FRQ #8b (Calculator) – <u>Parametric</u>, position/velocity/acceleration, tangent lines, parametric-vectors, displacement vs total distance traveled

2011 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
### SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\dfrac{dx}{dt} = 4t + 1$ and $\dfrac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

   (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

   (c) Find the position of the particle at time $t = 3$.

   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

FRQ #8c (Calculator) – Parametric, slope from a graph, position/velocity/acceleration, tangent lines, arclength/total distance traveled
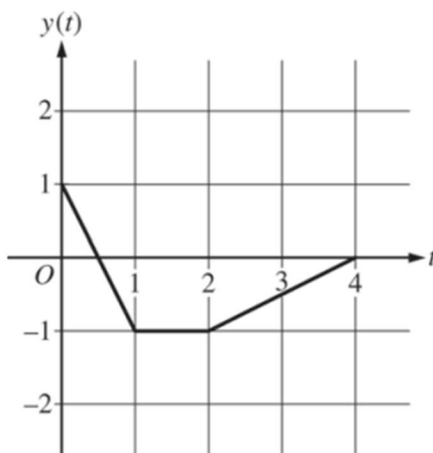
2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\dfrac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

   (a) Find the position of the particle at $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at $t = 3$.

   (c) Find the speed of the particle at $t = 3$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

FRQ #8d (Calculator) – Parametric, net change theorem, position/velocity/acceleration, tangent lines, arclength/total distance traveled, parametric derivatives and 2nd derivatives

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3. A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\dfrac{dy}{dt} = te^{t-3} - 1$.

   (a) Find the speed of the particle at time $t = 3$ seconds.

   (b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

   (c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

   (d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

       (i) The two values of t when that occurs

       (ii) The slopes of the lines tangent to the particle's path at that point

       (iii) The y-coordinate of that point, given $y(2) = 3 + \dfrac{1}{e}$

FRQ #8e (Calculator) – _Parametric_, position/velocity/acceleration, tangent lines, parametric-vectors, displacement vs total distance traveled

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.