FRQ #5b (NO Calculator) – Differential Equations, Euler's method, solving by separation of variables, evaluating limits (l'Hopital's rule)

2013 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

   (a) Find $\lim_{x \to 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

   (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

   (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

FRQ #5c (NO Calculator) – Differential Equations, Euler's method, solving by separation of variables, evaluating limits (l'Hopital's rule)

2010 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

   (a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

   (b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

   (c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

FRQ #5d (NO Calculator) – Differential Equations, Euler's method, evaluating limits (l'Hopital's rule), implicit differentiation, f'(x) applications, f"(x) applications

2016 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

   (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

   (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

   (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1} \left(\frac{g(x) - 2}{3(x+1)^2} \right)$. Show the work that leads to your answer.

   (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.