8th edition

Sample Examination II

Section I Part A

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

skip #2 (2nd semester topic)

Part I - M/C calculator not allowed

1. $\int_0^2 (2x^3 + 3)\, dx =$

$\left[\frac{1}{2}x^4 + 3x\right]_0^2$

$\left(\frac{1}{2}2^4 + 3(2)\right) - 0$

$8 + 6$

14

(A) 8

(B) 11

(C) 14

(D) 20

(E) 24

2. The slope field for the differential equation $\frac{dy}{dx} = \frac{3y}{xy + 5x}$ will have vertical segments when

(A) $x = 0$, only

(B) $y = 0$, only

(C) $y = -5$, only

(D) $y = 5$, only

(E) $x = 0$ or $y = -5$

SKIP (2nd semester topic)

In case you are interested... vertical segments occur when slope is undefined, when denominator = 0

$xy + 5x = 0$

$x(y+5) = 0$

$x = 0$, $y = -5$

3. Suppose that f is a continuous function defined for all real numbers x and $f(-5) = 3$ and $f(-1) = -2$. If $f(x) = 0$ for one and only one value of x, then which of the following could be x?

f is continuous so Intermediate Value Theorem applies.

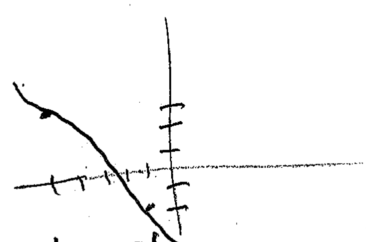
(A) -7

(B) -2

(C) 0

(D) 1

(E) 2



$f(x) = 0$
for $(-5, -1)$

4. If $f(x) = (2+3x)^4$, then the fourth derivative of f is

(A) 0

(B) $4!(3)$

(C) $4!(3^4)$

(D) $4!(3^5)$

(E) $4!(2+3x)$

$f'(x) = 4(2+3x)^3 \cdot 3$

$f''(x) = 4 \cdot 3(2+3x)^2 \cdot 3^2$

$f'''(x) = 4 \cdot 3 \cdot 2(2+3x)^1 \cdot 3^3$

$f^{(4)}(x) = 4 \cdot 3 \cdot 2(3) \cdot 3^3$

$= 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3^4$

$= \boxed{4! \cdot 3^4}$

5. At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum?

(A) 0 and -2 only

(B) 0 and 2 only

(C) 0 only

(D) -2 and 2 only

(E) -2, 0, and 2

$f'(x) = 4x^3 - 16x = 0$

$f''(x) = 12x^2 - 16$

$4x(x^2-4) = 0$

$4x(x-2)(x+2) = 0$

$x = 0$, $\boxed{x = 2, \; x = -2}$

$f''(0)$ $f''(2)$ $f''(-2)$

$(-)$ $(+)$ $(+)$

max min min

6. $\int \sqrt{x}(x+2)\,dx =$ $\int x^{1/2}(x^1+2)dx = \int (x^{3/2}+2x^{1/2})dx$

(A) $\sqrt{x^3}+2\sqrt{x}+C$ (B) $\frac{2}{5}x^{\frac{3}{2}}+\frac{4}{3}x^{\frac{1}{2}}+C$ (C) $\frac{3}{2}\sqrt{x}+\frac{1}{\sqrt{x}}+C$

$\boxed{\text{(D)}}$ $\frac{2}{5}x^{\frac{5}{2}}+\frac{4}{3}x^{\frac{3}{2}}+C$ (E) $\frac{x^2}{2}\left(\frac{2}{3}x^{\frac{3}{2}}+2x\right)+C$

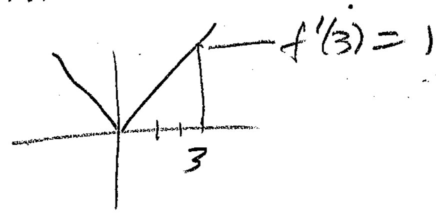
$= \frac{2}{5}x^{5/2}+2\frac{2}{3}x^{3/2}+c$

$= \frac{2}{5}x^{5/2}+\frac{4}{3}x^{3/2}+C$

7. The $\lim\limits_{h\to 0}\frac{|x+h|-|x|}{h}$ at $x=3$ is

limit is derivative of $f(x)=|x|$

so $f'(3)$



(A) -1

(B) 0

$\boxed{\text{(C)}}$ 1

(D) 3

(E) nonexistent

8. The function $y = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b?

(A) -6

(B) -1

(C) 1

(D) 4

(E) 6

horiz tan $= f'(x) = 0$

pt. of inflection $= f''(x) = 0$

$f'(x) = 4x^3 + 2bx + 8 = 0$

$f''(x) = 12x^2 + 2b = 0$

$-12x^2 = 2b$ substitution $-12x^2$ for $2b$ into $f'(x)$

$4x^3 + (-12x^2)x + 8 = 0$

$-8x^3 + 8 = 0$

$-8(x^3 - 1) = 0$

$x^3 = 1$

$x = 1$ so $2b = -12(1)^2$

$b = -6$

9. If $y = 7$ is a horizontal asymptote of a rational function f, then which of the following must be true?

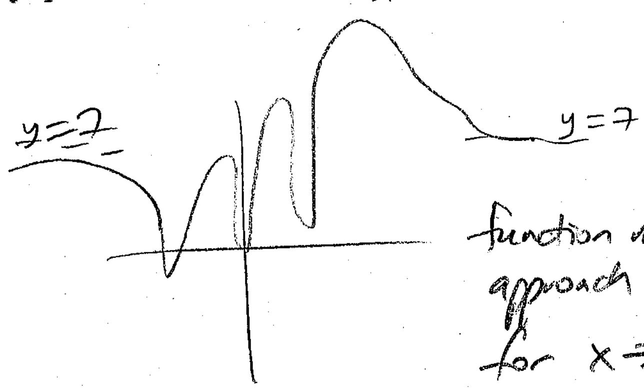
(A) $\lim\limits_{x \to 7} f(x) = \infty$

(B) $\lim\limits_{x \to -\infty} f(x) = -7$

(C) $\lim\limits_{x \to 0} f(x) = 7$

(D) $\lim\limits_{x \to 7} f(x) = 0$

(E) $\lim\limits_{x \to \infty} f(x) = 7$



function must approach $y=7$ for $x \to \infty$ or $x \to -\infty$

10. Let $f(x) = e^{(x^2)}$. At how many points in the interval $[-a, a]$, does the instantaneous rate of change of f equal the average rate of change of f?

(A) None

$\boxed{\text{(B)}}$ One

(C) Two

(D) Three

(E) More than three

inst. rate of change = avg rate of change

$$f'(x) = \frac{f(a)-f(-a)}{a-(-a)}$$

$$e^{x^2}(2x) = \frac{e^{a^2}-e^{(-a)^2}}{a+a} = \frac{e^{a^2}-e^{a^2}}{2a} = \frac{0}{2a} = 0$$

$$2x\underbrace{e^{x^2}}_{\text{never zero}} = 0$$

so $x = 0$ (one place)

11. Let f be the function given by $f(x) = x^3$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[-1, 2]$?

(A) 0 only

$\boxed{\text{(B)}}$ 1 only

(C) $\sqrt{3}$ only

(D) -1 and 1

(E) $-\sqrt{3}$ and $\sqrt{3}$

$f(-1) = (-1)^3 = -1$

$f(2) = (2)^3 = 8$

secant slope $= \dfrac{8-(-1)}{2-(-1)} = \dfrac{9}{3} = 3$

$f'(x) = 3x^2$, $f'(c) = 3$

$3x^2 = 3$

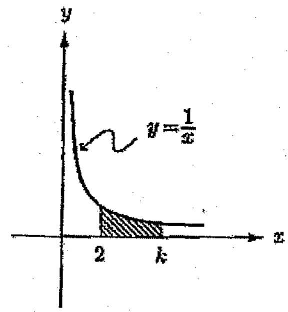
$x^2 = 1$

$x = -1, 1$

but Mean Value Theorem only guarantee values exist in the open interval $(-1, 2)$ so -1 is not included

(including -1 would be saying tangent line slope equals secant slope over the interval — not true except for linear func)

12.



$$A = \int_2^k \frac{1}{x}\,dx$$
$$= \left[\ln x\right]_2^k = \ln k - \ln 2 = \ln 4$$
$$\ln k = \ln 4 + \ln 2$$
$$\ln k = \ln(4 \cdot 2) = \ln(8)$$
$$k = 8$$

For the figure above, the area of the shaded region is $\ln 4$ when k is

(A) 4

$\boxed{\text{(B)}}$ 8

(C) e

(D) e^2

(E) e^3

13. If $x + y = xy$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{x-1}$ (B) $\frac{y-1}{x-1}$ $\boxed{\text{(C)}}$ $\frac{1-y}{x-1}$

(D) $x+y-1$ (E) $\frac{2-xy}{y}$

implicit differentiation: (product rule)

$$\frac{d}{dx}[x] + \frac{d}{dx}[y] = x\frac{d}{dx}[y] + y\frac{d}{dx}[x]$$
$$1 + 1\cdot\frac{dy}{dx} = x\left(1\,\frac{dy}{dx}\right) + y\,(1)$$
$$1 + \frac{dy}{dx} = x\frac{dy}{dx} + y$$
$$\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$$
$$\frac{dy}{dx}(1-x) = y - 1$$
$$\frac{dy}{dx} = \frac{y-1}{1-x} = \frac{-(1-y)}{-(x-1)} = \frac{1-y}{x-1}$$

14. If f and g are continuously differentiable functions defined for all real numbers, which of the following definite integrals is equal to $f(g(4)) - f(g(2))$?

(A) $\int_2^4 f'(g(x))\,dx$

(B) $\int_2^4 f(g(x))f'(x)\,dx$

(C) $\int_2^4 f(g(x))g'(x)\,dx$

$\boxed{\text{(D)}}$ $\int_2^4 f'(g(x))g'(x)\,dx$

(E) $\int_2^4 f(g'(x))g'(x)\,dx$

$\int_2^4 f'(g(x))\,g'(x)dx$

$u = g(x)$

$du = g'(x)dx$

$\int_{x=2}^{x=4} f'(u)\,du = f(g(4)) - f(g(2))$

15. The velocity of a particle moving along the y-axis is given by $v(t) = 8 - 2t$ for $t \geq 0$. The particle moves upward until it reaches the origin and then moves downward. The position of the particle at any time t is given by

$\boxed{\text{(A)}}$ $-t^2 + 8t - 16$

(B) $-t^2 + 8t + 16$

(C) $2t^2 - 8t - 16$

(D) $8t - 2t^2$

(E) $8t - t^2$

$s(t) = \int v(t)dt = \int (8-2t)dt = 8t - t^2 + C$

changes direction when $v=0$:

$8 - 2t = 0$

$2t = 8$

at $t = 4$

changes direction at $y=0$, so

$s(4) = 0$

$8(4) - (4)^2 + C = 0$

$32 - 16 + C = 0$

$16 + C = 0$

$C = -16$

So $s(t) = -t^2 + 8t - 16$

16. If the substitution $u = \sqrt{x-1}$ is made, the integral $\int_2^5 \frac{\sqrt{x-1}}{x}\,dx =$

(A) $\int_2^5 \frac{2u^2}{u^2+1}\,du$

(B) $\int_1^2 \frac{u^2}{u^2+1}\,du$

(C) $\int_1^2 \frac{u^2}{2(u^2+1)}\,du$

(D) $\int_2^5 \frac{u}{u^2+1}\,du$

$\boxed{\text{(E)}}$ $\int_1^2 \frac{2u^2}{u^2+1}\,du$

$u = (x-1)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(x-1)^{-1/2}$

$du = \frac{1}{2\sqrt{x-1}}\,dx$

So $dx = 2\sqrt{x-1}\,du$

$u = \sqrt{x-1}$

$u^2 = x-1$

$x = u^2+1$

$\int \frac{\sqrt{x-1}}{x}\,dx = \int \frac{\sqrt{x-1}}{u^2+1}\,2\sqrt{x-1}\,du$

$= \int \frac{2u^2}{u^2+1}\,du$

$x=5 \quad u=\sqrt{5-1}=2$

$x=2 \quad u=\sqrt{2-1}=1$

So $\int_1^2 \frac{2u^2}{u^2+1}\,du$

17. What are all values of x for which the function $f(x) = x^3 + 6x^2 + 9x + 1$ is increasing?

(A) $(-\infty, -3)$ only

(B) $(-3, -1)$ only

(C) $(-1, \infty)$ only

$\boxed{\text{(D)}}$ $(-\infty, -3) \cup (-1, \infty)$

(E) $(-\infty, -3) \cup (1, \infty)$

$f'(x) = 3x^2+12x+9$

critical when $f'(x)=0$ or DNE

$3x^2+12x+9=0$

$3(x^2+4x+3)=0$

$3(x+1)(x+3)=0$

$x=-1 \quad x=-3$

$-\infty$ test (-4) -3 test (-2) -1 test (0) ∞

$f'(-4) = (+)(-)(-) = +$

$f'(-2) = (+)(-)(+) = -$

$f'(0) = (+)(+)(+) = +$

increasing $(-\infty,-3)\cup(-1,\infty)$

18. If $\int_0^2 (2x^3 - kx^2 + 2k)\, dx = 12$, then k must be

(A) -3

(B) -2

(C) 1

(D) 2

$\boxed{\text{(E)}}$ 3

$$\left(\tfrac{1}{2}x^4 - \tfrac{k}{3}x^3 + 2kx\right]_0^2 = 12$$

$$\left(8 - \tfrac{8}{3}k + 4k\right) - 0 = 12$$

$$24 - 8k + 12k = 36$$

$$4k = 12$$

$$k = 3$$

19. $\int (\sec^2 x)(\tan^2 x)\, dx =$

$\boxed{\text{(A)}}$ $\frac{1}{3}\tan^3 x + C$

(B) $\tan^3 x + C$

(C) $\frac{1}{2}\tan^2 x + C$

(D) $\frac{1}{3}\sec^3 x + C$

(E) $\tan^2 x + C$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\, dx$

$\int u^2\, du$

$\frac{1}{3}u^3 + c$

$\frac{1}{3}\tan^3 x + c$

20. For $|x| < 1$, the derivative of $y = \ln\sqrt{1-x^2}$ is

(A) $\frac{x}{1-x^2}$ $\quad$ $\boxed{\text{(B)}}$ $\frac{x}{x^2-1}$ $\quad$ (C) $\frac{-1}{x^2-1}$ $\quad$ (D) $\frac{1}{2(1-x^2)}$ $\quad$ (E) $\frac{1}{\sqrt{1-x^2}}$

$$y' = \frac{1}{\sqrt{1-x^2}} \frac{d}{dx}\left[(1-x^2)^{1/2}\right]$$

$$\frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{-2x}{2(\sqrt{1-x^2})^2} = \frac{-x}{1-x^2} = \frac{-(x)}{-(x^2-1)} = \frac{x}{x^2-1}$$

21. A function whose derivative is a constant multiple of itself must be

(A) periodic

(B) linear

$\boxed{\text{(C)}}$ exponential (only family whose derivative is the same form as the function)

(D) quadratic

(E) logarithmic

23. The edge of a cube is increasing at the rate of 0.05 centimeters per second. In terms of the edge of the cube, s, what is the rate of change of the volume of the cube, in cubic centimeters per second?

(A) 0.05^3

(B) $0.05s^2$

(C) $0.05s^3$

$\boxed{\text{(D)}}$ $0.15s^2$

(E) $3s^2$

$V = x^3$

$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$

$\frac{dv}{dt} = 3(s)^2 0.05 = .15s^2$

24. The tangent line to the graph $y = e^{2-x}$ at the point $(1, e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

$\boxed{\text{(A)}}$ $2e$

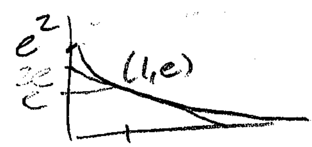
(B) $e^2 - 1$

(C) e^2

(D) $2e\sqrt{e}$

(E) $4e$

y int : (x=0) : $y = e^{2-0} = e^2$
of curve



tan line slope $m = y' = e^{(2-x)}(-1)$

at $x=1$ $m = -e^{2-1} = -e$

tan line equation:

$(y-e) = -e(x-1)$

$y - e = -ex + e$

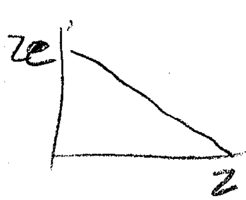
$y = -ex + 2e$

tan line y int $(x=0)$: $y = 2e$

tan line x int $(y=0)$: $-ex + 2e = 0$

$ex = 2e$

$x = 2$



$A_{triang} = \frac{1}{2}(2)2e = 2e$

25. If f is the function defined by $f(x) = \frac{5x^7}{7} + 4x^6 + 6x^5 + x + 1$, what are all the x-coordinates of the points of inflection of the graph of f?

(A) -2 only

$\boxed{\text{(B)}}$ 0 only

(C) 2 only

(D) -2 and 0 only

(E) -2, 0, and 2

$f'(x) = 5x^6 + 24x^5 + 30x^4 + 1$

$f''(x) = 30x^5 + 120x^4 + 120x^3 = 0$

$30x^3(x^2 + 4x + 4) = 0$

$30x^3(x+2)(x+2) = 0$

$x = 0 \quad x = -2$

test regions in $f''(x) = 30x^3(x+2)^2$

$-\infty$ ... -2 ... 0 ... ∞

(-3): $(+)(-)(+)$ $\quad (-1)$: $(+)(-)(+)$ $\quad (1)$: $(+)(+)(+)$

not an inflection point (sign doesn't change)

inflection point at $x = 0$

$\frac{24 \cdot 5}{120}$

26. A normal line to the graph of a function f at the point $(x, f(x))$ is defined to be the line perpendicular to the tangent line at that point. The equation of the normal line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

(A) $y + 12x = 38$

(B) $y - 4x = 10$

(C) $y + 2x = 4$

$\boxed{\text{(D)}}$ $y + 2x = 8$

(E) $y - 2x = -4$

tangent line slope:

$y' = m = \frac{1}{3}(x^2-1)^{-2/3}(2x)$ at $x = 3$

$= \frac{1}{3}(8)^{-2/3}(6)$

$= 2\frac{1}{8^{2/3}} = 2\frac{1}{(\sqrt[3]{8})^2} = 2\frac{1}{4} = \frac{1}{2}$

y-coord: $y = \sqrt[3]{3^2-1} = \sqrt[3]{8} = 2$

tangent line: $(y-2) = \frac{1}{2}(x-3)$

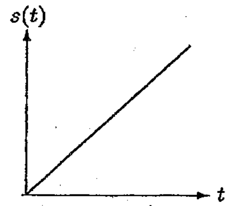
normal line: $(y-2) = -2(x-3)$

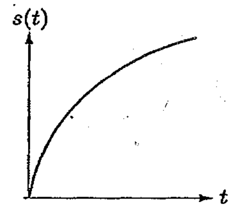
$y - 2 = -2x + 6$

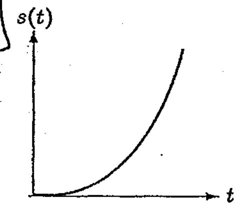
$y = -2x + 8$

$y + 2x = 8$

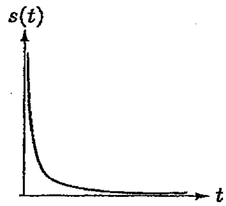
27. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?

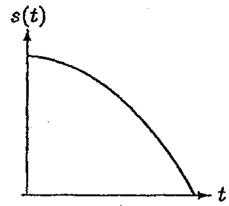
(A)


(B)


$\boxed{\text{(C)}}$


increasing
and
concave up

(D)


(E)


Answer

28. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n-1}{n}\right)^2\right] =$

Δx in a Riemann Sum:

$x_i = 0 + i\frac{1}{n}$ ← $\frac{0-1}{n}$

$f(x) = x^2$ $\int_0^1$

(A) $\int_0^1 \frac{1}{x^2}\,dx$

$\boxed{\text{(B)}}$ $\int_0^1 x^2\,dx$

(C) $\int_0^1 \frac{2}{x^2}\,dx$

(D) $\int_0^1 \frac{1}{x}\,dx$

(E) $\int_0^2 x^2\,dx$

x_i	$f(x)$	Δx
$1\cdot\frac{1}{n}$	$\left(\frac{1}{n}\right)^2$	$\frac{1}{n}$
$2\cdot\frac{1}{n}$	$\left(\frac{2}{n}\right)^2\cdot$	$\frac{1}{n}$
$3\cdot\frac{1}{n}$	$\left(\frac{3}{n}\right)^3\cdot$	$\frac{1}{n}$

so $\int_0^1 x^2\,dx$

CalcAB Sample Exam II (online version)

Part II - M/C calculator allowed

29. If f is a function such that $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

$\leftarrow = f'(x)$ at $x = a$

so $f'(a) = 0$

(A) $\lim_{x \to a} f(x)$ does not exist

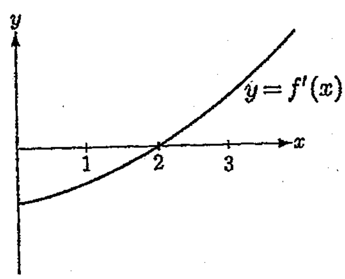
(B) $f(a)$ does not exist

$\boxed{\text{(C)}}$ $f'(a) = 0$

(D) $f(a) = 0$

(E) $f(x)$ is continuous at $x = 0$

30.



The graph of the derivative of a twice-differentiable function f is shown above. If $f(1) = -2$, which of the following is true?

$f'(2) = 0$

$-\infty$ — 2 — ∞

$f'(1) = (-)$ 2 $f'(3)$ $(+)$

$\boxed{\text{(A)}}$ $f(2) < f'(2) < f''(2)$

(B) $f''(2) < f'(2) < f(2)$

(C) $f'(2) < f(2) < f''(2)$

(D) $f(2) < f''(2) < f'(2)$

(E) $f'(2) < f''(2) < f(2)$

$f''(2) > 2$ because

$f'(x)$ is increasing at $x = 2$

f minimum at $x = 2$

so if $f(1) = -2$

$f(2)$ must be < -2

$\underset{(< -2)}{f(2)} < \underset{(=0)}{f'(2)} < \underset{(>0)}{f''(2)}$

31. Let f be a function that is everywhere differentiable. The value of $f'(x)$ is given for several values of x in the table below.

x	−10	−5	0	5	10
$f'(x)$	−2	−1	0	1	2

If $f'(x)$ is always increasing, which statement about $f(x)$ must be true?

$\boxed{\text{(A)}}$ $f(x)$ has a relative minimum at $x = 0$.

(B) $f(x)$ is concave downwards for all x.

(C) $f(x)$ has a point of inflection at $(0, f(0))$.

(D) $f(x)$ passes through the origin.

(E) $f(x)$ is an odd function.

$f'(0)=0$

$-\infty \quad (-) \quad 0 \quad (+) \quad \infty$

$f'(x)$ always increasing

f minimum at $x=0$

Answer

not B: f' increasing means $f''(x) > 0$ (concave up)

not C: f' always increasing so $f''(x)$ always > 0 (no sign change) so no points of inflection

not D: we aren't given any value for $f(x)$, so graph could shift up/down — don't know if/where it crosses the x-axis.

not E: $f'(x)$ is not even — the derivative of an odd function must be even.

32. A certain species of fish will grow from x million to $x(15 - x)$ million each year. In order to sustain a steady catch each year, a limit of $x(15 - x) - x$ million fish are to be caught, leaving x million fish to reproduce each year. What is the number of fish which should be left to reproduce each year so that the maximum catch may be sustained from year to year?

(A) 5 million

$\boxed{\text{(B)}}$ 7 million

(C) 7.5 million

(D) 10 million

(E) 15 million

catch is $x(15-x)-x$

$15x - x^2 - x$

$C = 14x - x^2$

maximize C:

$C' = 14 - 2x$

$\underline{C'=0}$ $\quad$ $\underline{C' \text{ DNE}}$

$14 - 2x = 0$

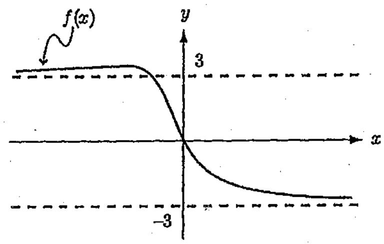
$2x = 14$

$\boxed{x = 7}$

$C'' = -2$ concave down, this is a maximum

Answer

33.



The figure above shows the graph of a function $f(x)$ which has horizontal asymptotes of $y = 3$ and $y = -3$. Which of the following statements are true?

I. $f'(x) < 0$ for all $x \geq 0$ true

II. $\lim_{x \to +\infty} f'(x) = 0$ true

III. $\lim_{x \to -\infty} f'(x) = 3$ false (limit of derivative 'slope' would also be zero)

(A) I only

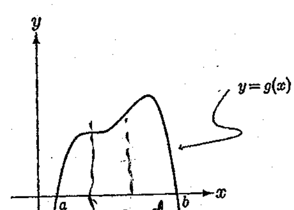
(B) II only

(C) III only

$\boxed{\text{(D)}}$ I and II only

(E) I, II, and III

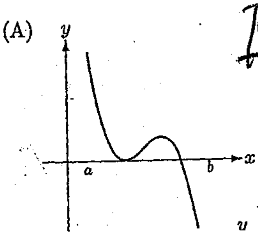
34.



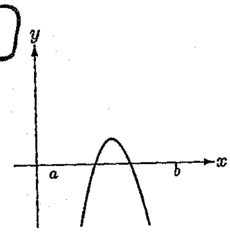
concave down up down

Let $g(x) = \int_a^x f(t)\,dt$, where $a \leq x \leq b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of $y = \frac{d}{dx} f(x)$ on $[a, b]$?

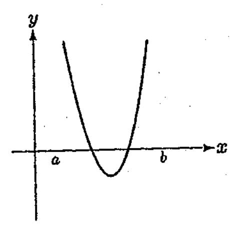
(A)



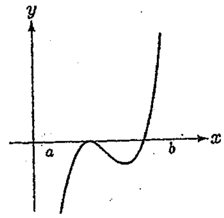
$\boxed{\text{(B)}}$



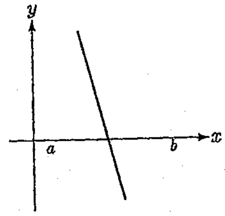
(C)



(D)



(E)



$g'(x) = \frac{d}{dx} \int_a^x f(t)\,dt$

$g'(x) = f(x)$

$y = \frac{d}{dx} f(x)$

would be

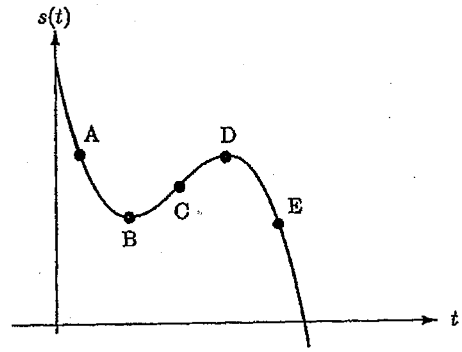
$y = g''(x)$

from provided graph concavity, sign of y

$-$ $+$ $-$

matches $\boxed{B}$

35.



The graph above shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time. Which of the points marked is closest to the point where the acceleration first becomes negative?

(A) A

(B) B

$\boxed{\text{(C)}}$ C

(D) D

(E) E

acceleration swithes to negative at an inflection point in the s(t) graph (when concavity switches from positive to negative) at point C

36. The <u>derivative</u> of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$ for $0 \le x \le 5$.

At what value of x is $f(x)$ an absolute minimum?

(A) For no value of x

(B) 0

(C) 0.618

(D) 1.623

$\boxed{\text{(E)}}$ 5

local extrema when $f'(x) = 0$ or DNE

$e^x(-x^3+3x)-3=0$

calculator graph shows zeros at $x = 0.61773884$

$x = 1.6234385$

0 — .6177 — 1.6234 — 5

test $f'(0.5)$ test $f'(1)$ test $f'(2)$

$-.733$ 2.4366 -17.78

local min at $x = .618$

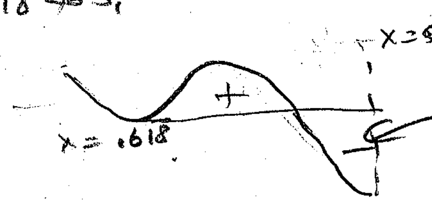
abs min might be at $x=5$ though

to check $x=5$, evaluate

$\int_{.618}^{5} (e^x(-x^3+3x)-3)\,dx$ in calculator (math 9)

$= -9218.575707$ is accumulation of $f(x)$ values from .618 to 5,

since this is negative...



this area must be bigger, so $f(5)$ must be lower than $f(.618)$

37.

x	$f(x)$
3.99800	1.15315
3.99900	1.15548
4.00000	1.15782
4.00100	1.16016
4.00200	1.16250

The table above gives values of a differentiable function f. What is the approximate value of $f'(4)$?

$$f'(4) \approx \frac{f(4.001) - f(4.000)}{4.001 - 4.000} = \frac{1.16016 - 1.15782}{.001}$$

$$= 2.34$$

(A) 0.00234

(B) 0.289

(C) 0.427

$\boxed{\text{(D)}}$ 2.340

(E) $f'(4)$ cannot be approximated from the information given.

38. Consider the function $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

In order for $f(x)$ to be continuous at $x = 0$, the value of k must be

(A) 0

(B) 1

(C) -1

(D) π

(E) a number greater than 1

$$k = \lim_{x \to 0} \frac{\sin x}{x} \quad \frac{0}{0}$$

by L'Hopital's rule.

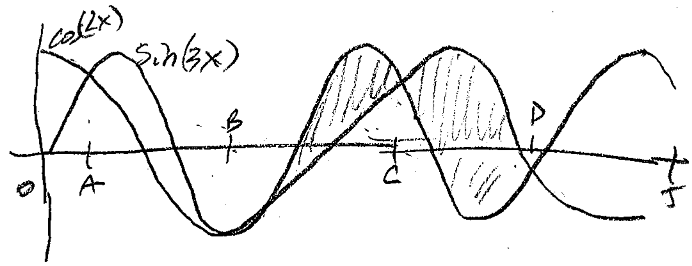
$$= \lim_{x \to 0} \frac{\cos x}{1} = \frac{1}{1}$$

$$= 1$$

so $k = 1$

39. In the interval $0 \le x \le 5$ the graphs of $y = \cos 2x$ and $y = \sin 3x$ intersect four times. Let A, B, C, and D be the x-coordinates of these points so that $0 < A < B < C < D < 5$. Which of the definite integrals below represents the largest number?

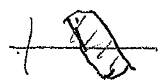
calculator graph:



(A) $\int_0^A (\cos 2x - \sin 3x)\, dx$ 

(B) $\int_A^B (\sin 3x - \cos 2x)\, dx$ 

(C) $\int_B^C (\sin 3x - \cos 2x)\, dx$ 

(D) $\boxed{\text{(D)}}$ $\int_C^D (\cos 2x - \sin 3x)\, dx$ 

(E) $\int_C^D (\sin 3x - \cos 2x)\, dx$ $-$ of E

visually, most + is either C or D, intersections

B: $x = 1.5708$ C: $x = 2.8274$ D: $x = 4.08407$

math 9: $\int_{1.5708}^{2.8274} (\sin 3x - \cos 2x)\, dx$ .489821

math 9: $\int_{2.8274}^{4.08407} (\cos 2x - \sin 3x)\, dx$ $\boxed{1.2823}$

40. The function $f(x) = \tan(3^x)$ has one zero in the interval $[0, 1.4]$. The derivative at this point is

(A) 0.411

(B) 1.042

$\boxed{\text{(C)}}$ 3.451

(D) 3.763

(E) undefined

$f'(x) = \sec^2(3^x) \cdot 3^x \ln(3)$

by calculator, zero at $x = 1.041978$

$f'(1.041978) = \dfrac{1}{(\cos(3^{1.041978}))^2}\, 3^{1.041979} \ln(3)$

$= 3.45139$

41.

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

For the function whose values are given in the table above, $\int_0^6 f(x)\ dx$ is approximated by a Riemann Sum using the value at the midpoint of each of three intervals of width 2. The approximation is

interval	x	f(x)	· width	= A
[0,2]	1	0.25	· 2	= .5
[2,4]	3	0.68	· 2	= 1.36
[4,6]	5	0.95	· 2	= 1.9
				3.76

(A) 2.64

(B) 3.64

(C) 3.72

(D) 3.76

(E) 4.64

Fundamental Thm of Calculus:

$$\frac{d}{dx}\int_a^{f(x)} g(t)dt = g(f(x))\cdot f'(x)$$

42. $\dfrac{d}{dx}\displaystyle\int_x^{x^3} \sin(t^2)\ dt =$ $\quad \frac{d}{dx}\int_x^0 \sin(t^2)dt + \frac{d}{dx}\int_0^{x^3}\sin(t^2)dt$

$$-\frac{d}{dx}\int_0^x \sin(t^2)dt + \frac{d}{dx}\int_0^{x^3}\sin(t^2)dt$$

$$-\sin(x^2)\cdot(1) + \sin((x^3)^2)\cdot 3x^2$$

$$-\sin(x^2) + 3x^2\sin(x^6)$$

(A) $\sin(x^6) - \sin(x^2)$

(B) $6x^2\sin(x^3) - 2\sin x$

(C) $3x^2\sin(x^6) - \sin(x^2)$

(D) $6x^5\sin(x^6) - 2\sin(x^2)$

(E) $2x^3\cos(x^6) - 2x\cos(x^2)$

43. A tank is being filled with water at the rate of $300\sqrt{t}$ gallons per hour with $t > 0$ measured in hours. If the tank is originally empty, how many gallons of water are in the tank after 4 hours?

(A) 600

(B) 900

(C) 1200

(D) 1600

(E) 2400

$\frac{dV}{dt} = 300\sqrt{t} = 300t^{1/2}$

$V(t) = \int 300t^{1/2}\,dt$

$V = 300\frac{2t}{3}^{3/2} + c$

$V = 200t^{3/2} + c \qquad V(0) = 0$

$0 = 200(0)^{3/2} + c, \qquad c = 0$

$V(t) = 200t^{3/2}$

$V(4) = 200(4)^{3/2} = 200(\sqrt{4})^3 = 200(8) = 1600$

44. The region in the first quadrant enclosed by the graphs of $y = x$ and $y = 2\sin x$ is revolved about the x-axis. The volume of the solid generated is

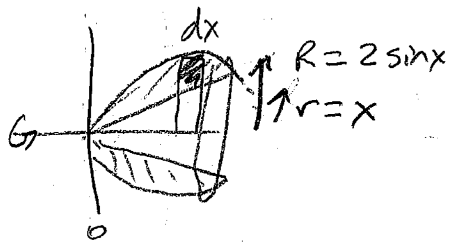
(A) 1.895

(B) 2.126

(C) 5.811

(D) 6.678

(E) 13.355



$V = \int \pi R^2\,dx - \int \pi r^2\,dx$

$= \pi \int_0^{1.8954943} ((2\sin x)^2 - x^2)\,dx$

$= 6.67773$

Intersections:

$x = 2\sin x$

$x > 0$ & $x = 1.8954943$

45. If $y = xe^x$, then $\frac{d^n y}{dx^n} =$

(A) e^x

(B) e^{nx}

(C) $(x+n)e^x$

(D) $x^n e^x$

(E) $(x+n^2)e^x$

$\frac{dy}{dx} = xe^x + e^x$

$\frac{d^2y}{dx^2} = xe^x + e^x + e^x = xe^x + 2e^x$

$\frac{d^3y}{dx^3} = xe^x + e^x + e^x + e^x = xe^x + 3e^x$

$\frac{d^ny}{dx^n} = xe^x + ne^x$

$= e^x(x+n)$

Section II Part A: Graphing Calculator MAY BE USED.

1. Let $f(x) = a(36 - x^2)$ for $0 < a < 1$ and let R be the region in the first quadrant bounded by the y-axis, the graph of f, and the graph of $g(x) = 36 - x^2$.

(a) Find the area of R in terms of a.

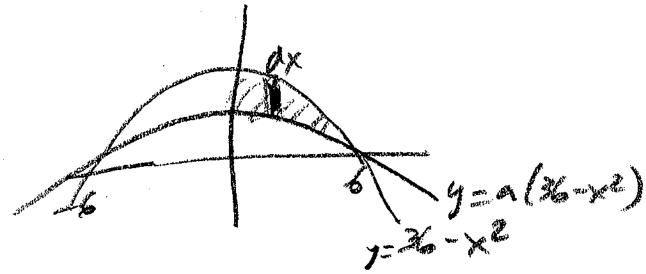
(b) Find, in terms of a, the equation of the line L, tangent to the graph of f at the point $(6, 0)$.

(c) Determine the value of a such that the line L divides the region R into two parts with equal areas. Show the analysis that leads to your conclusion.

(calculator OK)

① $f(x) = a(36 - x^2)$ $(0 < a < 1)$

(a) $g(x) = 36 - x^2$



$$A = \int_0^6 [(36 - x^2) - (a(36 - x^2))]\, dx$$

$$= \int_0^6 (36 - x^2 - 36a + ax^2)\, dx$$

$$= \left[36x - \tfrac{1}{3}x^3 - 36ax + \tfrac{1}{3}ax^3\right]_0^6$$

$$= \left(36(6) - \tfrac{1}{3}(6)^3 - 36a(6) + \tfrac{1}{3}a(6)^3\right) - 0$$

$$= 216 - 72 - 216a + 72a = \boxed{144 - 144a}$$

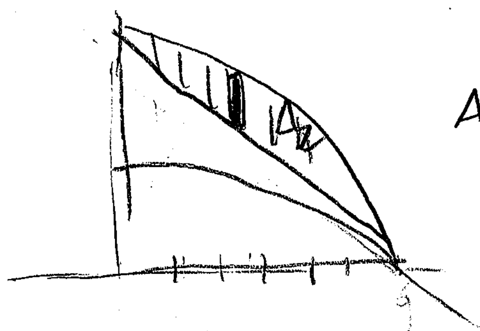
(b) $m = f'(6)$ $f(x) = 36a - ax^2$

$f'(x) = -2ax$, $f'(6) = -2a(6) = -12a = m$

$\boxed{(y - 0) = -12a(x - 6)}$ or $\boxed{y = -12a(x - 6)}$

$y = -12ax + 72$

(c)



$$A_2 = \int_0^6 [(36 - x^2) - (-12a(x - 6))]\, dx = \tfrac{1}{2}(144 - 144a)$$

$$\int_0^6 [36 - x^2 + 12ax - 72a]\, dx$$

$$\left(36x - \tfrac{1}{3}x^3 + 6ax^2 - 72ax\right]_0^6$$

$$216 - 72 + 216a - 432a = 72 - 72a$$

$$144a = 72$$

$$\boxed{a = 1/2}$$

(2) $T(x) = 73 - 14\cos\left(\frac{\pi(x-3.4)}{12}\right)$ x is time since midnight (in hours)

$C(x) = 0.16\int_0^{18}(T(x)-T_{on})\,dx$

(a) $70 = 73 - 14\cos\left(\frac{\pi(x-3.4)}{12}\right)$ use calculator $x = 8.575091$ hrs midnight

so 8:34–8:35 AM

(8:30 AM nearest half hour)

(b) calculator: $x = 10.50622$ so 10:30–10:31 AM (10:30 AM nearest half hour)

(c) turn on when $T = 70°C$ (at $x = 8.575091$)

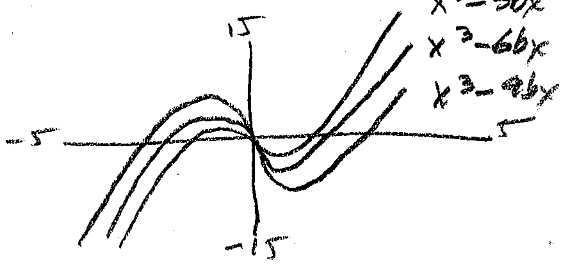
so $C = 0.16\int_{8.575091}^{18}\left(\left(73-14\cos\left(\frac{\pi(x-3.4)}{12}\right)\right)-70\right)dx$ $\overset{\text{fnInt}}{=}$ \$18.2659

(d) Same calc w/ $x = 10.50622$; and 77 here $= \$8.7884$

Savings $= \$18.2659 - \$8.7884 = \$9.4775$

(3) $f(x) = x^3 - 3bx \quad b>0$

(a)



(b)(c) $f'(x) = 3x^2 - 3b = 0$

$3x^2 = 3b$

$x^2 = b$

$x = \pm\sqrt{b}$

$f(\pm\sqrt{b}) = (\pm\sqrt{b})^3 - 3b(\pm\sqrt{b})$

$\pm b\sqrt{b} \mp 3b\sqrt{b}$

$\mp 2b\sqrt{b}$

$\boxed{\text{max}: (-\sqrt{b}, 2b\sqrt{b}) \quad \text{min}: (\sqrt{b}, -2b\sqrt{b})}$

(d) extrema sol'ns of $y = -ax^3$

$(-\sqrt{b}, 2b\sqrt{b})$

$2b\sqrt{b} = -a(-\sqrt{b})^3$

$2b\sqrt{b} = ab\sqrt{b}$

$a = 2$ ✓

$(\sqrt{b}, -2b\sqrt{b})$

$-2b\sqrt{b} = -a(\sqrt{b})^3$

$-2b\sqrt{b} = -ab\sqrt{b}$

$a = 2$

(4) (2nd semester topic — skipped)

(No calculator)

(5) $\tan y = x + y \qquad \overset{x}{(-2\pi, 2\pi)}$

(a) ~~implicit differentiation~~ $\left(\frac{dy}{dx}\right)$

$$\frac{d}{dx}[\tan(y)] = \frac{d}{dx}[x] + \frac{d}{dx}[y]$$

$$\sec^2(y)\cdot\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}\right)$$

$$\sec^2(y)\frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

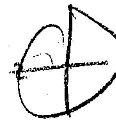
$$\frac{dy}{dx}\left(\sec^2(y) - 1\right) = 1 \qquad \sec^2\theta - 1 = \tan^2$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sec^2(y) - 1}} = \frac{1}{\tan^2 y} = \boxed{\cot^2 y}$$

(b) vertical tangent when $\frac{dy}{dx}$ undef, (when denominator $= 0$)

$$\sec^2(y) - 1 = 0 \qquad \cos^2(y) = 1$$

$$\sec^2(y) = 1 \qquad \cos y = \pm\sqrt{1} = \pm 1$$



$$\frac{1}{\cos^2(y)} = 1$$

$$y = n\pi = -2\pi, -\pi, 0, \pi, 2\pi$$

y	$x = \tan y - y$
-2π	$x = \tan(-2\pi) - (-2\pi) = 0 + 2\pi$ in interval
$-\pi$	$x = \tan(-\pi) - (-\pi) = 0 + \pi = \pi$
0	$x = \tan(0) - (0) = 0$
π	$x = \tan(\pi) - (\pi) = -\pi$
2π	$x = \tan(2\pi) - (2\pi) = -2\pi$

in the x interval $(-2\pi, 2\pi)$

vertical tangent at:

$$\boxed{(\pi, -\pi), (0, 0), (-\pi, \pi)}$$

(c) $\frac{dy}{dx} = (\cot y)^2$

$$\frac{d^2y}{dx^2} = 2(\cot y)^1(-\csc^2 y)\cdot\frac{dy}{dx} \quad \leftarrow \text{chain rule}$$

$$= -2\cot y\csc^2 y(\cot y)^2 = \boxed{-2\cot^3 y\csc^2 y}$$

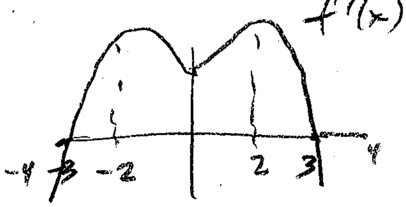
— or —

$$\frac{dy}{dx} = \frac{1}{\tan^2 y} = (\tan y)^{-2}$$

$$\frac{d^2y}{dx^2} = -2(\tan y)^{-3}(\sec^2 y)\frac{dy}{dx}$$

$$= \frac{-2}{\tan^3 y}\sec^2 y\,\frac{1}{\tan^2 y} = \boxed{\frac{-2\sec^2 y}{\tan^5 y}}$$

(6)



D f [-4, 4]

$f'(x)$ even and $f'(-3) = f'(3) = 0$

(a)(b) f relative extrema when $f'(x)=0$

so $f'(x)=0$ at $x=-3$, $x=3$

(a) max occurs when $f'(x)$ goes from increasing to decreasing

$+$ to $-$

which happens at $\boxed{x=3}$.

(b) min occurs when $f'(x)$ goes from decreasing to increasing

$-$ to $+$

which happens at $\boxed{x=-3}$

(c) f concave downward when $f''(x) < 0$ which occurs when $f'(x)$ is decreasing. On the graph this occurs for

$\boxed{-2 \le x < 0 \quad \text{and} \quad 2 < x \le 4}$

(d) $\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

If $f'(x)$ is even, $f(x)$ must be odd

so $\int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx$

therefore: $\int_{-a}^{a} f(x)\,dx = \boxed{0}$