**FRQ #9b (NO Calculator)** – Improper Integrals, f′(x) applications, derivative rules, tangent lines, infinite series convergence tests, evaluating integrals by u-substitution

2017 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

   (a) Find the slope of the line tangent to the graph of f at $x = 3$.

   (b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

   (c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\displaystyle\int_5^\infty f(x)\,dx$ or show that the integral diverges.

   (d) Determine whether the series $\displaystyle\sum_{n=5}^\infty \dfrac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

FRQ #9c (NO Calculator) – <u>Improper Integrals</u>, <u>Area</u>, evaluating integrals by the Fundamental Theorem of Calculus, u-substitution, and by parts

AP® Calculus BC 2023 Free-Response Questions
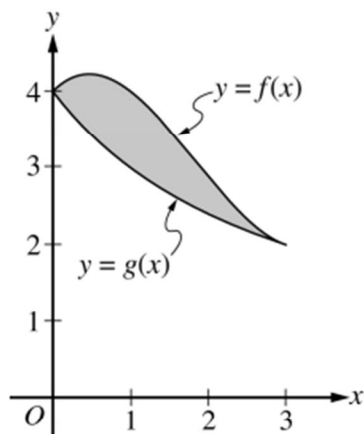
5. The graphs of the functions f and g are shown in the figure for $0 \le x \le 3$. It is known that $g(x) = \dfrac{12}{3+x}$ for $x \ge 0$. The twice-differentiable function f, which is not explicitly given, satisfies $f(3) = 2$ and $\int_0^3 f(x)\,dx = 10$.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Evaluate the improper integral $\int_0^\infty (g(x))^2\,dx$, or show that the integral diverges.

(c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x)\,dx$.

FRQ #9d (NO Calculator) – <u>Improper Integrals</u>, derivative rules, tangent lines, evaluating integrals by partial fraction expansion

2019 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the family of functions $f(x) = \dfrac{1}{x^2 - 2x + k}$, where k is a constant.

   (a) Find the value of k, for $k > 0$, such that the slope of the line tangent to the graph of f at $x = 0$ equals 6.

   (b) For $k = -8$, find the value of $\displaystyle\int_0^1 f(x)\,dx$.

   (c) For $k = 1$, find the value of $\displaystyle\int_0^2 f(x)\,dx$ or show that it diverges.