

**FRQ #5 (NO Calculator)** – Differential Equations, Euler’s method, solving by separation of variables, Taylor polynomials and error

AP<sup>®</sup> Calculus BC 2021 Free-Response Questions

5. Let  $y = f(x)$  be the particular solution to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition  $f(1) = 4$ . It can be shown that  $f''(1) = 4$ .
- (a) Write the second-degree Taylor polynomial for  $f$  about  $x = 1$ . Use the Taylor polynomial to approximate  $f(2)$ .
- (b) Use Euler’s method, starting at  $x = 1$  with two steps of equal size, to approximate  $f(2)$ . Show the work that leads to your answer.
- (c) Find the particular solution  $y = f(x)$  to the differential equation  $\frac{dy}{dx} = y \cdot (x \ln x)$  with initial condition  $f(1) = 4$ .