#1

Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.

$$AV = \frac{1}{2-(-1)} \int_{-1}^{2} (x^3 - 6x^2 + p)\, dx = 1$$

$$\frac{1}{3} \left[\frac{1}{4}x^4 - 2x^3 + px\right]_{-1}^{2} = 1$$

$$\frac{1}{3}\left(\left[\frac{1}{4}(2)^4 - 2(2)^3 + p(2)\right] - \left[\frac{1}{4}(-1)^4 - 2(-1)^3 + p(-1)\right]\right) = 1$$

$$\frac{1}{3}\left([4 - 16 + 2p] - \left[\frac{1}{4} + 2 - p\right]\right) = 1$$

$$\frac{1}{3}(-14{,}25 + 3p) = 1$$

$$-14{,}25 + 3p = 3$$

$$3p = 17{,}25$$

$$p = \frac{17{,}25}{3} = \boxed{5{,}750}$$

\#2 A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t=2$, the position of the particle is $x(2)=0$.

Find the total distance traveled by the particle.

$$\text{total distance} = \int_0^5 |v(t)|\,dt$$

0 — 1 — 3 — 5

$v(\tfrac{1}{2})$: $(+)(-)(-)$ → $+$; $v(2)$: $(+)(+)(-)$ → $-$; $v(4)$: $(+)(+)(+)$ → $+$

$$= \int_0^1 v(t)\,dt - \int_1^3 v(t)\,dt + \int_3^5 v(t)\,dt$$

$$\int v(t)\,dt = \int 3(t-1)(t-3)\,dt$$

$$= \int (3t^2 - 12t + 9)\,dt$$

$$= [t^3 - 6t^2 + 9t]$$

$$= [t^3-6t^2+9t]_0^1 - [t^3-6t^2+9t]_1^3 + [t^3-6t^2+9t]_3^5$$

$$= \boxed{[(1^3-6(1)^2+9(1)] - (0) - ([(3)^3-6(3)^2+9(3)] - [1^3-6(1)^2+9(1)]) + [((5)^3-6(5)^2+9(5)) - (3)^3-6(3)^2+ \ldots}$$

\#3 Find the average velocity of the particle over the interval $0 \le t \le 5$

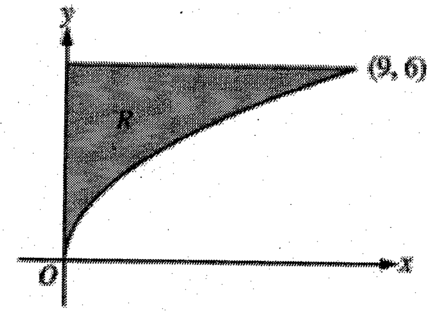
$$\text{Avg velocity} = \frac{1}{5-0}\int_0^5 3(t-1)(t-3)\,dt$$

$$= \frac{1}{5}\int_0^5 (3t^2 - 12t + 9)\,dt$$

$$= \frac{1}{5}[t^3 - 6t^2 + 9t]_0^5$$

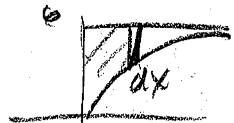
$$= \boxed{\frac{1}{5}\left[((5)^3 - 6(5)^2 + 9(5)) - (0^3 - 6(0)^2 + 9(0))\right]}$$

#4



Let *R* be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the *y*-axis, as shown in the figure above.

(a) Find the area of *R*.



$$A = \int_0^9 [6-2\sqrt{x}]\,dx = \int_0^9 6\,dx - 2\int_0^9 x^{1/2}\,dx$$

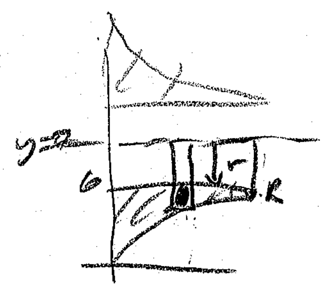
$$= 6(x)_0^9 - 2\left(\tfrac{2}{3}\right)\left(x^{3/2}\right)_0^9$$

$$= \boxed{6(9) - 0 - 3(9)^{3/2} - 2(0)^{3/2}}$$

(b) *Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line y = 7.*

$$V = \int_0^9 \pi(7-2\sqrt{x})^2\,dx - \int_0^9 \pi(1)^2\,dx$$

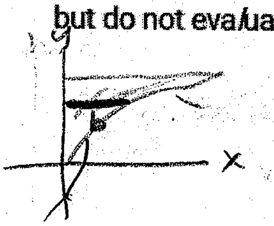
$$\boxed{V = \int_0^9 [\pi(7-2\sqrt{x})^2 - \pi(1)^2]\,dx}$$

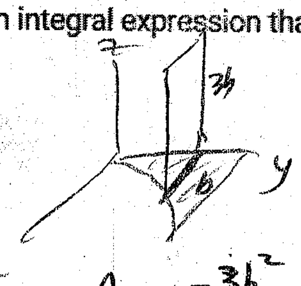


$R = 7 - 2\sqrt{x}$

$r = 7 - 6 = 1$

(c) *Region R is the base of a solid. For each y, where* $0 \le y \le 6$, *the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R.* Write, but do not evaluate, an integral expression that gives the volume of the solid.





$$\boxed{V = \int_0^6 \frac{3}{16} y^4\,dy}$$

$y = 2\sqrt{x}$

$y^2 = 4x$

$x = \frac{1}{4}y^2$

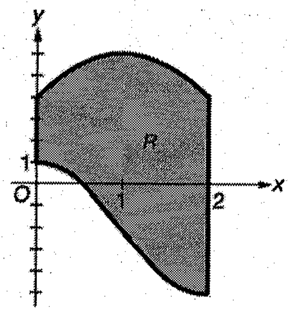
$b = \frac{1}{4}y^2$

$Area = 3b^2$

$= 3\left(\frac{1}{4}y^2\right)^2$

$= \frac{3}{16}y^4$

#5



Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

a) Find the area of R.

b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

#6

Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2+2}$ for $0 \leq x \leq \sqrt{6}$.

(a) *Find the area of R.*

$$A = \int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx \qquad u = x^2+2,\quad du = 2x\,du$$

$$A = \int_2^8 \frac{1}{u}\left(\tfrac{1}{2}du\right)$$

$$A = \tfrac{1}{2}\left[\ln|u|\right]_2^8$$

$$\boxed{A = \tfrac{1}{2}\ln|8| - \tfrac{1}{2}\ln(2)}$$

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

$$A_1 = A_2$$

$$\int_0^{k} \frac{x}{x^2+2}\,dx = \int_k^{\sqrt{6}} \frac{x}{x^2+2}\,dx \qquad u = x^2+2,\quad du = 2x\,du$$

$$\tfrac{1}{2}\int_2^{k^2+2} \frac{1}{u}\,du = \tfrac{1}{2}\int_{k^2+2}^{8} \frac{1}{u}\,du$$

$$\tfrac{1}{2}\ln|k^2+2| - \tfrac{1}{2}\ln|2| = \tfrac{1}{2}\ln|8| - \tfrac{1}{2}\ln|k^2+2|$$

$$\ln|k^2+2| = \tfrac{1}{2}\ln|8| + \tfrac{1}{2}\ln|2| = \tfrac{1}{2}\ln|(8)(2)| = \tfrac{1}{2}\ln|16) = \ln|(16)^{1/2})$$

$$\ln|k^2+2| = \ln|4|, \quad k^2+2 = 4, \quad k^2 = 2, \quad k = \pm\sqrt{2}, \ (0 \leq k \leq$$

$$\boxed{k = \sqrt{2}}$$

(c) What is the average value of $y = \frac{x}{x^2+2}$ on the interval $0 \leq x \leq \sqrt{6}$?

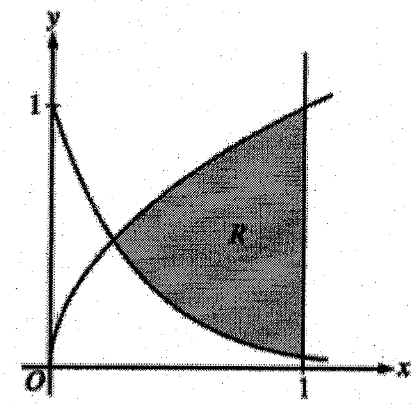
$$\text{avg value} = \frac{1}{\sqrt{6}-0}\int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx \quad \left(\text{part a: } \int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx = \tfrac{1}{2}\ln(8) - \tfrac{1}{2}\ln(2)\right)$$

$$= \boxed{\frac{1}{\sqrt{6}}\left[\tfrac{1}{2}\ln(8) - \tfrac{1}{2}(2)\right]}$$

#7

Intersection:

(0.23873413 , 0.4886092



Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

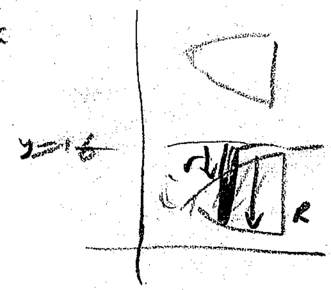
(a) Find the area of R. $\boxed{A = \int_{0.23873413}^{1} (\sqrt{x} - e^{-3x})\,dx = 0.443}$

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

$\boxed{V = \int_{0.23873413}^{1} \pi(1 - e^{-3x})^2 dx - \int_{0.23873413}^{1} \pi(1 - \sqrt{x})^2 dx}$

$r = 1 - \sqrt{x}$

$R = 1 - e^{-3x}$

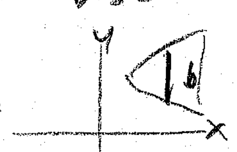


$V = 1.5962337702 - 0.17267522277$

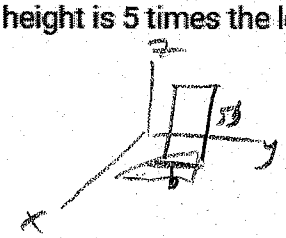
$\boxed{V = 1.424}$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

base



$b = \sqrt{x} - e^{-3x}$

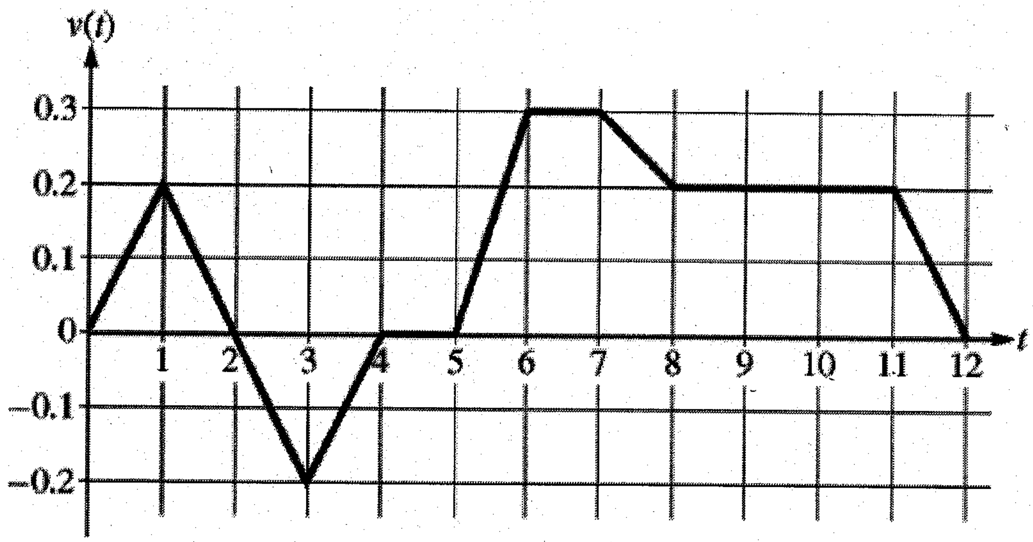


$A_{cross} = 5b^2$

$= 5(\sqrt{x} - e^{-3x})^2$

$\boxed{V = \int_{0.23873413}^{1} 5(\sqrt{x} - e^{-3x})^2 dx \quad V = 1.554}$

#8



Caren rides her bicycle along a straight road from home to school, starting at home at time t=0 minutes and arriving at school at time t=12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by w(t)=π/15sin(π/12t), where w(t) is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Caren distance $= \int_0^{12} v(t)\,dt$

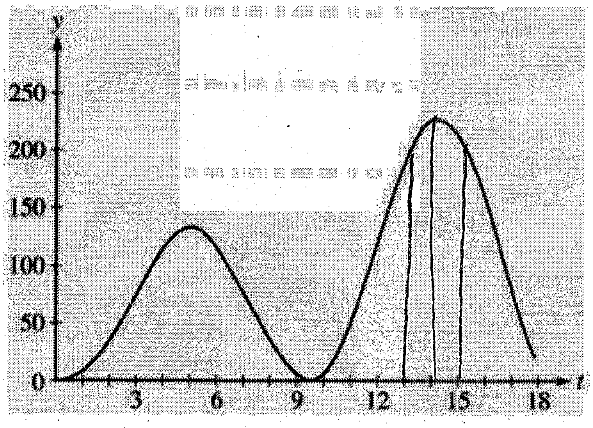
$= \frac{1}{2}(2)\cancel{(0.2)} - \frac{1}{2}(2)\cancel{(0.2)} + 0 + \frac{1}{2}(1)(0.3) + (1)(0.3) + (1)(0.2) + \frac{1}{2}(1)(0.1) + 3(0.2) + \frac{1}{2}(1$

$= 2$ miles

Larry distance $= \int_0^{12} \frac{\pi}{15} \sin(\frac{\pi}{12} t)\,dt = 1.6$ mile

∴ Larry lives closer to school because his distance traveled is smaller than Caren's distance traveled.

#9



At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

total cars turning left $= \int_0^{18} L(t)\,dt = 1657.824$

$\boxed{1658 \text{ cars}}$

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

$L(t) \ge 150$ for $\boxed{12.42831 \le t \le 16.121657}$

$L_{avg} = \frac{1}{16.121657 - 12.42831} \int_{12.42831}^{16.121657} L(t)\,dt = \boxed{199.426 \text{ cars per hour}}$

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

Signal required if $\int_{t_1}^{t_2} L(t) \cdot 500\,dt > 200{,}000$

or if $\int_{t_1}^{t_2} L(t)\,dt > \frac{200{,}000}{500} = 400$

Meaning if area under $L(t)$ graph > 400 for any 2 hr span

max 2hr area is from 13 – 15

$\int_{13}^{15} L(t)\,dt = 431.931 > 400$

so yes, this intersection does require a signal

#10

t (minutes)	0	2	5	9	10
H(t) (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

At time $t = 0$, biscuits with temperature $100^{\circ}C$ were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

$$\int_0^{10} B'(t)\,dt = B(10) - B(0)$$

$$-65.8172472 = B(10) - 100$$

$$B(10) = 34.1827528\,^{\circ}C$$

from table, $H(10) = 43^{\circ}C$)

The biscuits are $43 - 34.1827528 = 8.817^{\circ}C$ cooler than the tea at $t = 10$ minutes.

#11

A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t \cos t$. At time $t = 0$, the position of the particle is $y = 3$.

(a) For what values of t, $0 \leq t \leq 5$, is the particle moving upward?

The particle moves upward when $v(t) > 0$ which occurs for

$\boxed{0 < t < 1.571 \text{ and } 4.712 < t \leq 5}$

(b) Write an expression for the acceleration of the particle in terms of t.

$a(t) = t(-\sin t) + \cos t\,(1)$

$\boxed{a(t) = -t\sin(t) + \cos(t)}$

(c) Write an expression for the position $y(t)$ of the particle.

$u = t \quad dv = \cos t\,dt$

$du = dt \quad v = \sin t$

$y(t) = \int t\cos(t)\,dt$

$y(t) = [uv - \int v\,du] + c = [t\sin t + \int \sin t\,dt] + C$

$y(t) = t\sin(t) - \cos(t) + c$ but $y(0) = 3$

$3 = 0\sin(0) - \cos(0) + c$

$3 = -1 + c, \quad c = 4$

so $\boxed{y(t) = t\sin(t) - \cos(t) + 4}$

(d) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

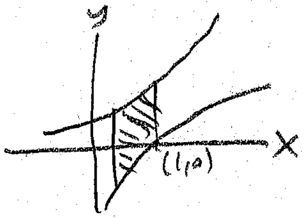
the first time, $t > 0$, where $v(t) = 0$ is at $t = 1.571$

$y(1.571) = (1.571)\sin(1.571) - \cos(1.571) + 4 = \boxed{5.571}$

#12

Let f and g be the functions given by $f(x) = e^{x}$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.



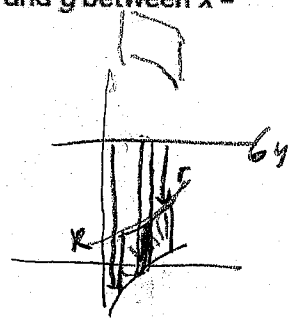
$$\boxed{A = \int_{1/2}^{1} [e^{x} - \ln(x)]\,dx = 1.223}$$

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

$r = 4 - e^{x}$

$R = 4 - \ln(x)$

disc $V = \int_{1/2}^{1} \pi(4 - \ln(x))^2 dx - \int_{1/2}^{1} \pi(4 - e^{x})^2 dx$



$V = 29.19808022 - 5.588587569$

$\boxed{V = 23.609}$

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

absolute min & max may occur at critical pts, where $h'(x) = 0$ or at interval endpts,

$h'(x) = f'(x) - g'(x)$

$h'(x) = e^{x} - \frac{1}{x}$

critical pts: $e^{x} - \frac{1}{x} = 0$

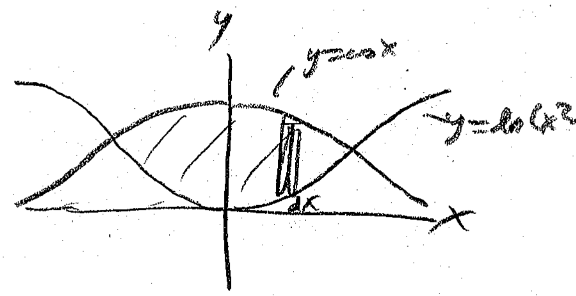
at $x = 0.56714329$

possible x	h(x) = f(x) - g(x)
0.56714329	0
$\frac{1}{2}$	−0.3512787293
1	1.718281828 46

$$\boxed{\begin{array}{l}\text{absolute minimum of } h(x) = -0.351 \text{ (at } x = \tfrac{1}{2}) \\ \text{absolute maximum of } h(x) = 1.718 \text{ (at } x = 1)\end{array}}$$

#13

Let R be the region enclosed by the graphs of $y = \ln(x^2+1)$ and $y = \cos x$.



(a) Find the area of R.

$$A_R = \int_{-0.91585766}^{.91585766} [\cos x - \ln(x^2+1)]\,dx = 1.168$$

(b) Write an expression involving one or more integrals that gives the length of the boundary of the region R. Do not evaluate.

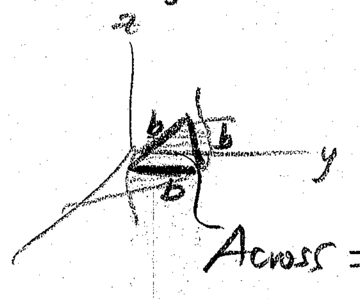
$$\text{arclength} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

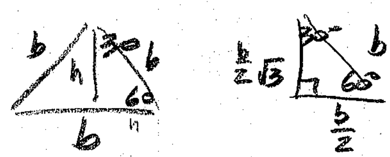
$$\boxed{\text{boundary} = \int_{-.91585766}^{.91585766} \sqrt{1+[-\sin(x)]^2}\,dx + \int_{-.91585766}^{.91585766} \sqrt{1+\left[\frac{1}{x^2+1}(2x)\right]^2}\,dx}$$

(c) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is an equilateral triangle. Write an expression involving one or more integrals that gives the volume of the solid. Do not evaluate.

base

$b = \cos x - \ln(x^2+1)$





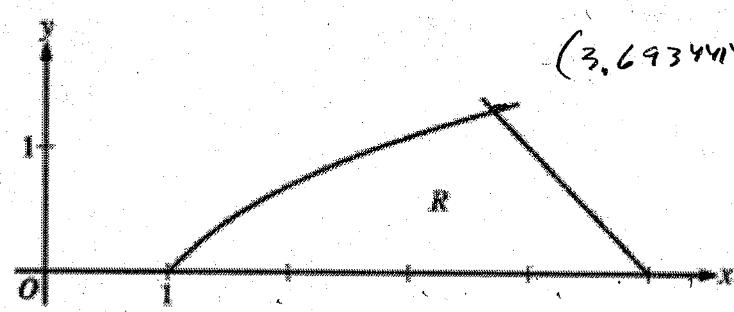
$$A_{cross} = \tfrac{1}{2}hb$$
$$= \tfrac{1}{2}\left(\tfrac{b}{2}\sqrt{3}\right)b = \tfrac{\sqrt{3}}{4}b^2$$
$$= \tfrac{\sqrt{3}}{4}\left(\cos(x) - \ln(x^2+1)\right)^2$$

$$V = \int_a^b A_{cross}\,dx$$

$$\boxed{V = \int_{-.91585766}^{.91585766} \frac{\sqrt{3}}{4}\left(\cos(x) - \ln(x^2+1)\right)^2 dx}$$

#14

$(3.693444, 1.3065586)$



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

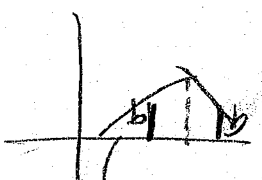
$$A_R = \int_1^{3.693444} \ln(x)\,dx + \int_{3.693444}^{5} (5-x)\,dx$$

$$A_R = 2.132256417 + 0.853547687 = 2.986$$

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

base

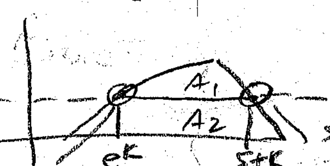
$$A_{cross} = b^2$$



$b_1 = \ln x \quad b_2 = 5-x$

$$\boxed{V = \int_1^{3.693444} (\ln x)^2\,dx + \int_{3.693444}^{5} (5-x)^2\,dx}$$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.



$5-x=k$

$x = 5-k$

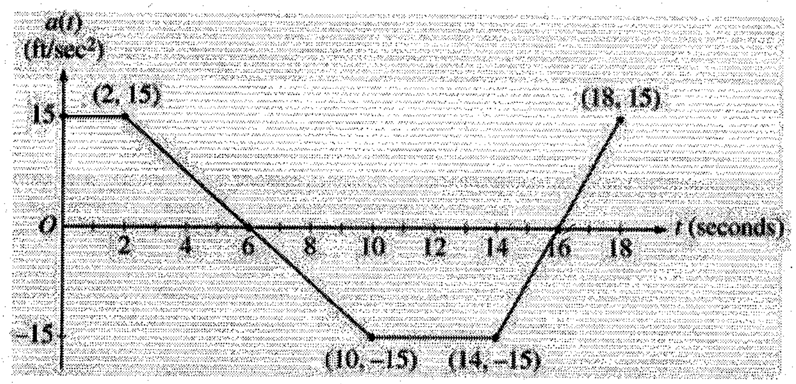
$\ln(x)=k$

$x=e^k$

$$A_1 = A_2$$

$$\boxed{\int_{e^k}^{3.693444} [\ln(x) - k]\,dx + \int_{3.693444}^{5-k} [(5-x)-k]\,dx = \int_1^{e^k} \ln(x)\,dx + \int_{e^k}^{5-k} k\,dx + \int_{5-k}^{5} (5-x)\,dx}$$

#15



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

yes, Since $a(2) > 0$, the velocity of the car is increasing at $t = 2$ seconds.

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

$\int_0^t a(t)dt = v(t) - v(0)$ so $v(t) = 55$ when $v(t) - v(0) = 55 - 55 = 0$

$\therefore$ this occurs when $\int_0^t a(t)dt = 0$, and $\int_0^6 a(t)dt = -\int_6^{12} a(t)dt$ (by graph symmetry)

$\therefore$ the velocity is again 55 ft/sec at $\boxed{t = 12 \text{ seconds}}$

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer. abs max may occur at critical pts or interval end-

critical pts when $v'(t) = 0$, which is when $a(t) = 0$, at $t = 6$ and $t = 16$

candidates:

t	$v(t) = 55 + \int_0^t a(t)dt$
6	$55 + 2(15) + \frac{1}{2}(4)(15) = 115$
16	$55 + 2(15) + \frac{1}{2}(4)(15) - \frac{1}{2}(4)(15) - (4)(15) - \frac{1}{2}(2)(15) = 10$
0	$55 + 0 = 55$
18	$55 + 2(15) + \frac{1}{2}(4)(15) - \frac{1}{2}(4)(15) - (4)(15) - \frac{1}{2}(2)(15) + \frac{1}{2}(2)(15) = 25$

abs max velocity = 115 ft/sec at $t = 6$ s

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

$v(t) = 55 + \int_0^t a(t)dt = 0$

so $\int_0^t a(t)dt = -55$ but the most negative accumulated area occurs at $t = 16$, when $v(t)$ is 10 ft/sec

$\therefore$ there is no time $0 \le t \le 18$ when the car's velocity $= 0$

#16

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

greatest distance between particle and origin is absolute max of $x(t)$ over interval $0 \le t \le 3$ which may occur at critical pts when $x'(t) = 0$ or at interval ends.

critical pts: $x'(t) = v(t) = 0$ which occurs at $t = 0$ and $t = 2.50662$

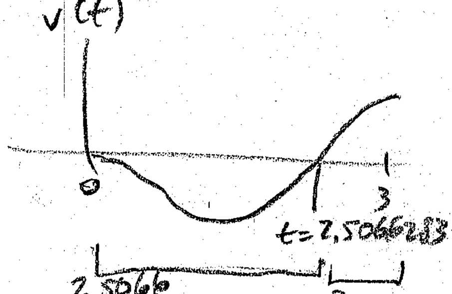
$$x(t) = x(0) + \int_0^t v(t)\,dt = 1 + \int_0^t v(t)\,dt$$

Candidates:

t	$x(t)$
0	1
2.5066283	$1 + \int_0^{2.5066283} v(t)\,dt = -2.26548 28$
3	$1 + \int_0^3 v(t)\,dt = -1.1971474 1$

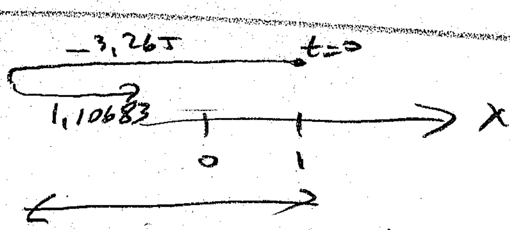
$\boxed{\text{greatest distance between particle and origin is } 2.265 \text{ (at } t = 2.507)}$

another approach:



$\int_0^{2.5066} v(t)\,dt$ $\quad$ $\int_{2.5066}^{3} v(t)\,dt$

-3.265 $\quad$ $1.06 83$



greatest distance between particle and origin is

$-3.265 + 1 = \boxed{-2.265}$

#17 A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

(a) Find the values of t for which the particle is at rest. (at rest when $v(t) = 0$)

$v(t) = \int (12t^2 - 4)\,dt = 4t^3 - 4t + C$ $\quad v(0) = 0$ "initially at rest"

$0 = 4(0)^3 - 4(0) + C \rightarrow C = 0$

so $v(t) = 4t^3 - 4t = 0$

$4t(t^2 - 1) = 0$ $\quad$ at $t = 0$, $\cancel{t = -1}$, $t = 1$

$\boxed{\text{The particle is at rest at } t = 0 \text{ and } t = 1}$

(b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.

$x(t) = \int v(t)\,dt = \int (4t^3 - 4t)\,dt = t^4 - 2t^2 + D$ $\quad x(1) = 3$

$3 = (1)^4 - 2(1)^2 + D = 1 - 2 + D = -1 + D$, $D = 4$

$\boxed{x(t) = t^4 - 2t^2 + 4}$

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

total distance $= \int_0^2 |v(t)|\,dt$

$v(t) = 0$

$4t(t^2 - 1) = 0$

$t = 0$, $t = 1$

0 ——— 1 ———

$v(\frac{1}{2})$ $\quad$ $v(2)$

$(+)(+)(-)$ $\quad$ $(+)(+)(+)$

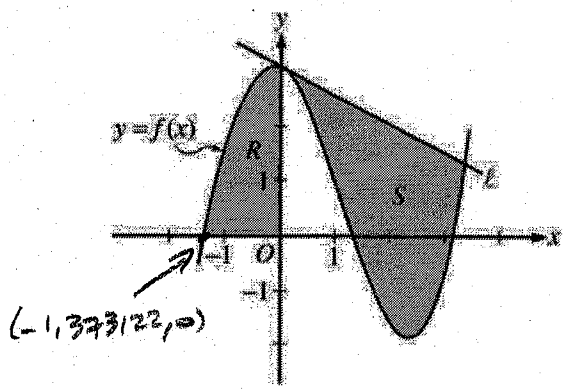
$v(\frac{1}{2}) < 0$ $\quad$ $v(2) > 0$

total distance $= -\int_0^1 (4t^3 - 4t)\,dt + \int_1^2 (4t^3 - 4t)\,dt$

$= -\left[t^4 - 2t^2\right]_0^1 + \left[t^4 - 2t^2\right]_1^2$

$= \boxed{-\left(\left[(1)^4 - 2(1)^2\right] - \left[(0)^4 - 2(0)^2\right]\right) + \left(\left[(2)^4 - 2(2)^2\right] - \left[(1)^4 - 2(1)^2\right]\right)}$

#18



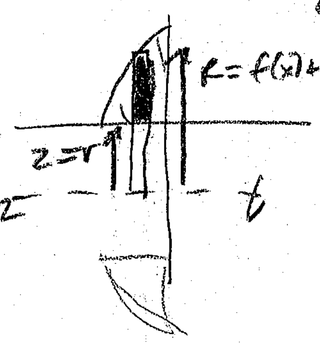
Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

$$\boxed{A_R = \int_{-1{,}373122}^{0} f(x)\,dx = 2{,}903}$$

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

disk $V = \int \pi R^2 dx - \int \pi r^2 dx$



$$V = \int_{-1{,}373122}^{0} \pi(f(x)+2)^2 dx - \int_{-1{,}373122}^{0} \pi[2]^2 dx$$

$V = 76.61656553 - 17{,}25515985 = 59{,}361$

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

$f'(x) = \frac{3}{4}x^2 - \frac{2}{3}x - \frac{1}{2}$

Intersection: $(3{,}3898677,\ 1{,}3050662)$

$f'(0) = -\frac{1}{2} = m$

$f(0) = 3\cos(0) = 3$

Line ℓ: $(y-3) = -\frac{1}{2}(x-0)$

$y = -\frac{1}{2}x + 3$

$$\boxed{A_S = \int_{0}^{3{,}3898677} \left[\left(-\tfrac{1}{2}x+3\right) - f(x)\right] dx}$$