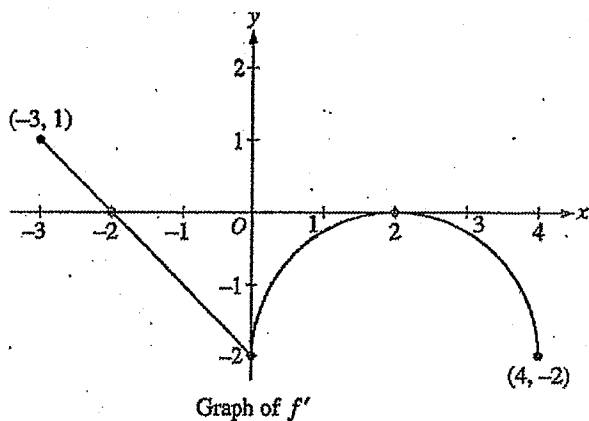
$\int_0^{90} R(t) dt \approx 3700 \text{ gallons}$

d) $\int_0^{90} R(t) dt = F(90) - F(0)$

$3700 = F(90) - 0$

$F(90) = 3700 \text{ gallons}$

#3. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of $f'(x)$, the derivative of $f(x)$, consists of one line segment and a semicircle, as shown in the figure.



- On what intervals, if any, is f increasing? Justify your answer.
- Find the x -coordinates of each of the following: critical points, inflection points, and locations where the function may have an absolute maximum or absolute minimum over the interval $-3 \leq x \leq 4$.
- Find an equation for the line tangent to the graph of f at $x = 0$.
- Find $f(-3)$ and $f(4)$.

a) $f(x)$ increases where $f'(x) > 0$ which occurs for $-3 \leq x < 2$

b) critical points when $f'(x) = 0$ or DNE, at $x = -2, x = 2$

inflection points when $f''(x) = 0$, which occurs when $f'(x)$ changes from increasing to decreasing or vice versa, at $x = 0$ and $x = 2$

abs max/min at critical points plus interval ends, at $x = -2, x = 2, x = -3, x = 4$

c) point $(0, 3)$ $m = f'(0) = -2$ $(y - 3) = -2(x - 0)$

$$d) \int_{-3}^0 f'(x) dx = f(0) - f(-3)$$

$$\frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = 3 - f(-3)$$

$$\frac{1}{2} - 2$$

$$-\frac{3}{2} = 3 - f(-3)$$

$$f(-3) = 3 + \frac{3}{2}$$

$$\int_0^4 f'(x) dx = f(4) - f(0)$$

rectangle - semicircle

$$-[(4)(2) - \frac{1}{2}\pi(2)^2] = f(4) - 3$$

$$-[8 - 2\pi] = f(4) - 3$$

$$f(4) = 2\pi - 5$$

#4. Find $\frac{dy}{dx}$ for $y - x^2y^2 = 6$

$$\frac{d}{dx}(y) - \left[x^2 \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x^2) \right] = \frac{d}{dx}(6)$$

$$\frac{dy}{dx} - [x^2(2y \frac{dy}{dx}) + y^2(2x)] = 0$$

$$\frac{dy}{dx} - 2x^2y \frac{dy}{dx} - 2xy^2 = 0$$

$$\frac{dy}{dx} (1 - 2x^2y) = 2xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{2xy^2}{1 - 2x^2y}}$$

#5. Find an equation of the tangent line to the curve $y^4 + xy = x^3 - x + 2$ at $(1,1)$.

$$\text{slope} = \frac{dy}{dx} \quad \frac{d}{dx}(y^4) + x \frac{d}{dx}(y) + y \frac{d}{dx}(x) = \frac{d}{dx}(x^3) - \frac{d}{dx}(x) + \frac{d}{dx}(2)$$

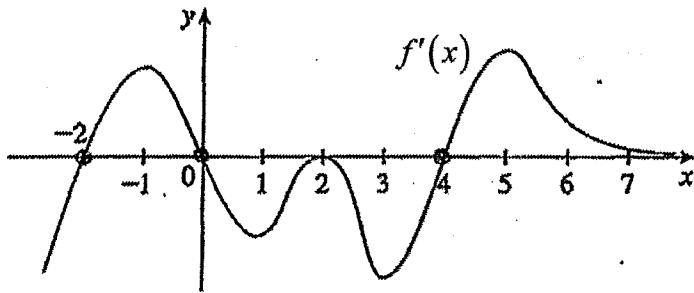
$$4y^3 \frac{dy}{dx} + x \frac{dy}{dx} + y(1) = 3x^2 - 1 + 0$$

$$\frac{dy}{dx} (4y^3 + x) = 3x^2 - 1 - y$$

$$m = \frac{dy}{dx} = \frac{3x^2 - 1 - y}{4y^3 + x} \Big|_{(1,1)} = \frac{3(1)^2 - 1 - 1}{4(1)^3 + 1} = \frac{1}{5}$$

$$\boxed{(y-1) = \frac{1}{5}(x-1)}$$

#6. The figure shows the graph of $f'(x)$, the derivative of $f(x)$.



Find the following (approximate coordinates to the nearest 0.5):

- All critical point x -values.
- x -intervals where $f(x)$ is increasing/decreasing.
- All inflection point x -values.
- x -intervals where $f(x)$ is concave up/down.
- All x -values where $f(x)$ has a local maximum or local minimum.

a) critical points where $f'(x) = 0$ or DNE, at $x = -2, x = 0, x = 2, x = 4$

b) $f(x)$ increasing where $f'(x) > 0$, which occurs for $-2 < x < 0$ and $4 < x < \infty$

$f(x)$ decreasing where $f'(x) < 0$, which occurs for $-\infty < x < -2, 0 < x < 2, 2 < x < 4$

c) Inflection points where $f''(x)$ changes sign, where $f'(x)$ changes from increasing/decreasing or vice versa, which occurs at $x = -1, x = 1, x = 2, x = 3, x = 5$

d) $f(x)$ is concave up when $f''(x) > 0$, when $f'(x)$ is increasing, which occurs for $-\infty < x < -1, 1 < x < 2, 3 < x < 5$

$f(x)$ is concave down when $f''(x) < 0$, when $f'(x)$ is decreasing, which occurs for $-1 < x < 1, 2 < x < 3, 5 < x < \infty$

e) local max or min at critical points (from a): $x = -2, x = 0, x = 2, x = 4$

#7. Given

$$g(x) = \begin{cases} x^2 & x < 2 \\ -3 & x = 2 \\ 3x & x > 2 \end{cases}$$

a) Is $g(x)$ continuous at $x = 2$?

1) $g(2) = -3$

2) $\lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^-} x^2 = (2)^2 = 4$

$\lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} 3x = 3(2) = 6$

$\therefore \lim_{x \rightarrow 2} g(x)$ DNE

$\therefore g(x)$ is not continuous at $x = 2$

b) Is $g(x)$ continuous at $x = 3$?

1) $g(3) = 3(3) = 9$

2) $\lim_{x \rightarrow 3^-} g(x) = \lim_{x \rightarrow 3^-} 3x = 3(3) = 9$

$\lim_{x \rightarrow 3^+} g(x) = \lim_{x \rightarrow 3^+} 3x = 3(3) = 9$

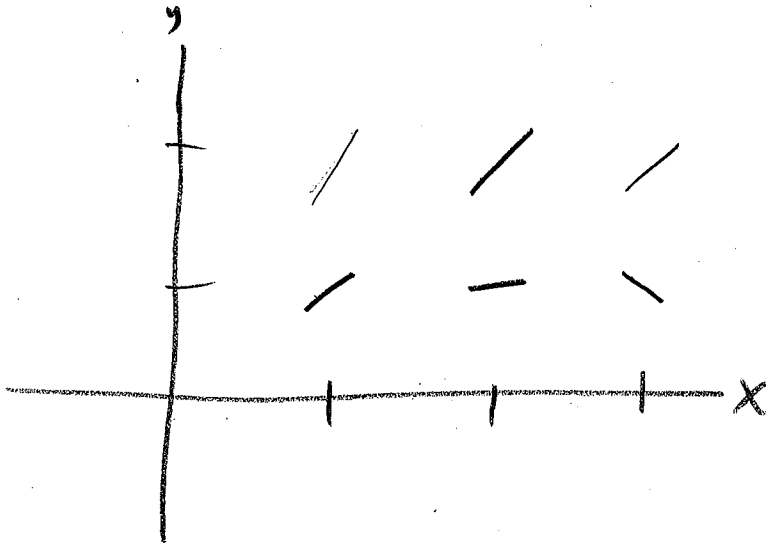
$\therefore \lim_{x \rightarrow 3} g(x) = 9$

3) $g(3) = \lim_{x \rightarrow 3} g(x)$

$9 = 9$

$\therefore g(x)$ is continuous at $x = 3$

#8. Sketch a slope field using at least 6 points in quadrant I for the differential equation $\frac{dy}{dx} = -x + 2y$



(x, y)	$\frac{dy}{dx}$
(1, 1)	1
(1, 2)	3
(2, 1)	0
(2, 2)	2
(3, 1)	-1
(3, 2)	1

Solve the differential equation by separation of variables:

#9. $\frac{dy}{dx} = xy^2$ if $y(1) = 3$.

$$\frac{1}{y^2} dy = x dx$$

$$\int y^{-2} dy = \int x dx$$

$$\frac{y^{-1}}{-1} = \frac{1}{2}x^2 + C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + C$$

$$-\frac{1}{(3)} = \frac{1}{2}(1)^2 + C$$

$$-\frac{1}{3} = \frac{1}{2} + C$$

$$C = -\frac{1}{3} - \frac{1}{2} = -\frac{2}{6} - \frac{3}{6} = -\frac{5}{6} \quad \nearrow$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + \left(-\frac{5}{6}\right)$$

$$\frac{1}{y} = -\frac{1}{2}x^2 + \frac{5}{6}$$

$$y = \frac{1}{-\frac{1}{2}x^2 + \frac{5}{6}}$$

Solve the differential equation by separation of variables:

#10. $\frac{dy}{dx} = (2+x)y^2$ if $y(2) = 4$.

$$\frac{1}{y^2} dy = (2+x) dx$$

$$\int y^{-2} dy = \int (2+x) dx$$

$$\frac{y^{-1}}{-1} = 2x + \frac{1}{2}x^2 + C$$

$$-\frac{1}{y} = 2x + \frac{1}{2}x^2 + C$$

$$-\frac{1}{4} = 2(2) + \frac{1}{2}(2)^2 + C$$

$$-\frac{1}{4} = 4 + 2 + C = 6 + C$$

$$C = -6 - \frac{1}{4} = -\frac{24}{4} - \frac{1}{4} = -\frac{25}{4}$$

$$-\frac{1}{y} = 2x + \frac{1}{2}x^2 - \frac{25}{4}$$

$$\frac{1}{y} = -2x - \frac{1}{2}x^2 + \frac{25}{4}$$

$$y = \frac{1}{-2x - \frac{1}{2}x^2 + \frac{25}{4}}$$

#11. $xyy' = 3+x^2$ if $y(1) = 2$.

$$y \frac{dy}{dx} = \frac{3+x^2}{x} = \frac{3}{x} + x$$

$$\int y dy = \int \left(\frac{3}{x} + x \right) dx$$

$$\frac{1}{2}y^2 = 3 \ln|x| + \frac{1}{2}x^2 + C$$

$$\frac{1}{2}(2)^2 = 3 \ln|1| + \frac{1}{2}(1)^2 + C$$

$$2 = 0 + \frac{1}{2} + C, \quad C = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\text{so } \frac{1}{2}y^2 = 3 \ln|x| + \frac{1}{2}x^2 + \frac{3}{2}$$

$$y^2 = 6 \ln|x| + x^2 + 3$$
$$y = \pm \sqrt{6 \ln|x| + x^2 + 3}$$

#12. For the function $g(t) = \frac{t}{t+2}$

a) Find the average rate of change over the interval $[1, 4]$

b) Show that the Mean Value Theorem (or Rolle's Theorem) guarantees a time value within the interval $[1, 4]$ where the instantaneous rate of change equals the average rate of change.

c) Find the time value where the instantaneous rate of change equals the average rate of change over the interval $[1, 4]$.

$$a) \text{ A.R.O.C} = \frac{g(4) - g(1)}{4 - 1} = \frac{\frac{4}{6} - \frac{1}{3}}{3} = \frac{\frac{2}{3} - \frac{1}{3}}{3} = \frac{\frac{1}{3}}{3} = \frac{1}{9}$$

b) $g(t)$ is continuous over $[1, 4]$ and differentiable over $(1, 4)$
 \therefore the Mean Value Theorem guarantees a value c , $1 < c < 4$,
such that $g'(c) = \frac{1}{9}$.

$$c) g'(t) = \frac{(t+2)(1) - t(1)}{(t+2)^2} = \frac{t+2-t}{(t+2)^2} = \frac{2}{(t+2)^2}$$

$$g'(c) = \frac{1}{9}$$

$$\frac{2}{(c+2)^2} = \frac{1}{9}$$

$$\frac{(c+2)^2}{2} = 9$$

$$(c+2)^2 = 18$$

$$c+2 = \pm\sqrt{18}$$

$$c = -2 \pm \sqrt{18}$$

$$c = -2 - \sqrt{18} \text{ (not in } 1 < t < 4)$$

$$\boxed{c = -2 + \sqrt{18}}$$

#13. A particle moves along the x-axis such that its acceleration in m/s^2 at time t in seconds is given by $a(t) = t^3 - 3t$. If $v(2) = 5$ and $x(1) = 4$, find the velocity and position functions for the particle at any time, t .

$$v(t) = \int a(t) dt = \int (t^3 - 3t) dt$$

$$v(t) = \frac{1}{4}t^4 - \frac{3}{2}t^2 + C \quad \& \quad v(2) = 5:$$

$$5 = \frac{1}{4}(2)^4 - \frac{3}{2}(2)^2 + C$$

$$5 = 4 - 6 + C = -2 + C$$

$$C = 7$$

$$v(t) = \frac{1}{4}t^4 - \frac{3}{2}t^2 + 7 \quad m/s$$

$$x(t) = \int v(t) dt = \int \left(\frac{1}{4}t^4 - \frac{3}{2}t^2 + 7 \right) dt$$

$$x(t) = \frac{1}{20}t^5 - \frac{1}{2}t^3 + 7t + D \quad \& \quad x(1) = 4:$$

$$4 = \frac{1}{20}(1)^5 - \frac{1}{2}(1)^3 + 7(1) + D$$

$$4 = \frac{1}{20} - \frac{1}{2} + 7 + D \quad (\text{multiply by } 20)$$

$$80 = 1 - 10 + 140 + 20D$$

$$80 = 131 + 20D$$

$$-51 = 20D$$

$$D = -\frac{51}{20}$$

$$\begin{array}{r} 131 \\ -80 \\ \hline 51 \end{array}$$

$$x(t) = \frac{1}{20}t^5 - \frac{1}{2}t^3 + 7t - \frac{51}{20}$$

#14. A particle moves along a line such that its position in meters at time t in seconds is given by

$$s(t) = t^3 - \frac{9}{2}t^2 + 6t$$

- Find the velocity at time t .
- What is the velocity after 1 second?
- Find the acceleration at time t .
- Find the acceleration of the particle at time $t = 4$ seconds.
- When is the acceleration of the particle zero?
- When is the particle at rest?
- At what times in the interval $0 \leq t \leq 8$ is the particle changing direction?

a) $v(t) = s'(t) = 3t^2 - 9t + 6 \text{ m/s}$

b) $v(1) = 3(1)^2 - 9(1) + 6 \text{ m/s} \quad (= 0 \text{ m/s})$

c) $a(t) = v'(t) = 6t - 9 \text{ m/s}^2$

d) $a(4) = 6(4) - 9 \text{ m/s}^2 \quad (= 15 \text{ m/s}^2)$

e) $a(t) = 0$
 $6t - 9 = 0, \quad 6t = 9, \quad t = \frac{9}{6} = \frac{3.3}{3.2} = \frac{3}{2} \text{ s}$

f) $v(t) = 0$
 $3t^2 - 9t + 6 = 0$
 $3(t^2 - 3t + 2) = 0$
 $3(t-1)(t-2) = 0 \quad \text{at } t = 1 \text{ s}, t = 2 \text{ s}$

g) $0 \quad | \quad 1 \quad | \quad 2 \quad | \quad 8 \quad v(t) = 3(t-1)(t-2)$

$v(\frac{1}{2})$	$v(1.5)$	$v(3)$
$(+)(-)(-)$	$(+)(+)(-)$	$(+)(+)(+)$
$v(\frac{1}{2}) > 0$	$v(1.5) < 0$	$v(3) > 0$

The particle changes directions when the sign of $v(t)$ changes which occurs at $t = 1 \text{ s}$, and $t = 2 \text{ sec}$

Multiple-Choice Part: Although the posted solution for these MCQs show full solution work, on the actual final exam, you'll only need to choose an answer, so no work is required. However, this may mean that you need to simplify your work answer to obtain a listed answer. **CALCULATOR REQUIRED** for the MCQ part of the final exam.

#15. If $f(x) = \frac{e^{5x}}{x^3}$, find $f'(x)$

(A) $f'(x) = \frac{5e^{5x}}{3x^2}$ (B) $f'(x) = \frac{e^{5x}(3-5x)}{x^4}$ (C) $f'(x) = \frac{e^{5x}(x-3)}{x^4}$ **(D) $f'(x) = \frac{e^{5x}(5x-3)}{x^4}$**

Quotient rule
(and chain rule)

$$f'(x) = \frac{x^3(e^{5x} \cdot 5) - e^{5x}(3x^2)}{(x^3)^2}$$

$$= \frac{e^{5x}(5x^3 - 3x^2)}{x^6}$$

$$= \frac{e^{5x} x^2(5x-3)}{x^4}$$

$$= \frac{e^{5x}(5x-3)}{x^2}$$

#16. If $f(x) = \sin(4x) \ln(6x^2 + 3x)$, find $f'(x)$

(A) $f'(x) = \frac{4x+1}{x(x+1)} \sin(4x) + 4 \ln(6x^2 + 3x) \cos(4x)$

(B) $f'(x) = \frac{4(4x+1)\cos(4x)}{x(2x+1)}$

(C) $f'(x) = \frac{4\sin(4x)}{x} + 4 \ln(6x^2 + 3x) \cos(4x)$

(D) $f'(x) = \frac{\sin(4x)}{3x(2x+1)} + \ln(6x^2 + 3x) \cos(4x)$

product rule
(and chain rule)

$$f'(x) = \sin(4x) \frac{1}{6x^2+3x} (12x+3) + \ln(6x^2+3x) \cos(4x) \cdot 4$$

$$= \frac{\sin(4x) 3(4x+1)}{3x(2x+1)} + 4 \ln(6x^2+3x) \cos(4x)$$

$$= \frac{4x+1}{x(2x+1)} \sin(4x) + 4 \ln(6x^2+3x) \cos(4x)$$

#17. Let f be the function given by $f(x) = e^x \cos(2x)$. For what value of x in the interval $1 \leq x \leq 3$ is the slope of the tangent line to the graph of f equal to 1.2?

(A) 1.803

(B) 1.630

(C) 1.845

(D) 2.410

$$f'(x) = e^x (-\sin(2x) \cdot 2) + \cos(2x)e^x = 1.2 \quad (\text{calc. graph solution})$$

$$\text{at } x = 1.845$$

#18. Evaluate $\lim_{x \rightarrow 0} \frac{\ln(1-x) + x + \frac{1}{2}x^2}{x^3}$ $\left(\frac{0}{0}\right)$

(A) 0

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) undefined

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{1-x}(-1) + 1 + x}{3x^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \rightarrow 0} \frac{-(1-x)^{-1} + 1 + x}{3x^2} = \frac{(1-x)^{-2}(-1) + 1}{6x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \rightarrow 0} \frac{2(1-x)^{-3}(-1)}{6} = \frac{-2}{6} = -\frac{1}{3}$$

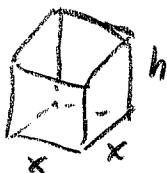
#19. Evaluate $\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{\ln x}$ $\left(\frac{\infty}{\infty}\right)$

- (A) 0 (B) -1 (C) 1 (D) undefined

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\ln x} = 0$$

#20. There are $40,000 \text{ cm}^2$ of material available to form into an open-topped square-bottom box. What is the maximum volume that can be enclosed within the box?

- (A) $5,569.907 \text{ cm}^3$ (B) 115.470 cm^3 (C) 43.089 cm^3 (D) $769,800.359 \text{ cm}^3$



objective function

$$\max V = x^2 h$$

$$V = x^2 \frac{(40000 - x^2)}{4x}$$

$$V = \frac{1}{4} x (40000 - x^2)$$

$$V = 10000x - \frac{1}{4} x^3$$

$$V' = 10000 - \frac{3}{4} x^2 = 0$$

$$\frac{3}{4} x^2 = 10000$$

$$x^2 = \frac{40000}{3}$$

$$x = \sqrt{\frac{40000}{3}} = 115.470$$

$$V_{\max} = 10000(115.470) - \frac{1}{4}(115.470)^3 = 769800.3589$$

constraint

$$A = 4xh + x^2 = 40000$$

$$4xh = 40000 - x^2$$

$$\leftarrow h = \frac{40000 - x^2}{4x}$$

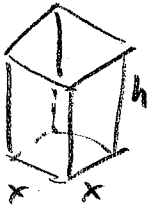
#21. An open-topped square-bottom box must enclose $50,000 \text{ cm}^3$. What is the minimum amount of material area that can be used to enclose this volume?

(A) $6,839.904 \text{ cm}^2$

(B) 46.416 cm^2

(C) $6,464.304 \text{ cm}^2$

(D) 129.099 cm^2



Objective function

$$\min A = 4xh + x^2$$

$$A = 4x\left(\frac{50000}{x^2}\right) + x^2$$

$$A = 20000x^{-1} + x^2$$

$$A' = -200000x^{-2} + 2x = 0$$

$$2x = \frac{200000}{x^2}$$

$$2x^3 = 200000$$

$$x^3 = 100000$$

$$x = \sqrt[3]{100000} = 46.41588834$$

$$A_{\min} = 20000 \frac{1}{46.41588834} + (46.41588834)^2 = 6463.30407$$

constraint

$$V = x^2h = 50000$$

$$\leftarrow h = \frac{50000}{x^2}$$

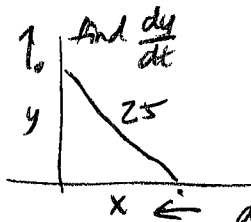
#22. A 25-foot long ladder is leaning against the wall of a house. The base of the ladder is pushed horizontally toward the wall at a rate of 2 feet per second. How fast is the top of the ladder moving up the wall when the bottom of the ladder is 15 feet from the wall?

(A) $2.667 \frac{\text{ft}}{\text{s}}$

(B) $1.5 \frac{\text{ft}}{\text{s}}$

(C) $0.667 \frac{\text{ft}}{\text{s}}$

(D) $0.375 \frac{\text{ft}}{\text{s}}$



$$x^2 + y^2 = 25^2$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[25^2]$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(15)(-2) + 2(20) \frac{dy}{dt} = 0$$

$$-60 + 40 \frac{dy}{dt} = 0$$

$$40 \frac{dy}{dt} = 60$$

$$\frac{dy}{dt} = \frac{60}{40} = 1.5 \text{ ft/s}$$

instantaneous:

$$x^2 + y^2 = 25^2$$

$$15^2 + y^2 = 25^2$$

$$y = \sqrt{25^2 - 15^2} = 20$$

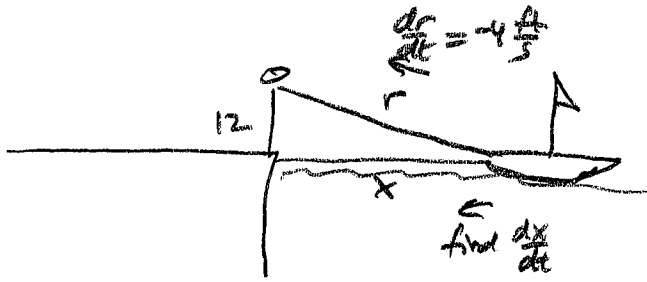
#23. A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat. Suppose the winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat moving horizontally when there is 13 feet of rope out.

(A) $10.4 \frac{ft}{s}$

(B) $0.096 \frac{ft}{s}$

(C) $1.538 \frac{ft}{s}$

(D) $0.65 \frac{ft}{s}$



In a right triangle:

$$x^2 + 12^2 = r^2$$

$$x^2 + 12^2 = 13^2$$

$$x = \sqrt{13^2 - 12^2} = 5$$

$$x^2 + 12^2 = r^2$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[12^2] = \frac{d}{dt}[r^2]$$

$$2x \frac{dx}{dt} + 0 = 2r \frac{dr}{dt}$$

$$2(5) \frac{dx}{dt} = 2(13)(-4)$$

$$10 \frac{dx}{dt} = -104$$

$$\frac{dx}{dt} = \frac{-104}{10} = -10.4$$

$$\text{"speed"} = \left| \frac{dx}{dt} \right| = 10.4 \frac{ft}{s}$$

#24. Evaluate $\int \frac{x}{\sqrt{x^7}} dx$

(A) $\frac{2}{7}x^{\frac{7}{2}} + C$

(B) $\frac{2}{7}x^{\frac{-7}{2}} + C$

(C) $\frac{2}{5}x^{\frac{-5}{2}} + C$

(D) $-\frac{2}{3}x^{\frac{-3}{2}} + C$

$$\int \frac{x}{\sqrt{x^7}} dx = \int \frac{x}{x^{7/2}} dx = \int x^{(1-7/2)} dx = \int x^{-5/2} dx$$

$$\frac{x^{-3/2}}{(-3/2)} = -\frac{2}{3}x^{-3/2} + C$$

#25. Evaluate $\int (4x+6)e^{(x^2+3x)} dx$

- (A) $e^{(x^2+3x)} + C$ (B) $2e^{(x^2+3x)} + C$ (C) $(4x+2)e^{(x^2+3x)} + C$ (D) $(2x+3)e^{(x^2+3x)} + C$

$$\int (4x+6)e^{(x^2+3x)} dx = \int 2(2x+3)e^{(x^2+3x)} dx = 2 \int (2x+3)e^{(x^2+3x)} dx$$

u sub $u = x^2 + 3x$
 $\frac{du}{dx} = 2x + 3$
 $du = (2x+3)dx$

$$= 2 \int e^u du = 2e^u = 2e^{(x^2+3x)} + C$$

#26. Evaluate $\int \frac{1}{x(\ln x)^2} dx$

- (A) $\frac{-1}{\ln(x)} + C$ (B) $\frac{1}{2x^2} + C$ (C) $-\ln(x) + C$ (D) $\ln(x) + C$

u sub $u = \ln x$
 $\frac{du}{dx} = \frac{1}{x}$
 $du = \frac{1}{x} dx$

$$\int \frac{1}{(\ln x)^2} \cdot \frac{1}{x} dx$$

$$= \int u^{-2} du$$

$$= \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{\ln x} + C$$

#27. Evaluate $\int x^2 e^x dx$

- (A) $\frac{1}{3}x^3 e^x + C$
 (B) $x^2 e^x - 2x e^x + C$
 (C) $x^2 e^x - 2x e^x - e^x + C$
 (D) $x^2 e^x - 2x e^x + 2e^x + C$

by parts twice: $\int x^2 e^x dx$

$uv - \int v du$
 $(x^2)(e^x) - \int e^x 2x dx$

$u = x^2 \quad dv = e^x dx$
 $\frac{du}{dx} = 2x \quad \int dv = \int e^x dx$
 $du = 2x dx \quad v = e^x$

$$x^2 e^x - 2 \int x e^x dx$$

$u = x \quad dv = e^x dx$
 $\frac{du}{dx} = 1 \quad \int dv = \int e^x dx$
 $du = dx \quad v = e^x$

$$x^2 e^x - 2[uv - \int v du]$$

$$x^2 e^x - 2[xe^x - \int e^x dx]$$

$$x^2 e^x - 2[xe^x - e^x] + C$$

$$x^2 e^x - 2x e^x + 2e^x + C$$

#28. Evaluate $\int x \sin(2x) dx$

(A) $\frac{1}{2} x \cos(2x) - \frac{1}{4} \sin(2x) + C$

(B) $-x \cos(2x) + \sin(2x) + C$

(C) $-\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) + C$

(D) $-\frac{1}{2} \cos(2x) + C$

by parts: $u = x \quad dv = \sin(2x) dx$

$\frac{du}{dx} = 1 \quad \int dv = \int \sin(2x) dx$

$du = dx \quad v = -\frac{1}{2} \cos(2x)$

$uv - \int v du$

$(x)(-\frac{1}{2} \cos(2x)) - \int (-\frac{1}{2} \cos(2x)) dx$

$-\frac{1}{2} x \cos(2x) + \frac{1}{2} \int \cos(2x) dx$

$-\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) + C$

#29. Evaluate $\int \frac{1}{x^2 + x - 20} dx = \int \frac{1}{(x-4)(x+5)} dx$ Factorable \rightarrow Partial Fraction Expansion:

(A) $\frac{2}{9} \arctan\left(\frac{2x+1}{9}\right) + C$

(B) $-\frac{1}{9} \ln|x-4| + \frac{1}{9} \ln|x+5| + C$

(C) $\frac{1}{9} \ln|x-4| - \frac{1}{9} \ln|x+5| + C$

(D) $\ln|x^2 + x - 20| + C$

$\frac{1}{(x-4)(x+5)} = \frac{A}{x-4} + \frac{B}{x+5}$

$\frac{1}{(x-4)(x+5)} = \frac{A(x+5)}{(x-4)(x+5)} + \frac{B(x-4)}{(x-4)(x+5)} \rightarrow A(x+5) + B(x-4) = 1$

$Ax + 5A + Bx - 4B = 1$

$(A+B)x + (5A-4B) = (0)x + (1)$

$\begin{cases} A+B=0 & 4A+4B=0 \\ 5A-4B=1 & 5A-4B=1 \end{cases}$

$9A = 1, A = \frac{1}{9}$

$A+B=0, B = -\frac{1}{9}$ (or use REF)

$= \frac{1}{9} \int \frac{1}{x-4} dx - \frac{1}{9} \int \frac{1}{x+5} dx$

$= \frac{1}{9} \ln|x-4| - \frac{1}{9} \ln|x+5| + C$

#30. Evaluate $\int \frac{1}{x^2 + 4x + 13} dx$

Not Factorable, complete the square to arctan form:

$x^2 + 4x + \frac{16}{4} + 13 - \frac{16}{4}$

$(x+2)^2 + 9$

$u = x+2 \quad a=3$

(A) $\ln|x^2 + 4x + 13| + C$

(B) $-\frac{1}{8} \ln|x-2| + \frac{1}{8} \ln|x+6| + C$

(C) $\frac{1}{8} \ln|x-2| - \frac{1}{8} \ln|x+6| + C$

(D) $\frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + C$

$= \int \frac{1}{(x+2)^2 + 9} dx = \int \frac{1}{(x+2)^2 + (3)^2} dx$

$\frac{du}{dx} = 1$
 $du = dx$

$= \int \frac{1}{u^2 + a^2} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$

$= \frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + C$

vertical asymptote at $x=1$ (improper integral)

#31. Evaluate $\int_0^5 \frac{1}{(x-1)^{1/5}} dx = \lim_{b \rightarrow 1^-} \int_0^b (x-1)^{-1/5} dx + \lim_{c \rightarrow 1^+} \int_c^5 (x-1)^{-1/5} dx$

side:
 $\int (x-1)^{-1/5} dx = \frac{(x-1)^{4/5}}{(4/5)} = \frac{5}{4}(x-1)^{4/5}$

- (A) 0
- (B) 2.539**
- (C) 1.625
- (D) diverges

$$= \lim_{b \rightarrow 1^-} \left[\frac{5}{4} (x-1)^{4/5} \right]_0^b + \lim_{c \rightarrow 1^+} \left[\frac{5}{4} (x-1)^{4/5} \right]_c^5$$

$$= \lim_{b \rightarrow 1^-} \left[\left(\frac{5}{4} (b-1)^{4/5} \right) - \left(\frac{5}{4} (0-1)^{4/5} \right) \right] + \lim_{c \rightarrow 1^+} \left[\left(\frac{5}{4} (5-1)^{4/5} \right) - \left(\frac{5}{4} (c-1)^{4/5} \right) \right]$$

$$= 0 - \frac{5}{4} (-1)^{4/5} + \frac{5}{4} (4)^{4/5} - 0 = 2.539$$

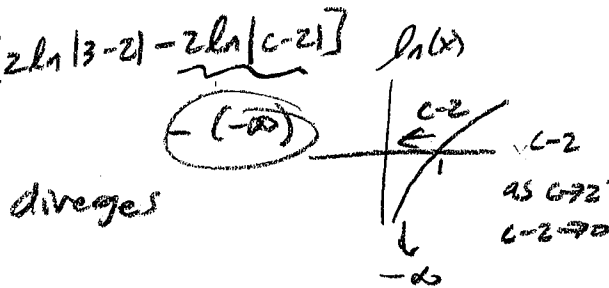
#32. Evaluate $\int_1^3 \frac{2}{x-2} dx$ vertical asymptote at $x=2$ (improper integral)

- (A) 0
- (B) 1
- (C) 2
- (D) diverges**

$$= \lim_{b \rightarrow 2^-} \int_1^b \frac{2}{x-2} dx + \lim_{c \rightarrow 2^+} \int_c^3 \frac{2}{x-2} dx$$

$$= \lim_{b \rightarrow 2^-} \left[2 \ln|x-2| \right]_1^b + \lim_{c \rightarrow 2^+} \left[2 \ln|x-2| \right]_c^3$$

$$= \lim_{b \rightarrow 2^-} \left[2 \ln|b-2| - 2 \ln|1-2| \right] + \lim_{c \rightarrow 2^+} \left[2 \ln|3-2| - 2 \ln|c-2| \right]$$



#33. Evaluate $\int_4^\infty \frac{x}{x^{7/2}} dx$ (improper integral)

- (A) 0.083**
- (B) 2,304
- (C) 12.048
- (D) diverges

$$= \lim_{b \rightarrow \infty} \int_4^b x^{-5/2} dx$$

$$= \lim_{b \rightarrow \infty} \left[\frac{x^{-3/2}}{(-3/2)} \right]_4^b = \lim_{b \rightarrow \infty} \left[-\frac{2}{3} x^{-3/2} \right]_4^b$$

$$= \lim_{b \rightarrow \infty} \left[\left(-\frac{2}{3} b^{-3/2} \right) - \left(-\frac{2}{3} 4^{-3/2} \right) \right]$$

$$= 0 + \frac{2}{3} (4)^{-3/2} = 0.0833$$

#34. Find the derivative of the function $F(x) = \int_2^{e^{3x}} \cos(t^2) dt$ when $x = 0.2$

- (A) -0.984
- (B) -5.379
- (C) -1.793
- (D) 2.345

$$F'(x) = \frac{d}{dx} \left[\int_2^{e^{3x}} \cos(t^2) dt \right] = \cos((e^{3x})^2) \cdot \underbrace{e^{3x} \cdot 3}_{\text{chain rule for } e^{3x}}$$

$$F'(0.2) = \cos((e^{3(0.2)})^2) \cdot e^{3(0.2)} \cdot 3$$
$$= -5.379$$

#35. Find the average value of the function $f(x) = -x^4 + 2x^2 + 4$ on the closed interval $[-2, 1]$.

- (A) 11.4
- (B) 3.000
- (C) 3.8
- (D) 0.5

$$AV = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{1-(-2)} \int_{-2}^1 (-x^4 + 2x^2 + 4) dx$$

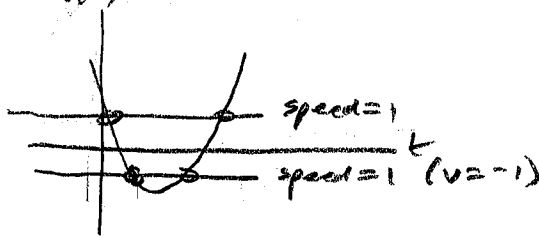
$$= 3.8$$

(For #36-#38) A particle moves along a line such that its position in meters at time t in seconds is given by $s(t) = 2t^3 - 5t^2 + 2t$

#36. When is the speed of the particle $1 \frac{m}{s}$?

- (A) $t = 2.142 \text{ s}$
- (B) $t = 0.107 \text{ s}, t = 1.560 \text{ s}$
- (C) $t = -0.281 \text{ s}, t = 1.000 \text{ s}, t = 1.781 \text{ s}, t = 2.142 \text{ s}$
- (D) $t = 0.107 \text{ s}, t = 0.392 \text{ s}, t = 1.274 \text{ s}, t = 1.560 \text{ s}$

$$v(t) = s'(t) = 6t^2 - 10t + 2$$



#37. How is the particle moving at time $t = 0.4$ seconds?

- (A) The particle is moving left
- (B) The particle is moving right
- (C) The particle is at rest
- (D) Cannot be determined from the information given

$$v(0.4) = -1.04 < 0 \quad (\text{left})$$

#38. Is the particle speeding up or slowing down at time $t = 0.4$ seconds?

- (A) The particle is slowing down
- (B) The particle is speeding up
- (C) The particle is at rest
- (D) Cannot be determined from the information given

$$a(t) = 12t - 10$$

$$a(0.4) = -5.2$$

$$v(0.4) = -1.04$$

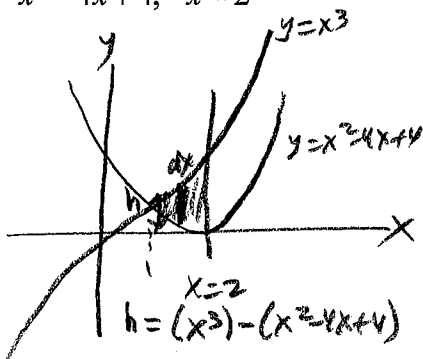
same sign

= speeding up

For #39-40, find the area bounded by the given curves (use math9 to evaluate the integrals)

#39. $y = x^3$, $y = x^2 - 4x + 4$, $x = 2$

- (A) 1.333
- (B) 3.417**
- (C) 56.369
- (D) 53.228



$$A = \int h \, dx$$

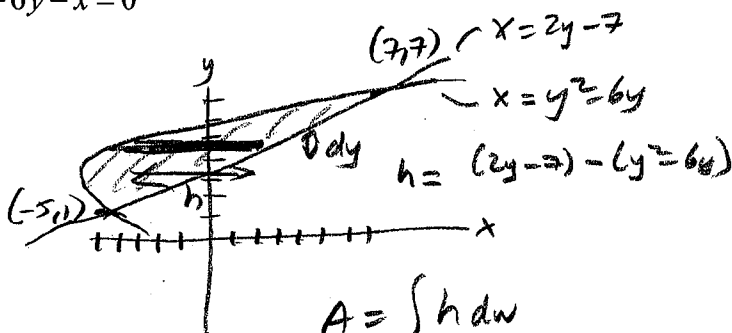
$$= \int_1^2 [x^3 - (x^2 - 4x + 4)] \, dx$$

$$= 3.417$$

intersection (by calculator)
at (1, 1)

#40. $x - 2y + 7 = 0$, $y^2 - 6y - x = 0$

- (A) 144
- (B) 36**
- (C) 32.667
- (D) 588.106



$$A = \int h \, dy$$

$$= \int_1^7 [(2y - 7) - (y^2 - 6y)] \, dy$$

$$= 36$$

intersections:

$$x - 2y + 7 = 0$$

$$y^2 - 6y - x = 0$$

$$x = y^2 - 6y, \text{ into } x - 2y + 7 = 0$$

$$y^2 - 6y - 2y + 7 = 0$$

$$y^2 - 8y + 7 = 0$$

$$(y - 1)(y - 7) = 0$$

$$y = 1 \quad y = 7 \quad (\& x = 2y - 7)$$

$$x = 2(1) - 7 \quad x = 2(7) - 7$$

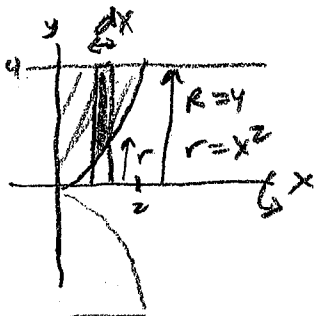
$$x = -5 \quad x = 7$$

$$(-5, 1) \quad (7, 7)$$

For #41-43, use the disk method to find the volume generated by rotating the region bounded by the given curves about the specified axis (use math9 to evaluate the integrals).

#41. $y = x^2$, $y = 4$, $x = 0$ about the x -axis

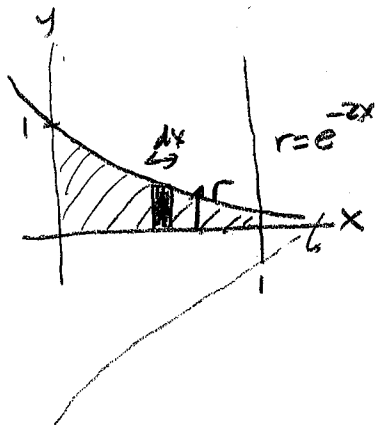
- (A) 80.425
- (B) 25.600
- (C) 53.617
- (D) 442.336



$$\begin{aligned}
 V &= \int \pi R^2 dx - \int \pi r^2 dx \\
 &= \int_0^2 \pi (4)^2 dx - \int_0^2 \pi (x^2)^2 dx \\
 &= 80.425
 \end{aligned}$$

#42. $y = e^{-2x}$, $y = 0$, $x = 1$, $x = 0$ about the x -axis

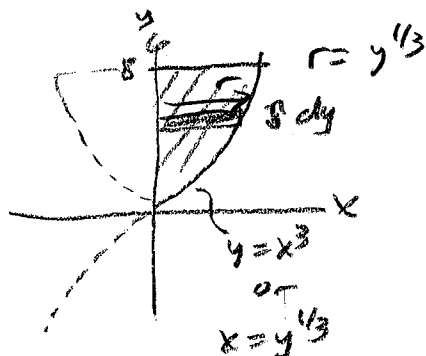
- (A) 1.358
- (B) 0.245
- (C) 0.771
- (D) 0.443



$$\begin{aligned}
 V &= \int \pi r^2 dx \\
 &= \int_0^1 \pi (e^{-2x})^2 dx \\
 &= 0.771
 \end{aligned}$$

#43. $y = x^3$, $y = 8$, $x = 0$, about the y -axis

- (A) 57.446
- (B) 20.106
- (C) 18.286
- (D) 60.319



$$\begin{aligned}
 V &= \int \pi r^2 dy \\
 &= \int_0^8 \pi (y^{1/3})^2 dy \\
 &= 60.319
 \end{aligned}$$