Sample Examination III

Section I Part A

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$ is

(A) 36

(B) 34

(C) 31

(D) 26

(E) 12

2. $\lim_{x \to 3} \dfrac{(3-x)^2}{(x-3)}$ is

(A) -2

(B) -1

(C) 0

(D) 1

(E) nonexistent

3. For a car traveling at a speed of s miles per hour, the fuel consumption of the car, $C(s)$, is measured in gallons per mile. What are the units of $\int_a^b C(s)\, ds$?

(A) gallons

(B) hours per gallon

(C) gallons per hour

(D) miles per hour per gallon

(E) gallons per miles per hour

4. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

(A) 40π

(B) 80π

(C) 800

(D) 800π

(E) 3200π

5. A particle moves along a straight line so that its velocity is given by $v(t) = t^2$. How far does the particle travel between $t = 1$ and $t = 3$?

(A) $\frac{1}{3}$ (B) $\frac{26}{3}$ (C) 8 (D) 26 (E) 27

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6. The derivative of $(4x)^3 \cdot (2x)^6$ is

(A) $72x^8$

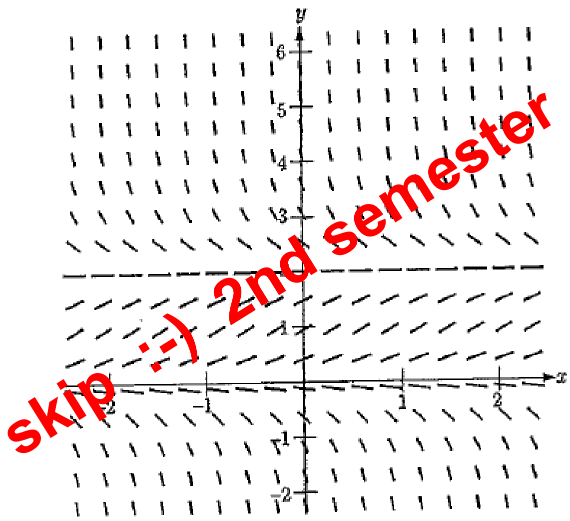
(B) $124x^{17}$

(C) $30x(4x)^2(2x)^5$

(D) $72x(4x)^2(2x)^5$

(E) $144(4x)^2(2x)^5$

7.



The slope field for a differential equation $\frac{dy}{dx} = f(y)$ is shown in the figure above.

Which statement is true about $y(x)$?

I. If $y(0) > 2$, then $\lim_{x\to\infty} y(x) \approx 2$

II. If $0 < y(0) < 2$, then $\lim_{x\to\infty} y(x) \approx 2$

III. If $y(0) < 0$, then $\lim_{x\to\infty} y(x) \approx 2$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

8. $\int_1^3 \frac{x}{x^2+1}\,dx =$

(A) $\ln 5$ (B) $\ln 10$ (C) $2\ln 2$ (D) $\frac{1}{2}\ln 5$ (E) $\ln\left(\frac{5}{2}\right)$

9. $\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right) =$

(A) $\frac{1}{x^2-1}$

(B) $-\frac{2x}{x^2-1}$

(C) $\frac{2x}{x^2-1}$

(D) $2x^3-2x$

(E) $2x-2x^3$

10. For $0 \le x < \frac{\pi}{2}$, an antiderivative of $2\tan x$ is

(A) $\ln(\sec 2x)$

(B) $2\sec^2 x$

(C) $\ln(\sec^2 x)$

(D) $2\ln(\cos x)$

(E) $\ln(2\sec x)$

11. If the substitution $u = x^2 + 1$ is made, the integral $\int_0^2 \frac{x^3}{x^2+1}\,dx =$

(A) $\int_1^5 \frac{u-1}{2u}\,du$

(B) $\int_0^2 \frac{u-1}{2u}\,du$

(C) $\int_1^5 \frac{u+1}{2u}\,du$

(D) $\int_1^5 \frac{2(u-1)}{u}\,du$

(E) $\int_0^2 \frac{2(u-1)}{u}\,du$

12. The equation of the line tangent to the curve $y = \frac{kx+8}{k+x}$ at $x = -2$ is $y = x + 4$. What is the value of k?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

13.

x	5	6	9	11	12
$f(x)$	10	7	11	12	8

The function f is continuous on the closed interval $[5, 12]$ and differentiable on the open interval $(5, 12)$ and f has the values given in the table above. Using the subintervals $[5, 6]$, $[6, 9]$, $[9, 11]$, and $[11, 12]$, what is the right-hand Riemann sum approximation to $\int_5^{12} f(x)\, dx$?

(A) 64

(B) 65

(C) 66

(D) 68.5

(E) 72

14. A function $f(x)$ has a vertical asymptote at $x = 2$. The derivative of $f(x)$ is positive for all $x \neq 2$. Which of the following statements are true?

I. $\lim_{x \to 2} f(x) = +\infty$

II. $\lim_{x \to 2^+} f(x) = +\infty$

III. $\lim_{x \to 2^-} f(x) = +\infty$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

15. If $y = 5^{(x^3-2)}$, then $\frac{dy}{dx} =$

(A) $(x^3 - 2)5^{(x^3-2)}$

(B) $3x^2(\ln 5)5^{(x^3-2)}$

(C) $(3x^2)5^{(x^3-2)}$

(D) $(\ln 5)5^{(x^3-2)}$

(E) $x^3(\ln 5)5^{(x^3-2)}$

16. If n is a positive integer, then $\lim\limits_{n\to+\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)$ is

(A) 0

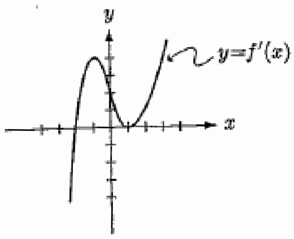
(B) $\frac{2}{\pi}$

(C) $\frac{\pi}{2}$

(D) 2

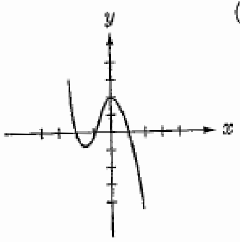
(E) 2π

17.



The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)



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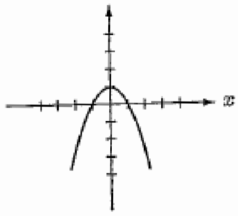

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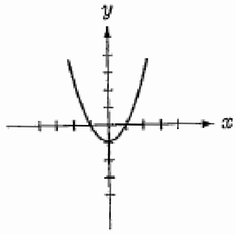

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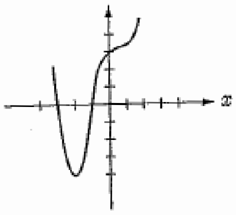
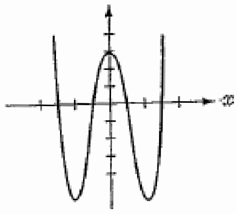


(E)



18. $\frac{d}{dx}\int_x^0 \frac{du}{1+u^2} =$

(A) $\frac{1}{x^2+1}$

(B) $\frac{-1}{x^2+1}$

(C) x^2+1

(D) $-x^2+1$

(E) $\text{Arctan}\, x$

19. Let $f(x)$ be a differentiable function defined only on the interval $-2 \le x \le 10$. The table below gives the value of $f(x)$ and its derivative $f'(x)$ at several points of the domain.

x	-2	0	2	4	6	8	10
$f(x)$	26	27	26	23	18	11	2
$f'(x)$	1	0	-1	-2	-3	-4	-5

The line tangent to the graph of $f(x)$ and parallel to the segment between the endpoints intersects the y-axis at the point

(A) $(0, 27)$

(B) $(0, 28)$

(C) $(0, 31)$

(D) $(0, 36)$

(E) $(0, 43)$

20. At each point (x, y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0, 4)$, then its equation is

(A) $y = e^{2x^2} + 4$

(B) $y = e^{2x^2} + 3$

(C) $y = 4e^{2x^2}$

(D) $y^2 = 2x^2 + 4$

(E) $y = 2x^2 + 4$

skip :-) 2nd semester

21. The equation of the tangent line to the curve $y = x^3 - 6x^2$ at its point of inflection is

(A) $y = -12x + 8$

(B) $y = -12x + 40$

(C) $y = 12x - 8$

(D) $y = -12x + 12$

(E) $y = 12x - 40$

22. If $0 < k < \pi$, then $\int_0^k \cos(2x)\, dx = \frac{1}{2}$ when $k =$

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{4}$

(C) $\frac{5\pi}{12}$

(D) $\frac{\pi}{2}$

(E) $\frac{3\pi}{4}$

23. Which of the following properties of the definite integral are true?

I. $\int_a^b kf(x)\, dx = k \int_a^b f(x)$ dx, k is a constant

II. $\int_a^b xf(x)\, dx = x \int_a^b f(x)\, dx$

III. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

24. If $f(x) = \sqrt{e^{2x} + 1}$, then $f'(0) =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

25. If $x^2y + yx^2 = 6$, then $\frac{d^2y}{dx^2}$ at the point $(1, 3)$ is

(A) -18

(B) -6

(C) 6

(D) 12

(E) 18

26. The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle?

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{8\pi}{3}$ (D) $\frac{32\pi}{3}$ (E) $\frac{64\pi}{3}$

27. A particle moves along the x-axis so that at any time t its position is given by $x(t) = (t+1)(t-3)^3$. For what values of t is the <u>velocity</u> of the particle increasing?

(A) $t > 3$ only

(B) $0 < t < 3$ only

(C) $1 < t < 3$ only

(D) $t < 1$ or $t > 3$

(E) $0 < t < 3$ or $t > 3$

28. At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where y is the number of bacteria present and k is a constant. The initial population is 1,500 and the population is quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days?

(A) 4

(B) 5

(C) 6

(D) 8

(E) 10

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Section I Part B

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. A graphing calculator is required for some questions on this part of the examination.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices, the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

39. How many zeros does the function $y = \sin(\ln x)$ have for $0 < x \leq 1$?

(A) One

(B) Two

(C) Three

(D) Four

(E) More than four

40. Let $f(x) = x^3 - 7x^2 + 25x - 39$ and let g be the inverse function of f. What is the value of $g'(0)$?

(A) $-\frac{1}{25}$ (B) $\frac{1}{25}$ (C) $\frac{1}{10}$ (D) 10 (E) 25

42. A rectangle is to be inscribed in a semicircle of radius 8, with one side lying on the diameter of the circle. What is the maximum possible area of the rectangle?

(A) $4\sqrt{2}$

(B) $8\sqrt{2}$

(C) 32

(D) $32\sqrt{2}$

(E) 64

44. At how many points do the graphs of the functions $f(x) = 5 + x^4$ and $g(x) = 5e^{0.2x}$ intersect?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

45. The line $y = mx + b$ with $b \geq 2$ is tangent to the graph of $f(x) = -2(x-2)^2 + 2$ at a point in the first quadrant. What are all possible values of b?

(A) $b = 2$ only

(B) $2 \leq b < 10$

(C) $2 \leq b < 12$

(D) $2 \leq b < 14$

(E) $2 \leq b < 20$

29. The average value of a continuous function $f(x)$ on the closed interval $[3, 7]$ is 12. What is the value of $\int_3^7 f(x)\, dx$?

(A) 3

(B) 4

(C) 12

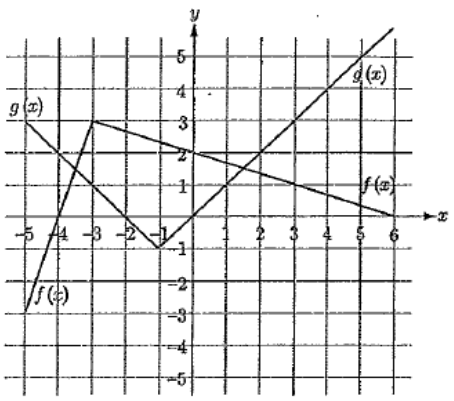
(D) 36

(E) 48

30. The region in the first quadrant enclosed by the x-axis, the line $x = \pi$, and the curve $y = \cos(\cos x)$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 1.921 (B) 3.782 (C) 6.040 (D) 8.130 (E) 23.781

31.



The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(3) =$

(A) $-\frac{2}{9}$

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{1}{3}$

(E) $\frac{8}{27}$

32. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = 0$ and $x = \pi/2$ is divided into two regions by the line $x = c$. If the area of the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \pi/2$, then c must be

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{9}$

(E) $\frac{5\pi}{18}$

33. For what values of x is the function $f(x) = 5 + 15x + 6x^2 - x^3$ decreasing?

(A) $-1 < x < 5$

(B) $-5 < x < 1$

(C) $x < -5$ or $x > 1$

(D) $x < -1$ or $x > 5$

(E) All real numbers

34. If f is an antiderivative of $\frac{\tan^2 x}{x^2+1}$ such that $f(1) = \frac{1}{2}$, then $f(0) =$

(A) 0

(B) 0.155

(C) 0.345

(D) 0.845

(E) 1

35. The second derivative of a function is given by $f''(x) = 0.5 + \cos x - e^{-x}$. How many points of inflection does the function $f(x)$ have on the interval $0 \leq x \leq 20$?

(A) None

(B) Three

(C) Six

(D) Seven

(E) Ten

36. The local linear approximation of a function f will always be greater than or equal to the function's value if, for all x in an interval containing the point of tangency,

(A) $f' < 0$

(B) $f' > 0$

(C) $f'' < 0$

(D) $f'' > 0$

(E) $f' = f'' = 0$

37. Let f be the function given by $f(x) = 5 + 5.8\sin\left(\frac{\pi x}{4}\right) - 15.7\cos\left(\frac{\pi x}{3}\right)$. For $0 \le x \le 12$, f is increasing most rapidly when $x =$

(A) 1.328

(B) 4.434

(C) 6.000

(D) 7.566

(E) 10.672

38. If $\lim_{x \to 3} \frac{g(3) - g(x)}{3 - x} = -0.628$, then near the point where $x = 3$, the graph of $g(x)$

(A) is decreasing

(B) is increasing

(C) is concave upwards

(D) is concave downwards

(E) has a point of inflection

41. For all $x > 0$, if $f(\ln x) = x^2$, then $f(x) =$

(A) $\sqrt{e^x}$

(B) $2 \ln x$

(C) $e^{\sqrt{x}}$

(D) $\sqrt{\ln x}$

(E) e^{2x}

43. If $f(x) = \begin{cases} e^{-x} + 2, & \text{for } x < 0 \\ ax + b, & \text{for } x \geq 0 \end{cases}$ is differentiable at $x = 0$, then $a + b =$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

SECTION II - FREE-RESPONSE QUESTIONS
GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numerical or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 45 Minutes, Questions 1,2,3

> A graphing calculator is required to some problems or parts of problems.
>
> During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided for that part in the pink test booklet.
>
> On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clarly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.
>
> Do not go on to Part B until you are told to do so .

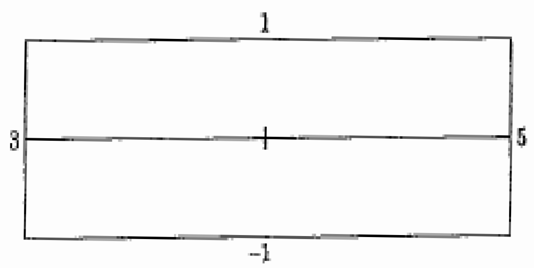
Section II Part A: Graphing Calculator MAY BE USED.

1. A particle is moving along the y-axis so that at any time t its velocity is given by

$$v(t) = (t-4)^4 \sin(t-4)$$

(a) Find the velocity, speed, and acceleration of the particle at $t = 3$.

(b) In the viewing window provided below, sketch the graph of $v(t)$.



(c) In the interval $3 \leq t \leq 5$ is the velocity increasing or decreasing? Use your graph to justify your answer.

(d) Find the distance the particle travels from $t = 3$ to $t = 5$. Show how you arrived at your answer.

Section II Part A: Graphing Calculator MAY BE USED.

2. Sand is being poured into a bin that is initially empty. During the work day, for $0 \leq t < 9$ hours, the sand pours into the bin at the rate given by

$$S(t) = \frac{5000}{t^3 + 50} \text{ cubic meters per hour.}$$

After one hour, for $1 \leq t < 9$, sand is removed from the bin at the rate of

$$R(t) = 23.967\sqrt{t} \text{ cubic meters per hour.}$$

(a) How much sand is put into the bin during the work day? Include units of measure.

(b) Find $S(6) - R(6)$; include units of measure. Explain what this number means in the context of the problem.

(c) Explain why the maximum amount of sand is in the bin when $S(t) = R(t)$.

(d) How much sand is in the bin at the end of the work day?

Section II Part A: Graphing Calculator MAY BE USED.

3. It is estimated that at the current rate of consumption, r gallons per year, the oil supply of the earth will last 200 years. However, the rate of consumption, $R(t)$, is increasing at the rate of 5% per year; that is $\frac{dR}{dt} = 0.05R$.

(a) In terms of r, how many gallons of oil is currently available?

(b) Use the given differential equation to find $R(t)$.

(c) If no additional oil is discovered, how long, to the nearest year, will the current oil supply actually last? Show how you arrived at your solution.

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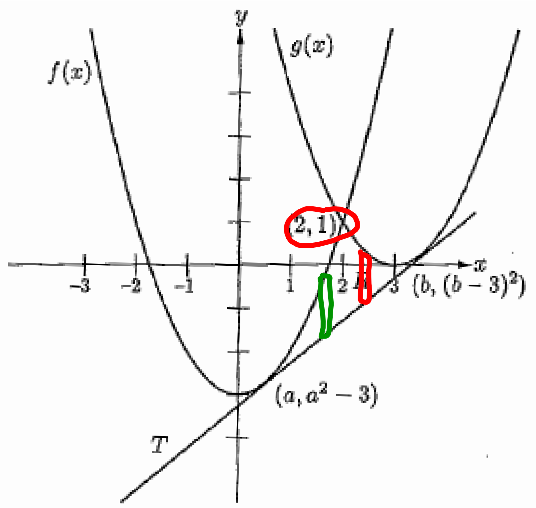
SECTION II PART B: 45 Minutes, Questions 4,5,6

Write you solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

4.



The region, R, is bounded by the graphs of $f(x) = x^2 - 3$, $g(x) = (x-3)^2$, and the line, T, as shown in the figure above. T is tangent to the graph of f at the point $(a, a^2 - 3)$ and tangent to the graph of g at the point $(b, (b-3)^2)$.

(a) Show that $a = b - 3$.

(b) Find the numerical value of a and b.

(c) Write an equation of the line T.

(d) Set up, but do not evaluate, an integral expression that gives the area of region R.

Section II Part B: Graphing Calculator MAY NOT BE USED.

5. Let $f(x)$ be the function defined by $f(x) = k + 12x + 3x^2 - 2x^3$, where k is a constant.

(a) On what interval is the function increasing? Justify your answer.

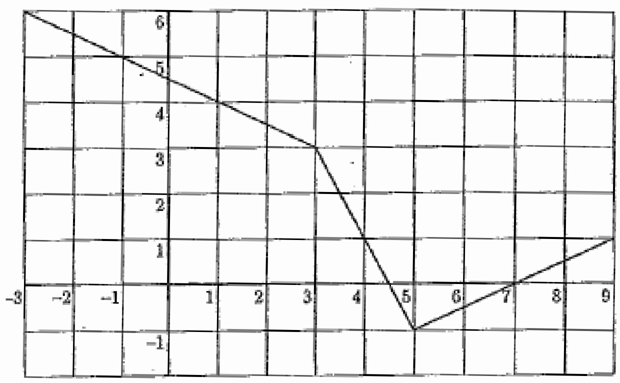
(b) If the relative maximum value of f is 4, what is the value of k?

(c) Find the interval where the function is concave up. Justify your answer.

(d) Find the relative minimum value of the function.

Section II Part B: Graphing Calculator MAY NOT BE USED.

6.



The graph of f

Let f be a function defined on the closed interval $[-3, 9]$. The graph of f, consisting of three line segments is shown above. Let $g(x) = \int_0^x f(t)\, dt$.

(a) Find $g(4.5)$, $g'(4.5)$, and $g''(4.5)$.

(b) Find the average value of f on the interval $[-3, 5]$. Show the work that leads to your answer.

(c) Find the x-coordinate of any points of inflection of g. Justify your answer.

(d) Find the coordinates of all maximum points of g.