**FRQ #6 (NO Calculator)** – <u>Infinite Series</u>

AP® Calculus BC 2021 Free-Response Questions

6. The function $g$ has derivatives of all orders for all real numbers. The Maclaurin series for $g$ is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3}$$ on its interval of convergence.

(a) State the conditions necessary to use the integral test to determine convergence of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$.

Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.

(b) Use the limit comparison test with the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ to show that the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$

converges absolutely.

(c) Determine the radius of convergence of the Maclaurin series for $g$.

(d) The first two terms of the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ are used to approximate $g(1)$. Use the alternating

series error bound to determine an upper bound on the error of the approximation.