Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

$3: \begin{cases} 1: \text{uses initial condition} \\ 1: f(-6) \\ 1: f(5) \end{cases}$

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

$2: \begin{cases} 1: \text{considers } x = 2 \\ 1: \text{answer with justification} \end{cases}$

(d) $f''(-5) = \dfrac{2 - 0}{-6 - (-2)} = -\dfrac{1}{2}$

$\lim_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} = 2$ and $\lim_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3} = -1$

$f''(3)$ does not exist because
$\lim_{x \to 3^-} \dfrac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \to 3^+} \dfrac{f'(x) - f'(3)}{x - 3}$.

$2: \begin{cases} 1: f''(-5) \\ 1: f''(3) \text{ does not exist, with explanation} \end{cases}$