FRQ #14b (NO Calculator) – <u>Slope field / Differential Equations solution behavior</u>, f'(x) applications f"(x) applications, <u>implicit differentiation</u>
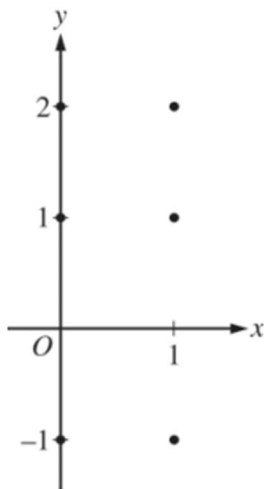
2015 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

   (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

   (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

FRQ #14c (NO Calculator) – <u>Slope field / Differential Equations solution behavior</u>, derivative rules, f"(x) applications, solving a differential equation by separation of variables

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.
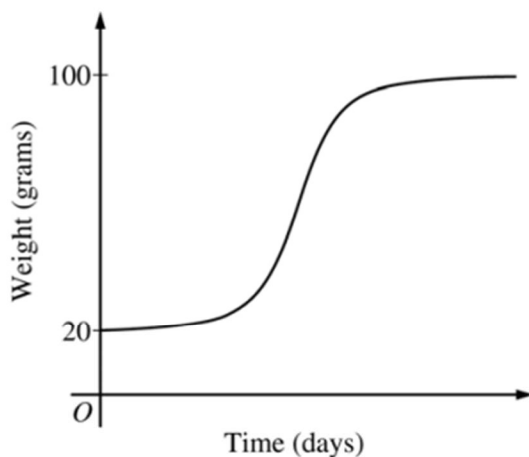
(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.