Solutions for Riemann Sum Review

(1) $\int_2^6 (3+x^3)\,dx$

(a) $n=4$ RHS

$\Delta x = \frac{6-2}{4} = 1$, $f(x) = 3+x^3$

intervals	x_i	$f(x_i)$	$\cdot\,\Delta x$	= area
[2,3]	3	30	$\cdot$ 1	= 30
[3,4]	4	67	$\cdot$ 1	= 67
[4,5]	5	128	$\cdot$ 1	= 128
[5,6]	6	219	$\cdot$ 1	= 219

sum = $\boxed{444}$

(b) $n=4$, LHS

$\Delta x = \frac{6-2}{4} = 1$, $f(x) = 3+x^3$

intervals	x_i	$f(x_i)$	$\cdot\,\Delta x$	= area
[2,3]	2	11	$\cdot$ 1	= 11
[3,4]	3	30	$\cdot$ 1	= 30
[4,5]	4	67	$\cdot$ 1	= 67
[5,6]	5	128	$\cdot$ 1	= 128

sum = $\boxed{236}$

(c) $n=4$, Midpoint Rule

$\Delta x = \frac{6-2}{4} = 1$, $f(x) = 3+x^3$

intervals	x_i	$f(x_i)$	$\cdot\,\Delta x$	= area
[2,3]	2.5	18.625	$\cdot$ 1	= 18.625
[3,4]	3.5	45.875	$\cdot$ 1	= 45.875
[4,5]	4.5	94.125	$\cdot$ 1	= 94.125
[5,6]	5.5	169.375	$\cdot$ 1	= 169.375

sum = $\boxed{328}$

(2) $\int_4^7 (x^3-5)dx$

(a) $n=4$, RHS

$\Delta x = \frac{7-4}{4} = \frac{3}{4}$, $f(x)=x^3-5$

interval	x_i	$f(x_i)$	$\cdot \Delta x$	= area
[4, 4.75]	4.75	102.171875	$\cdot$ 3/4	= 76.62890625
[4.75, 5.5]	5.5	161.375	$\cdot$ 3/4	= 121.03125
[5.5, 6.25]	6.25	239.14	$\cdot$ 3/4	= 179.355
[6.25, 7]	7	338	$\cdot$ 3/4	= 253.5

Sum = $\boxed{630.515}$

(b) $n=4$, LHS

$\Delta x = \frac{7-4}{4} = 3/4$, $f(x)=x^3-5$

interval	x_i	$f(x_i)$	$\cdot \Delta x$	= area
[4, 4.75]	4	59	$\cdot$ 3/4	= 44.25
[4.75, 5.5]	4.75	102.171875	$\cdot$ 3/4	= 76.62890625
[5.5, 6.25]	5.5	161.375	$\cdot$ 3/4	= 121.03125
[6.25, 7]	6.25	239.14	$\cdot$ 3/4	= 179.355

Sum = $\boxed{421.265}$

(c) $n=4$, Midpoint Rule

$\Delta x = \frac{7-4}{4} = 3/4$, $f(x)=x^3-5$

interval	x_i	$f(x_i)$	$\cdot \Delta x$	= area
[4, 4.75]	4.375	78.740234375	$\cdot$ 3/4	= 59.05517578
[4.75, 5.5]	5.125	129.611328125	$\cdot$ 3/4	= 97.20849609
[5.5, 6.25]	5.875	197.779296875	$\cdot$ 3/4	= 148.3344727
[6.25, 7]	6.625	285.775390625	$\cdot$ 3/4	= 214.331543

Sum = $\boxed{518.9297}$

(3) $\lim_{n\to\infty} \frac{5}{n}\left[\left(3+\frac{5}{n}\right)^2+\left(3+\frac{10}{n}\right)^2+\left(3+\frac{20}{n}\right)^2+\ldots+\left(3+\frac{5n}{n}\right)^2\right]$

width $= \frac{5}{n}$

Lower Bound (LB) $= 3$

$\frac{5}{n} = \frac{UB-LB}{n}$

$5 = UB - 3$

$UB = 8$

structure $= x^2$

$$\boxed{\int_3^8 x^2\,dx}$$

(4) $\lim_{n\to\infty} \frac{2}{n}\left[(\sin(0))+\left(\sin\left(\frac{2}{n}\right)\right)+\left(\sin\left(\frac{4}{n}\right)\right)+\ldots+(\sin(2))\right]$

$0+\frac{2}{n}$

width $= \frac{2}{n} = \frac{UB-LB}{n}$ lower bound $= 0$

so $UB - 0 = 2$, upperbound $= 2$

structure $= \sin(x)$

$$\boxed{\int_0^2 \sin(x)\,dx}$$

(5) RHS

interval	x_i	$f(x_i)$	$\cdot\ \Delta x$	= area
[0,27]	27	50	$\cdot$ 27	= 1350
[27,48]	48	40	$\cdot$ 21	= 840
[48,60]	60	30	$\cdot$ 12	= 360
[60,70]	70	22	$\cdot$ 10	= 220
[70,88]	88	10	$\cdot$ 18	= 180
[88,100]	100	0	$\cdot$ 12	= 0

sum $= \boxed{2950}$

varies according to interval

AP Calculus BC

Name: Solutions

Review for Quiz 5.1-5.2

#1) Given $\int_2^6 \left(3+x^3\right) dx$ evaluate using a Riemann Sum…

(a) Using RHS endpoints and n=4 rectangles

(b) Using LHS endpoints and n=4 rectangles

(c) Using Midpoints and n=4 rectangles

#2) Given $\int_4^7 \left(x^3-5\right) dx$ evaluate using a Riemann Sum…

(a) Using RHS endpoints and n=4 rectangles

(b) Using LHS endpoints and n=4 rectangles

(c) Using Midpoints and n=4 rectangles

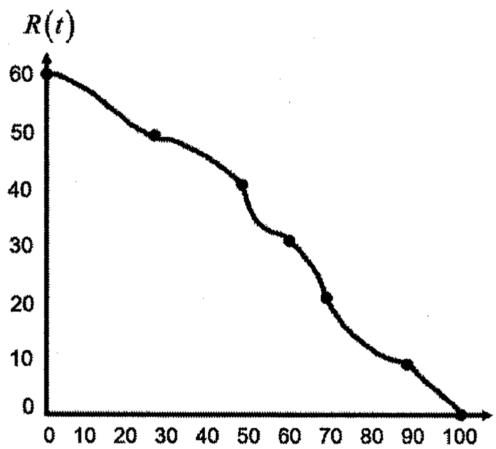
#3) Find the definite integral which the following Riemann Sum approximates:

$$\lim_{n\to\infty}\frac{5}{n}\left[\left(3+\frac{5}{n}\right)^2+\left(3+\frac{10}{n}\right)^2+\left(3+\frac{20}{n}\right)^2+\ldots.+\left(3+\frac{5n}{n}\right)^2\right]$$

#4) Find the definite integral which the following Riemann Sum approximates:

$$\lim_{n\to\infty}\frac{2}{n}\left[\left(\sin(0)\right)+\left(\sin\left(\frac{2}{n}\right)\right)+\left(\sin\left(\frac{4}{n}\right)\right)+\ldots.+\left(\sin(2)\right)\right]$$

#4) Water is draining out of a large tank, but occasionally debris partially obscures the drainage pipe, and as the water drains the hydrostatic pressure decreases forcing the water out at a slower rate. The result is that the rate at which the water is draining out is changing over time. The drainage rate measured in gallons per minute is recorded and is given by a twice-differentiable and strictly decreasing function R of time t. The graph of R and a table of selected values of $R(t)$ for the time interval $0 \le t \le 100$ are shown below:



t (minutes)	R(t) (gallons per minute)
0	60
27	50
48	40
60	30
70	22
88	10
100	0

Use the data values in the table to approximate the value of $\int_0^{100} R(t)\,dt$ using a right Riemann Sum with the six subintervals indicated by the data in the table.