Derivatives

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}\log_b x = \frac{1}{x\ln b}$

$\frac{d}{dx}e^x = e^x$

$\frac{d}{dx}b^x = b^x \cdot \ln b$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \cdot \tan x$

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int \frac{1}{x} dx = \ln|x| + C$

Trig Identities

$\tan x = \frac{\sin x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$

Volume

Disc

$V = \pi \int_a^b r^2\, dx$

Washer

$V = \pi \int_a^b \left(R^2 - r^2\right) dx$

Shell

$V = 2\pi \int_a^b rh\, dx$

Cross Section

$V = \int_a^b A\, dx$

Integration Rules

U-Substitution

$\int f(g(x))dx$ let u = g(x)

Integration by Parts

$\int u\, dv = uv - \int v\, du$

Decomposing into P.F.

$\frac{1}{(cx+d)(hx+k)} = \frac{A}{(cx+d)} + \frac{B}{(hx+k)}$

L'Hopital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $= \frac{\infty}{\infty}$,

then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

First Fundamental Theorem

$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x)) \cdot g'(x)$

Second Fundamental Theorem

$\int_a^b f(t)dt = F(b) - F(a)$ where F'(x) = f(x)

Derivatives (Differentiation Rules)

Prod.

$\frac{d}{dx}(f \cdot g) = f'g + fg'$

Quot.

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$

Chain

$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

Position, Vel, Acc

$v(t) = \frac{d}{dt}(pos)$ $a(t) = \frac{d}{dt}(v(t))$

$displacement = \int_a^b v(t)dt$

$T.D.T. = \int_a^b |v(t)|dt$

$speed = |vel|$

Alt. Series Error: error $\leq |a_{n+1}|$ (the next term)

Lagrange Error:

error $\leq \left|\frac{f^{(n+1)}(c)(b-a)^{n+1}}{(n+1)!}\right|$ where

$\left|f^{(n+1)}(c)\right|$

is the maximum value of $f^{n+1}(x)$ on [a,b].

Definition of Derivative

$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$

Inv Fun Theorem

f(x) (a,b) slope = m

$f^{-1}(x)$ (b,a) slope = $1/m$

McLaurin Series to have memorized

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$

Logistic

$\frac{dP}{dt} = \frac{k}{M}P(M-P)$

$P = \frac{M}{1 + Ce^{-kt}}$

M = carrying capacity

Pt Slope Form

$y - y_1 = m(x - x_1)$

Taylor Series

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$

Maclaurin Series (Taylor series with $a = 0$)

Euler's Method

(x, y)	$y_{n+1} = y_n + h\left(\frac{dy}{dx}\right)$

Average Rate of Change: $\text{AROC} = \dfrac{f(b)-f(a)}{b-a}$ (slope between two points)

Inst. Rate of Change: $\text{IROC} = f'(c)$ (slope at a single point)

Mean Value Thm Part 1: $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Rolles Thm.: if f(a) = f(b), then f'(c) = 0

Average Value of a Function: $f_{avg} = \dfrac{\int_a^b f(x)\,dx}{b-a}$

Mean Value Thm Part 2: $f(c) = \dfrac{\int_a^b f(x)\,dx}{b-a}$

Intermediate Value Thm. A function *f(x)* that is continuous on $[a,b]$ takes on every y-value between $f(a)$ and $f(b)$.

Extreme Value Thm: If *f(x)* is continuous on $[a,b]$, then f(x) must have both an absolute min and absolute max on the interval $[a,b]$.

Arc Length $= \underbrace{\int_a^b \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx}_{\text{cartesian}}$

Arc Length $= \underbrace{\int_{t_1}^{t_2} \sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}\,dt}_{\text{parametric}}$

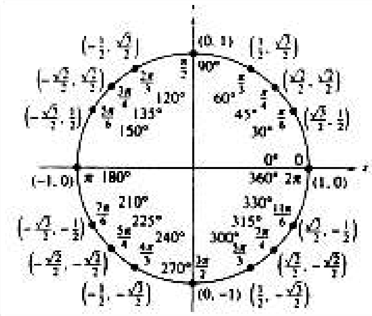
Speed = $= \sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}$

T.D.T. = $\int_{t_1}^{t_2} \sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}\,dt$

Polar Area $= \dfrac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta$

Parametric Derivatives: $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$, $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Polar Conversions: $r^2 = x^2 + y^2$, $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \arctan\dfrac{y}{x}$



Area of Trapezoid

$A = 1/2h(b_1 + b_2)$

$$\cos^2(x) = \frac{1}{2}(1+\cos(2x)) \qquad \sin^2(x) = \frac{1}{2}(1-\cos(2x))$$

n^{th} term test	div. if $\lim_{n\to\infty} a_n \neq 0$ (cannot be used to show convergence)
Geom. series test	$\sum_{n=0}^{\infty} ar^n$ $\quad |r|<1 \to$ conv. , $|r|\geq 1 \to$ div. , $S = \dfrac{a}{1-r}$
p-series	$\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ $\quad p>1 \to$ conv. , $p \leq 1 \to$ div.
Alternating series	decr. terms and $\lim_{n\to\infty} a_n = 0 \to$ conv.
Integral test	$a_n = f(x)$ $\quad \sum_{n=1}^{\infty} a_n$ conv. if $\int_1^{\infty} f(x)\,dx$ conv., $\sum_{n=1}^{\infty} a_n$ div. if $\int_1^{\infty} f(x)\,dx$ div.
Ratio test	$\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| < 1 \to$ conv. , $\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| > 1 \to$ div. , (inconclusive if $\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1$) (works well for factorials and exponentials)
Direct Comparison	a series with terms **smaller** than a known convergent series also converges a series with terms **larger** than a known divergent series also diverges
Limit Comparison	if $\lim_{n\to\infty}\dfrac{a_n}{b_n}$ is finite and positive both series converge or both diverge (use with "messy" algebraic series, usually compared to a p-series)