AP Calculus BC

Unit 1 Review

Name: KEY

Find the limit for each. Show ALL work and reasoning.

1) $\lim_{x\to 3}\sqrt{x+1}$

$=\sqrt{3+1}$

$=\sqrt{4} = \boxed{2}$

2) $\lim_{x\to -4}\frac{x+4}{x^2+9x+20} = \frac{0}{16-36+20} = \frac{0}{0}$

$\lim_{x\to -4}\frac{(x+4)}{(x+4)(x+5)}$

$\lim_{x\to -4}\frac{1}{x+5} = \frac{1}{1} = \boxed{1}$

3) $\lim_{x\to -3}\frac{x^3+27}{x+3}$

$\lim_{x\to -3}\frac{(x+3)(x^2-3x+9)}{(x+3)}$

$\lim_{x\to -3} x^2-3x+9 = 9+9+9 = \boxed{27}$

4) $\lim_{x\to 0}\frac{\sin(3x)}{x}\cdot\frac{3}{3}$

$\lim_{x\to 0} 3\left(\frac{\sin 3x}{3x}\right)$

$= 3(1) = \boxed{3}$

5) $\lim_{x\to 0}\frac{5\tan(x)}{\tan(3x)} = \frac{0}{0}$

$\lim_{x\to 0}\frac{5\sin x}{\cos x}\cdot\frac{\cos 3x}{\sin 3x}\cdot\left(\frac{3x}{3x}\right)$

$\lim_{x\to 0}\frac{5}{3}\,\frac{\sin x}{x}\cdot\frac{3x}{\sin 3x}\cdot\frac{\cos 3x}{\cos x}$

$=\frac{5}{3}(1)\cdot(1)\cdot\frac{1}{1} = \boxed{\frac{5}{3}}$

6) $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \frac{0}{0}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$\lim_{x\to 3}\frac{x+1-4}{(x-3)\sqrt{x+1}+2}$

$\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

7) $\lim_{x\to 1}\frac{e^x}{2x^2-x}$

$=\frac{e^1}{2-1} = \boxed{e}$

8) $\lim_{x\to 0}\frac{3(1-\cos(x))}{x} = \frac{0}{0}$

$\lim_{x\to 0}\frac{-3(\cos x-1)}{x}$

$=-3(0) = \boxed{0}$

9) $\lim_{x\to 0}\frac{4(e^{2x}-1)}{e^x-1} = \frac{0}{0}$

$\lim_{x\to 0}\frac{4(e^x+1)(e^x-1)}{(e^x-1)}$

$\lim_{x\to 0} 4(e^x+1) = 4(1+1)$

$=\boxed{8}$

10) Find the value for each using the graph at the right.

a. $f(-2)$ DNE

b. $\lim_{x\to 1} f(x)$ 2

c. $f(0)$ 4

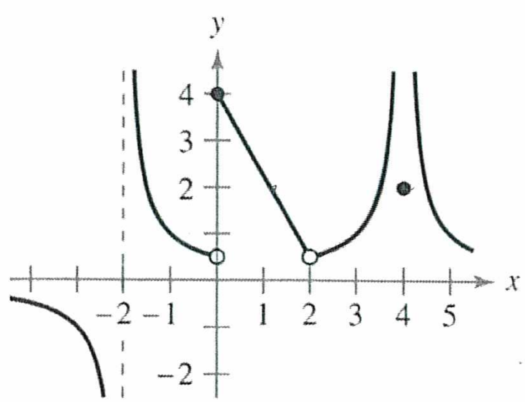
d. $\lim_{x\to 0} f(x)$ DNE

e. $f(2)$ DNE

f. $\lim_{x\to 2} f(x)$ 1/2

g. $f(4)$ 2

h. $\lim_{x\to 4} f(x)$ DNE



11) Given the function $f(x) = \begin{cases} 2x+4, & x < -2 \\ x^2 - 4, & -2 \le x < 2 \\ 4, & x \ge 2 \end{cases}$ Find the following limits.

a) $\lim_{X \to -2} f(x)$

$\lim_{x \to -2^-} = -4+4 = 0$

$\lim_{x \to -2^+} = 4-4 = 0$

$\therefore \boxed{\lim_{x \to -2} f(x) = 0}$

b) $\lim_{X \to 2} f(x)$

$\lim_{x \to 2^-} = 4-4 = 0$

$\lim_{x \to 2^+} = 4$

$\therefore \boxed{\lim_{x \to 2} f(x) = DNE}$

12) Let f be a function defined by $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ -x, & x \ge 1 \end{cases}$

Show that f is continuous at $x = 1$

$f(1) = -1$

$\lim_{x \to 1^-} = 1-2 = -1$

$\lim_{x \to 1^+} = -1$

$> \lim_{x \to 1} = -1$ and since $\lim_{x \to 1} f(x) = f(1)$ then $f(x)$ is continuous at $x = 1$

13) Use the Intermediate Value Theorem to show that for $f(x) = x^3 - 5$ there is at least one x-value on the interval $[-2, 2]$ where $f(x) = 0$

$f(x)$ is continuous (poly)

$f(-2) = (-2)^3 - 5 = -13$

$f(2) = (2)^3 - 5 = 3$

$\Rightarrow$ Since $-13 < 0 < 3$ there must be a value of c in $(-2, 2)$ such that $f(c) = 0$

14) State the functions vertical asymptotes, horizontal asymptotes and holes.

$f(x) = \frac{x^2-x-12}{x^2-9} = \frac{(x-4)(x+3)}{(x-3)(x+3)} = \frac{x-4}{x-3}$

Vertical

$\boxed{x = 3}$

Horizontal

$\lim_{x \to \infty} = 1$

$\lim_{x \to -\infty} = 1$

$\boxed{y = 1}$

Hole at $x = -3$

$\frac{-3-4}{-3-3} = \frac{-7}{-5}$

$\boxed{(-3, 7/5)}$