AP Calculus BC Name: solutions

Review for Quiz 3.1-3.6

These problems provide an overview, but we recommend that you also review all homework problems from the unit.

#1) Find y'' using implicit differentiation. $\sqrt{x}+\sqrt{y}=1$ $x^{1/2}+y^{1/2}=1$

$\frac{dy}{dx}$: $\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx}=0$

$\frac{1}{2}y^{-1/2}\frac{dy}{dx}=-\frac{1}{2}x^{-1/2}$

$\frac{dy}{dx}=-\frac{x^{-1/2}}{y^{-1/2}}=\frac{-y^{1/2}}{x^{1/2}}$

$\frac{d^2y}{dx^2}=\frac{(x^{1/2})(-\frac{1}{2}y^{-1/2}\boxed{\frac{dy}{dx}})-(-y^{1/2})(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}$

$=\frac{-\frac{1}{2}x^{1/2}y^{-1/2}(-\frac{y^{1/2}}{x^{1/2}})+\frac{1}{2}x^{-1/2}y^{1/2}}{x}=\frac{-\frac{1}{2}+\frac{1}{2}x^{-1/2}y^{1/2}}{x}$

$=-\frac{1}{2}x^{-1}+\frac{1}{2}x^{-3/2}y^{1/2}=-\frac{1}{2}x^{-1}(1-x^{-1/2}y^{1/2})=\boxed{\frac{-1}{2x}\left(1-\frac{\sqrt{y}}{\sqrt{x}}\right)}$

#2) Find y'' using implicit differentiation. $x^4+y^4=16$

$\frac{dy}{dx}$: $4x^3+4y^3\frac{dy}{dx}=0$

$4y^3\frac{dy}{dx}=-4x^3$

$\frac{dy}{dx}=\frac{-x^3}{y^3}$

$\frac{d^2y}{dx^2}=\frac{(y^3)(-3x^2)-(-x^3)(3y^2\boxed{\frac{dy}{dx}})}{y^6}=\frac{-3x^2y^3+3x^3y^2(-\frac{x^3}{y^3})}{y^6}$

$=\frac{-3x^2y^3-3x^6y^{-1}}{y^6}=-3x^2y^{-3}-3x^6y^{-7}$

$=-3x^2y^{-3}(1+x^4y^{-4})=\boxed{\frac{-3x^2}{y^3}\left(1+\frac{x^4}{y^4}\right)}$

#3) Find the derivative: $y=(x^2+1)\sqrt[3]{x^2+2}$

$y'=(x^2+1)(\frac{1}{3}(x^2+2)^{-2/3}(2x))+\sqrt[3]{x^2+2}\,(2x)$

$=\boxed{\frac{2x(x^2+1)}{3\sqrt[3]{(x^2+2)^2}}+2x\sqrt[3]{x^2+2}}$

#4) Find the derivative: $f(t)=\sin^2\left(e^{\sin^2 t}\right)=\sin(e^{\sin^2 t})^2$

$\boxed{f'(t)=2(\sin(e^{\sin^2 t}))\cos(e^{\sin^2 t})\,e^{\sin^2 t}(2\sin t)\cos t}$

#5) Find the derivative: $y=2^{\sin(\pi x)}$ (logarithmic)

$\ln y=\ln(2^{\sin(\pi x)})$

$\frac{d}{dx}$: $\frac{1}{y}\frac{dy}{dx}=\frac{1}{2^{\sin(\pi x)}}(\ln 2)2^{\sin(\pi x)}\cos(\pi x)\pi$

$\boxed{\frac{dy}{dx}=(\ln 2)\cos(\pi x)\pi\left(2^{\sin(\pi x)}\right)}$

#6) Find the equation of the tangent line to the curve at the given point: $y=\sin(\sin x)$ $(\pi,0)$

$y'=\cos(\sin x)\cos x\Big|_{x=\pi}=\cos(\sin(\pi))\cdot\cos(\pi)$

$=\cos(0)(-1)$

$=(1)(-1)$

$m=-1$

$\boxed{(y-0)=-(x-\pi)}$

Use the table for #7, #8, and #9:

x	F(x)	F'(x)	F"(x)	G(x)	G'(x)	G"(X)
3	5	4	-3	2	7	-2
5	8	6	10	-6	-4	11

#7) If $H(x)=(F(x))^2$, *then* $H'(3)=$

A) 0 B) 10 C) 25 D) 40 E) 100

$H'(x)=2(F(x))F'(x)$
$H'(3)=2(F(3))F'(3)$
$=2(5)(4)$
$=40$

#8) If $H(x)=\dfrac{F(x)}{G(x)}$, *then* $H'(3)=$

A) $-\dfrac{27}{4}$ B) $-\dfrac{3}{2}$ C) 0 D) $\dfrac{4}{7}$ E) $\dfrac{43}{4}$

$H'(x)=\dfrac{G(x)F'(x)-F(x)G'(x)}{[G(x)]^2}$

$H'(3)=\dfrac{G(3)F'(3)-F(3)G'(3)}{[G(3)]^2}$

$=\dfrac{(2)(4)-(5)(7)}{(2)^2}=-27/4$

#9) If $H(x)=\ln(F(x))$, *then* $H'(3)=$

A) 0.2 B) 0.25 C) 0.333 D) 0.621 E) 0.8

$H'(x)=\dfrac{1}{F(x)}F'(x)$

$H'(3)=\dfrac{1}{F(3)}F'(3)=\dfrac{1}{5}(4)=\dfrac{4}{5}=.8$

#10) If $y=e^{x^2}$, *then* $\dfrac{d^2y}{dx^2}=$

A) $(2x)(x^2-1)e^{x^2-2}$ B) e^{x^2} C) $2xe^{2x}$ D) $(2+2x)e^{x^2}$ E) $(2+4x^2)e^{x^2}$

$\dfrac{dy}{dx}=e^{x^2}(2x)$

$\dfrac{d^2y}{dx^2}=e^{x^2}(2)+2x(e^{x^2}2x)$
$=2e^{x^2}+4x^2e^{x^2}$
$=(2+4x^2)e^{x^2}$

#11) Find an equation of the tangent line at the point $P=(1,1)$ to the curve: $y^4+xy=x^3-x+2$

$4y^3\dfrac{dy}{dx}+x\dfrac{dy}{dx}+y(1)=3x^2-1$

$\dfrac{dy}{dx}(4y^3+x)=3x^2-y-1$

$\dfrac{dy}{dx}=\dfrac{3x^2-y-1}{4y^3+x}\Bigg|_{(1,1)}=\dfrac{3-1-1}{4+1}=\dfrac{1}{5}=m$

$\boxed{(y-1)=\frac{1}{5}(x-1)}$

#12) Find the slope of the tangent line at the point $P = (1,1)$ to the curve: $e^{x-y} = 2x^2 - y^2$

$e^{x-y}\left(1 - 1\frac{dy}{dx}\right) = 4x - 2y\frac{dy}{dx}$

$e^{x-y} - e^{x-y}\frac{dy}{dx} = 4x - 2y\frac{dy}{dx}$

$\frac{dy}{dx}\left(2y - e^{x-y}\right) = 4x - e^{x-y}$

$\frac{dy}{dx} = \frac{4x - e^{x-y}}{2y - e^{x-y}}\Big|_{(1,1)} = \frac{4 - e^0}{2 - e^0} = \frac{3}{1} = \boxed{3}$

#13) If $y - x^2y^2 = 6$, then $\frac{dy}{dx} =$

$\boxed{\text{A) } \frac{2xy^2}{1-2x^2y}}$ B) $\frac{1-2x^2y}{2xy^2}$ C) $\frac{2xy^2}{2x^2y+1}$ D) $\frac{5}{4xy}$ E) $\frac{6+2xy^2}{1+2x^2y}$

$1\frac{dy}{dx} - \left(x^2\left(2y\frac{dy}{dx}\right) + (y^2)(2x)\right) = 0$

$\frac{dy}{dx}(1 - 2x^2y) = 2xy^2$ $\frac{dy}{dx} = \frac{2xy^2}{1-2x^2y}$

#14) If $x^2 + y^2 = 6$, then $\frac{d^2y}{dx^2} =$ both correct

$\boxed{\text{A) } \frac{-6}{y^3} \quad \text{B) } -\frac{(x^2+y^2)}{y^3}}$ C) $\frac{6}{y^3}$ D) $\frac{6}{y^2}$ E) $\frac{x-y}{y^2}$

$\frac{dy}{dx}: 2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y}$

$\frac{d^2y}{dx^2}: \frac{(y)(-1) - (-x)\boxed{\frac{dy}{dx}}}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} \frac{(y)}{(y)}$

$= \frac{-y^2 - x^2}{y^3} = \boxed{\frac{-(x^2+y^2)}{y^3}} = \boxed{\frac{-6}{y^3}}$ because $x^2+y^2=6$

#15) Find the derivative: $y = \sqrt{x^4+1}$

$y' = \frac{1}{2}(x^4+1)^{-1/2}(4x^3)$

$y' = \boxed{\frac{2x^3}{\sqrt{x^4+1}}}$

#16) Find the derivative: $y = \tan\left(\frac{x}{x+1}\right)$

$y' = \sec^2\left(\frac{x}{x+1}\right) \cdot \left[\frac{(x+1)(1) - (x)(1)}{(x+1)^2}\right]$

$= \sec^2\left(\frac{x}{x+1}\right)\left(\frac{x+1-x}{(x+1)^2}\right)$

$= \boxed{\frac{\sec^2\left(\frac{x}{(x+1)}\right)}{(x+1)^2}}$

#17) Find the derivative: $y=\left(x^2+7x+2\right)^{-\frac{1}{3}}$

$$y'=\boxed{-\tfrac{1}{3}\left(x^2+7x+2\right)^{-4/3}(2x+7)}$$ this is ok too

$$=\boxed{\frac{-(2x+7)}{3\sqrt[3]{(x^2+7x+2)^4}}}$$

#18) Find the derivative: $y=e^{\cos t}$

$$\boxed{y'=e^{\cos t}\cdot(-\sin t)}$$

#19) Find the derivative: $y=\sqrt{1+\sqrt{x^2+1}}$

$$y'=\tfrac{1}{2}\left(1+(x^2+1)^{1/2}\right)^{-1/2}\left(\tfrac{1}{2}(x^2+1)^{-1/2}(2x)\right)$$

$$=\boxed{\frac{1}{2}\,\frac{1}{\sqrt{1+\sqrt{x^2+1}}}\,\frac{1}{\sqrt{x^2+1}}\,\frac{x}{1}}$$

#20) Find the derivative: $y=7^{3x^2}$

(logarithmic diff.)

$$\ln y=\ln\left(7^{3x^2}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{7^{3x^2}}\ln(7)\,7^{3x^2}(6x)$$

$$\frac{1}{y}\frac{dy}{dx}=\ln(7)\,6x$$

$$\frac{dy}{dx}=6x\ln(7)\,y$$

$$=\boxed{6x\ln(7)\cdot 7^{3x^2}}$$