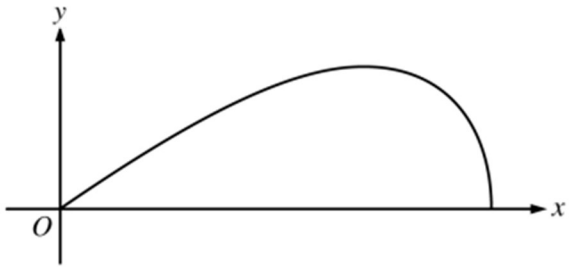
FRQ #3 (NO Calculator) – Areas and Volumes, f'(x) applications

AP® Calculus BC 2021 Free-Response Questions



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

   (a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

   (b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

   (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?