EXAM VI
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which $f(x)$ is a real number.

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1. Which of the following is an antiderivative of $\sqrt{4-2x}$?

(A) $-\frac{1}{3}(4-2x)^{3/2}$ (B) $\frac{2}{3}(4-2x)^{3/2}$ (C) $-\frac{1}{6}(4-2x)^{3}$

(D) $\frac{1}{2}(4-2x)^{2}$ (E) $\frac{4}{3}(4-2x)^{3/2}$

Ans ☐

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2. A particle moves along a straight line with its position at any time $t \geq 0$ given by $s(t) = \int_0^t (\sqrt{x} - x + 1)\, dx$, where s is measured in meters and t in seconds. What is the velocity of the particle when its acceleration is zero?

(A) $-\frac{1}{4}$ m/s (B) $\frac{1}{4}$ m/s (C) $\frac{1}{2}$ m/s (D) 1 m/s (E) $\frac{5}{4}$ m/s

Ans ☐

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3. If $\frac{dy}{dx} = \frac{\cos x}{e^y}$ and $y(0) = 0$, then $y\left(\frac{\pi}{2}\right) =$

(A) 0

(B) ln 2

(C) 1

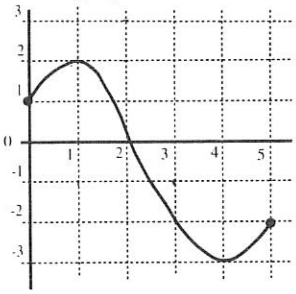
(D) $\frac{1}{2}$

(E) -1

Ans

4. The function h is defined on the interval $0 \le x \le 5$ and a graph of its derivative function h' is shown in the figure. Which of the following are true?

graph of h'



I. The function h is decreasing on the interval (1, 2).

II. The function h has a local maximum at the point where $x = 2$.

III. Given $h(1) = -1$, an equation of the tangent line to the graph of h at the point $(1, -1)$ is $y = 2x - 3$.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

5. $\int_1^{\infty} \frac{x}{1+x^2}\,dx =$

(A) 0 (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{\pi}{2}$ (E) divergent

Ans

6. If $x = \sin t$ and $y = \cos^2 t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $-\frac{1}{4}$ (D) –2 (E) 2

Ans

7. In an effort to enhance fishing, 100 trout were initially put in a small lake. Fishery Department biologists predict that the rate of growth of the trout population is modeled by the logistic differential equation $\frac{dP}{dt} = 0.12P\left(1 - \frac{P}{600}\right)$, where time t is measured in months.

I. The growth rate of the fish population is greatest at $P = 600$.

II. If $P > 600$, the population of fish is decreasing.

III. $\lim_{t \to \infty} P(t) = 600$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Ans

8. If f and g are continuously differentiable functions for all real numbers, which of the following definite integrals is equal to $f(g(5)) - f(g(3))$?

(A) $\int_3^5 f(g'(x)) \cdot g(x)\, dx$

(B) $\int_3^5 f'(g(x)) \cdot g(x)\, dx$

(C) $\int_3^5 f'(g(x)) \cdot g'(x)\, dx$

(D) $\int_3^5 f(g(x)) \cdot f'(x)\, dx$

(E) $\int_3^5 f'(g(x))\, dx$

Ans

9. $\lim\limits_{x \to +\infty} \dfrac{x - \frac{1}{2x}}{2x - \frac{1}{6x}} =$

(A) -3 (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 2

Ans

10. An equation of the line tangent to the graph of $y = \dfrac{3x+4}{4x-3}$ at the point where $x = 1$ is

(A) $y + 25x = 32$ (B) $y - 31x = -24$ (C) $y - 7x = 0$
(D) $y + 5x = 12$ (E) $y - 25x = -18$

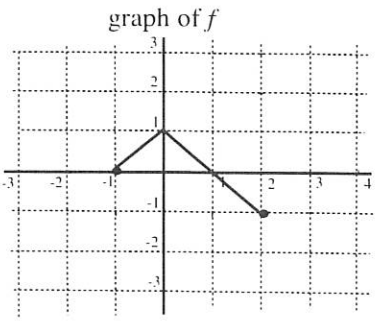
Ans

11. If $\frac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0, 4]$ is

(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

Ans

12. A graph of the function f, consisting of two line segments, is shown in the figure. If $g(x) = \int_1^x f(t)\,dt$, then the maximum value of g on the closed interval $[-1,2]$ is



(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) 1 (E) 2

Ans

13. The total area of the region enclosed by the polar graph of $r = 1 + \cos\theta$ is given by which of the following expressions?

(A) $\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

(B) $\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

(C) $\frac{1}{2}\int_0^{2\pi}(1+\cos\theta)\,d\theta$

(D) $2\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$

(E) $\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$

Ans

14. A particle moves along the x-axis with acceleration given by $a(t) = \cos t$ ft/sec^2 for $t \geq 0$. At time $t = 0$ seconds the velocity of the particle is 2 ft/sec. The total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 1 ft (B) $\frac{\pi}{2}$ ft (C) π ft (D) $\pi+1$ ft (E) $\pi+2$ ft

Ans ☐

15. A curve is parametrically defined by the equations $x = 2\cos t$ and $y = 2\sin t$. The length of the arc from $t = 0$ to $t = 2$ is

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Ans ☐

16. $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{2k} =$

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{9}{8}$ (E) ∞

Ans ☐

17. The shortest distance from the curve $xy = 4$ to the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) $\frac{1}{2}\sqrt{2}$

Ans

18. If the function f is defined so that $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

(A) f is not continuous at $x = 0$

(B) $f(a) = 0$

(C) $f'(a) = 0$

(D) $f(a)$ does not exist

(E) $\lim_{x \to a} f(a)$ does not exist

Ans

19. A particle moves in the xy-plane so that its velocity vector a time t is $v(t) = (t^2, \sin \pi t)$ and the particle's position vector at time $t = 0$ is (1, 0). The position vector of the particle when $t = 3$ is

(A) $(9, \frac{1}{\pi})$ (B) $(10, \frac{2}{\pi})$ (C) $(6, -2\pi)$ (D) $(10, 2\pi)$ (E) (10, 2)

Ans

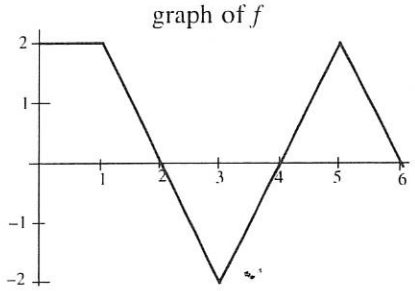
20. The density of cars (in cars per mile) down a 20-mile stretch of the Jersey Turnpike starting at a toll plaza is given by $f(x) = 500 + 100\sin(\pi x)$ where x is the distance in miles from the toll plaza and $0 \le x \le 20$. The total number of cars down the 20-mile stretch is

(A) 8500 (B) 9000 (C) 9500 (D) 10,000 (E) 10,500

Ans

21. The function G is defined on the interval [0, 6] by $G(x) = \int_0^x f(t)\,dt$ where f is the function graphed in the figure. A linear approximation of G near $x = 3$ is

graph of f



(A) $6 - x$ (B) $8 - x$ (C) $5 - 2x$ (D) $8 - 2x$ (E) $9 - 2x$

Ans

22. What is the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{3^n(x+2)^n}{n+1}$?

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) 2

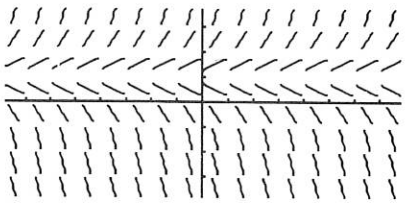
Ans

23. The infinite first quadrant region bounded above by the curve $y = e^{-2x}$ is rotated about the x-axis to generate a solid of revolution. The volume of the solid is

(A) $\frac{\pi}{6}$ units^3 (B) $\frac{\pi}{4}$ units^3 (C) $\frac{\pi}{3}$ units^3 (D) $\frac{\pi}{2}$ units^3 (E) ∞

Ans

24. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. Which of the following statements are true?



I. A solution curve that contains the point (0, 2) also contains the point (–2, 0).

II. As y approaches 1 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of y.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

25. If the definite integral $\int_1^7 \ln x \, dx$ is approximated by 3 circumscribed rectangles of equal width on the x-axis, then the approximation is

(A) $\frac{1}{2}(\ln 3 + \ln 5 + \ln 7)$

(B) $\frac{1}{2}(\ln 1 + \ln 3 + \ln 5)$

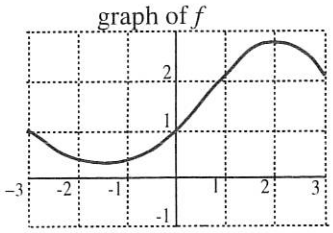
(C) $2(\ln 3 + \ln 5 + \ln 7)$

(D) $2(\ln 3 + \ln 5)$

(E) $\ln 1 + 2\ln 3 + 2\ln 5 + \ln 7$

Ans

26. The function f is defined on the interval $-3 \le x \le 3$ and its graph is shown in the figure. Which of the following statements are true?



I. $f(2) > f'(2)$

II. $\int_0^1 f(x)\,dx > f''(2)$

III. $1 - x + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{15} - \cdots$ is a Maclaurin series representation of the function f

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II and III

Ans

27. Given the differential equation $\frac{dy}{dx} = \frac{10x}{x + y}$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) $\frac{16}{3}$

Ans

28. The coefficient of $(x - 1)^5$ in the Taylor series for $x \ln x$ about $x = 1$ is

(A) $-\frac{1}{20}$ (B) $\frac{1}{20}$ (C) $-\frac{1}{24}$ (D) $\frac{1}{24}$ (E) $-\frac{1}{120}$

Ans

EXAM VI
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem. Calculators may be used on this part of the examination.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which $f(x)$ is a real number.

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1. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t+1)(t-4)^3$. For what values of t, $2 \leq t \leq 4$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [2, 4]?

(A) 2.54 (B) 2.64 (C) 2.74 (D) 2.84 (E) 2.94

Ans ☐

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2. If the function f defined by $f(x) = x^4 + ax^2 + 8x - 5$ has a horizontal tangent line and a point of inflection at the same value of x, then a is

(A) –12 (B) –6 (C) 0 (D) 1 (E) 3

Ans ☐

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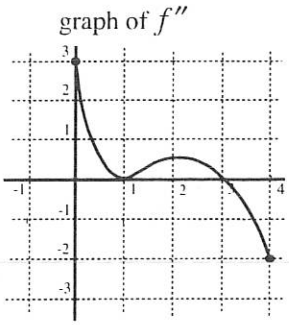
3. A graph of the second derivative of a function f is shown in the figure. Use the graph to determine which of the following are true.

I. The f-graph is concave down on the interval $(1, 3)$.

II. f has points of inflection at $x = 1$ and $x = 3$.

III. If $f'(2) = 0$, f is increasing at $x = 3$.



(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

4. The average value of $f(x) = 1 - \sqrt{1 - x^2}$ over the interval $[-1, 1]$ is approximately

(A) 0.140 (B) 0.165 (C) 0.190 (D) 0.215 (E) 0.240

Ans

5. The base of a solid is the region bounded below by the curve $y = x^2$ and above by the line $y = d$, where d is a positive constant. Every cross-section of the solid perpendicular to the y-axis is a square. If the volume of the solid is 72, what is the value of d?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Ans

6. The function g is defined by $g(x) = \int_{\pi/2}^{x} \cos t \, dt$. The maximum value of g on the closed interval $[-\pi, \pi]$ is

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

Ans

7. The Cartesian equation for the polar curve $r = \sin\theta$ is

(A) $x^2 + y^2 = x$

(B) $x^2 + y^2 = y$

(C) $x^2 + y^2 = x + y$

(D) $(x + y)^2 = y$

(E) $(x + y)^2 = x$

Ans

8. The function f is defined for all real numbers by $f(x) = \begin{cases} e^{-x} + 3, & \text{for } x > 0, \\ ax + b & \text{for } x \le 0. \end{cases}$ If f is differentiable at $x = 0$, then $a + b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans

9. If the derivative of the function f is given by $f'(x) = \cos\left(\frac{x}{2}\right) \cdot \ln x$ for $0 < x < 3\pi$, then the graph of f is increasing and concave down on the interval

(A) (0, 1.97) (B) (1.97, 3.14) (C) (3.14, 6.60)
(D) (6.60, 9.42) (E) (3.14, 9.42)

Ans

10. The least integer value of a for which the the series $\sum_{n=1}^{\infty} \frac{1}{n^{a-27}}$ converges is

(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

Ans

11. The graph of the function represented by the Maclaurin series $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \frac{x^6}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ intersects the graph of $y = 1 + x^2$ at the point where $x =$

(A) 0.718 (B) 0.738 (C) 0.758 (D) 0.778 (E) 0.798

Ans

12. A particle is traveling along the circle $x^2 + y^2 = 4$. When it is at the point $(1, \sqrt{3})$, $\frac{dx}{dt} = 2$. Find $\frac{dy}{dt}$ at this instant.

(A) $-\frac{2}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) 0 (D) $\frac{1}{\sqrt{3}}$ (E) $\frac{2}{\sqrt{3}}$

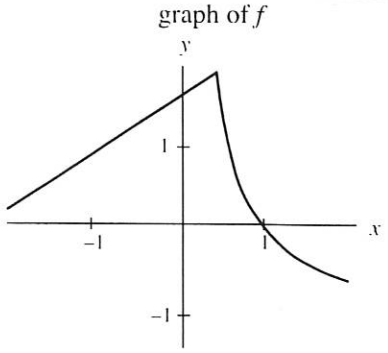
Ans

13. Suppose interest on money in a bank account accumulates at an annual rate of 4% per year compounded continuously. If the balance $B = B(t)$ in the account satisfies the equation $\frac{dB}{dt} = .04B$, then approximately how much money should be invested today so that 5 years from now it would be worth \$4000?

(A) \$3600
(B) \$3300
(C) \$3000
(D) \$2700
(E) \$2400

Ans

14. A graph of the function f, is shown in the figure. Which of the following are true?



(A) $f''(1) < f'(1) < f(1)$
(B) $f'(1) < f(1) < f''(1)$
(C) $f(1) < f'(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f(1) < f''(1) < f'(1)$

Ans

15. If $\int f(x)\cdot\sec^2 x\,dx = f(x)\cdot\tan x - \int 9x^2\cdot\tan x\,dx$, then $f(x)$ could be

(A) $x^3\cdot\sec^2 x$ (B) $x^3\cdot\tan x$ (C) $3x^3$ (D) $3x^2$ (E) $6x^3$

Ans

16. If the substitution $\sqrt{x} = u - 1$ is made in the integrand of $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$, the resulting integral is

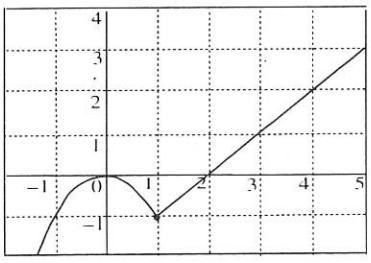
(A) $\int_1^4 \frac{2}{u}\,du$ (B) $\int_1^4 \frac{1}{u(u-1)}\,du$ (C) $\int_1^4 \frac{1}{u}\,du$

(D) $\int_2^3 \frac{2}{u}\,du$ (E) $2\int_2^3 \frac{1}{u(u-1)}\,du$

Ans

17. Using the graph of f, determine which of the following statements are true.

graph of f



I. $\lim_{x\to 2} \frac{f(x)}{\sin(x-2)} = 1$

II. $\lim_{x\to 1} \frac{f(x-1)}{f(x+1)} = 1$

III. $\lim_{x\to 2} \frac{[f(x)]^2}{(x-2)^2} = 1$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

Ans