FRQ #13b (NO Calculator) – <u>Mixed ideas</u>, derivative rules, f"(x) applications, tangent lines, solving a differential equation by separation of variables

2011 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\dfrac{dW}{dt} = \dfrac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \dfrac{1}{4}$).

   (b) Find $\dfrac{d^2W}{dt^2}$ in terms of W. Use $\dfrac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \dfrac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\dfrac{dW}{dt} = \dfrac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

FRQ #13c (NO Calculator) – <u>Mixed ideas</u>, f'(x) applications, derivative rules, tangent lines, evaluating integrals by partial fraction expansion

5. Consider the function $f(x) = \dfrac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$$f'(x) = \frac{k - 2x}{\left(x^2 - kx\right)^2}.$$

(a) Let $k = 3$, so that $f(x) = \dfrac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x-coordinate is 4.

(b) Let $k = 4$, so that $f(x) = \dfrac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.

(c) Find the value of k for which f has a critical point at $x = -5$.

(d) Let $k = 6$, so that $f(x) = \dfrac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f.

Find $\displaystyle\int f(x)\, dx$.