EXAM V
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS

Time–55 minutes

Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. $\int_{2}^{3} \frac{x}{x-1}\, dx =$

(A) $1 - \ln 2$

(B) $\ln 2$

(C) $1 + \ln 2$

(D) $2 + \ln 2$

(E) $5 + \ln 2$

Ans ☐

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2. The radius of convergence of the series $\frac{x}{4} + \frac{x^2}{4^2} + \frac{x^3}{4^3} + \ldots + \frac{x^n}{4^n} + \ldots$

(A) ∞

(B) 0

(C) 1

(D) 2

(E) 4

Ans ☐

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3. The position vector of a particle moving in the xy-plane at time t is given by $\mathbf{p} = (3t^2 - 4t)\,\mathbf{i} + (t^2 + 2t)\,\mathbf{j}$. The speed of the particle at $t = 2$ is

(A) 2 units/sec

(B) $2\sqrt{10}$ units/sec

(C) 10 units/sec

(D) 14 units/sec

(E) 20 units/sec

Ans

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4. If $f(x) = \ln(x^2 - e^{2x})$, then $f'(1) =$

(A) 0

(B) 1

(C) 2

(D) e

(E) undefined

Ans

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5. The length of the curve $y = \int_0^x \sqrt{\frac{u}{3}}\, du$ from $x = 0$ to $x = 9$ is

(A) 16

(B) 14

(C) $10\frac{1}{3}$

(D) $9\sqrt{3}$

(E) $4\frac{2}{3}$

Ans

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6. If a population of wolves grows according to the logistic equation

$$\frac{dN}{dt} = 0.05N - 0.0005N^2$$

where N is the number of wolves and t is measured in years, then $\lim_{t\to\infty} N(t) =$

(A) 50 (B) 75 (C) 100 (D) 150 (E) 200

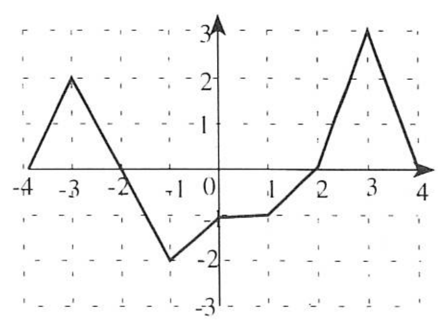
Ans

7. The slope of the line tangent to the graph of $y = 2[\text{Arc}\tan\sqrt{x}]^2$ at the point $\left(1, \frac{\pi^2}{8}\right)$ is

(A) 0 (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$ (E) π

Ans

8. The graph of the function f on the interval $[-4,4]$ is shown at the right.



$$\int_{-4}^{4} |f(x)|\, dx =$$

(A) 1

(B) 2

(C) 5

(D) 8

(E) 9

Ans

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9. Both x and y are functions of a third variable t and $y^2 + x^2 + y = 10$. If $\frac{dx}{dt} = -5$ when $x = 2$ and $y = 2$, then $\frac{dy}{dt} =$

(A) -1

(B) 1

(C) 2

(D) 3

(E) 4

Ans

10. The substitution $u = \ln x$ transforms the definite integral $\int_1^e \frac{1 - \ln x}{x^2}\, dx$ into

(A) $\int_0^1 (1-u)\, du$ (B) $\int_0^e (1-u)\, du$

(C) $\int_0^1 \frac{1-u}{e^u}\, du$ (D) $\int_0^1 \frac{1-u}{e^{2u}}\, du$ (E) $\int_0^e \frac{1-u}{e^u}\, du$

Ans

11. The number of cells of a certain type of bacteria increases continuously at a rate equal to two more than three times the number of bacteria present. If there are 10 present at the start and 42 present t hours later, the value of t is

(A) $3 \ln 4$

(B) $\ln 4$

(C) $\frac{1}{2} \ln 4$

(D) $\frac{1}{3} \ln 4$

(E) $\frac{1}{4} \ln 4$

Ans

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12. If $\dfrac{dy}{dx} = x \cdot \sec y$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ and $y = 0$ when $x = \sqrt{2}$, then when $x = 1$ the value of y is

(A) $-\dfrac{\pi}{6}$

(B) 0

(C) $\dfrac{\pi}{6}$

(D) $\dfrac{\pi}{4}$

(E) $\dfrac{\pi}{2}$

Ans

13. Which of the following are asymptotes of $y + xy - 2x = 0$?

I. $x = -1$ II. $x = 1$ III. $y = 2$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

Ans

14. The curve passing through (1, 0) satisfies the differential equation $\dfrac{dy}{dx} = 4x + y$. An approximation to $y(2)$ using Euler's Method with two equal steps is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Ans

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15. The function $f(x)=\begin{cases}4-x^2 & \text{for } x\le 1\\ mx+b & \text{for } x>1\end{cases}$ is continuous and differentiable for all real numbers. The values of m and b are

(A) $m=2,\ b=1$

(B) $m=2,\ b=5$

(C) $m=-2,\ b=1$

(D) $m=-2,\ b=5$

(E) none of these

Ans

16. $\int \frac{8}{(x-1)(x+3)}\,dx =$

(A) $2\ln\left|\frac{x+3}{x-1}\right| + C$

(B) $2\ln\left|\frac{x-1}{x+3}\right| + C$

(C) $2\ln|(x+3)(x-1)| + C$

(D) $2\ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

(E) $8\ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

Ans

17. If $f(x)=\frac{x-k}{x+k}$ and $k\neq 0$, then $f''(0)=$

(A) $-\frac{4}{k^2}$ (B) $-\frac{2}{k}$ (C) 0 (D) $\frac{2}{k}$ (E) $\frac{4}{k^2}$

Ans

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18. The base of a solid is a right triangle whose perpendicular sides have lengths 6 and 4. Each plane section of the solid perpendicular to the side of length 6 is a semicircle whose diameter lies in the plane of the triangle. The volume of the solid is

(A) 2π units^3

(B) 4π units^3

(C) 8π units^3

(D) 16π units^3

(E) 24π units^3

Ans

19. $\lim\limits_{h \to 0} \dfrac{(1+h)^3 - 1}{h} =$

(A) undefined

(B) 3

(C) 2

(D) 1

(E) 0

Ans

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20. Suppose a function f is defined so that it has derivatives $f'(x) = x^2(1-x)$ and $f''(x) = x(2-3x)$. Over which interval is the graph of f both increasing and concave up?

(A) $x < 0$ (B) $0 < x < \frac{2}{3}$ (C) $\frac{2}{3} < x < 1$ (D) $x > 1$ (E) none of these

Ans

21. The average value of the function $f(x) = \sqrt[3]{x^2}$ on the interval $[0,8]$ is

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{4}$ (D) $\frac{12}{5}$ (E) $\frac{17}{6}$

Ans

22. Let $f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x+2 & \text{if } x \geq 0 \end{cases}$ and let $F(x) = \int_{-2}^{x} f(t)\, dt$. Which of the following statements are true?

I. $F(1) = 6.5$ II. $F'(1) = 3$ III. $F''(1) = 1$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

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23. Which of the following three improper integrals converge?

I. $\int_1^{\infty} \frac{1}{x^3}\, dx$ II. $\int_0^{1} \frac{1}{\sqrt{x}}\, dx$ III. $\int_0^{1} \frac{1}{x^3}\, dx$

(A) II only (B) I and II only (C) I and III only (D) II and III only (E) I, II, III

Ans

24. The acceleration of a particle moving along the x-axis at time $t > 0$ is given by $a(t) = \frac{1}{t^2}$.

When $t = 1$ second, the particle is at $x = 2$ and moving with velocity -1 unit per second. The position when $t = \mathrm{e}$ seconds is

(A) $x = -2$

(B) $x = -1$

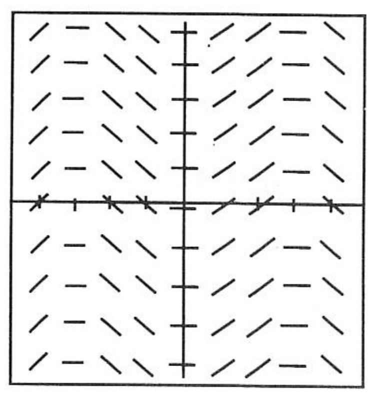
(C) $x = 0$

(D) $x = 1$

(E) $x = 2$

Ans

25. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = \tan x \cdot \sec x$

(B) $\frac{dy}{dx} = \sin x$

(C) $\frac{dy}{dx} = \sec^2 x$

(D) $\frac{dy}{dx} = \ln x$

(E) $\frac{dy}{dx} = e^{2x}$

Ans

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26. The area enclosed by the two curves $y = x^2 - 4$ and $y = x - 4$ is given by

(A) $\int_0^1 (x - x^2)\, dx$ (B) $\int_0^1 (x^2 - x)\, dx$

(C) $\int_0^2 (x - x^2)\, dx$ (D) $\int_0^2 (x^2 - x)\, dx$

(E) $\int_0^4 (x^2 - x)\, dx$

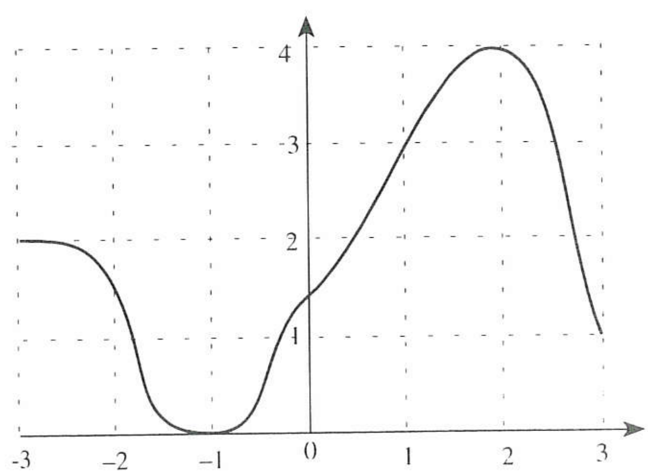
Ans

27. The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

Ans

28. The graph of f is shown at the right. Approximate $\int_{-3}^{3} f(x)\, dx$ using the Trapezoid Rule with 3 equal subdivisions.



(A) $2\frac{1}{4}$

(B) $4\frac{1}{2}$

(C) 9

(D) 18

(E) 36 s

Ans

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EXAM V
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem. Calculators may be used on this part of the examination.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. The graph of $g(x)=\int_0^x \sin(2t)\,dt$ is both decreasing and concave up everywhere on the interval

(A) $(0, \frac{\pi}{4})$ (B) $(\frac{\pi}{4}, \frac{\pi}{2})$ (C) $(\frac{\pi}{2}, \frac{3\pi}{4})$ (D) $(\frac{3\pi}{4}, \pi)$ (E) none of these

Ans

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2. The position at time $t > 0$ of a particle moving on the x-axis is given by $x(t)=\sin(t)\cdot\cos(t)$ At the first instant when the acceleration is 1 unit per sec^2, the particle has velocity

(A) – 1 unit per sec
(B) –0.866 units per sec
(C) 0 units per sec
(D) 0.866 units per sec
(E) 1 unit per sec

Ans

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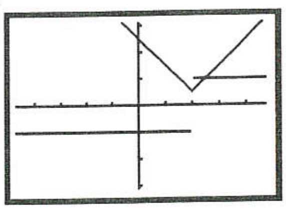
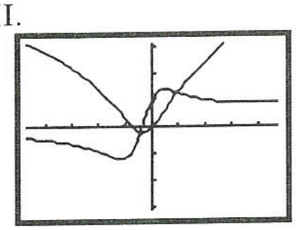
3. The average value of $f(x) = x \cdot \ln x$ on the interval $1 \le x \le e$ is

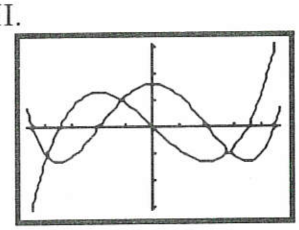
(A) 0.772 (B) 1.221 (C) 1.359 (D) 1.790 (E) 2.097

Ans

4. A pair of functions is graphed in each of the following viewing rectangles. In which of these is one function the derivative of the other?

I. 

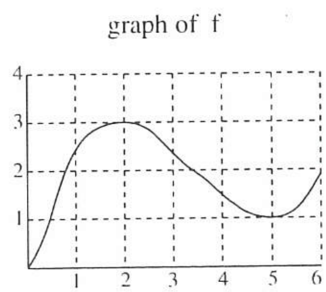
II. 

III. 

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

5. A graph of the function f is shown at the right. Which of the following statements is are true?



I. $\lim_{h \to 0} \dfrac{f(2+x) - f(2)}{h} = f'(5)$

II. $\dfrac{f(5) - f(2)}{5-2} = \dfrac{2}{3}$

III. $f''(1) \le f''(5)$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, III (E) none

Ans

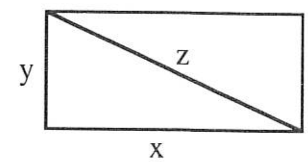
6. Let the function f be differentiable on the interval $[0, 2.5]$ and define g by $g(x) = f[f(x)]$. Use the table to estimate $g'(1)$.

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.7	1.8	2.0	2.4	3.1	4.4

(A) 0.8 (B) 1.2 (C) 1.6 (D) 2.0 (E) 2.4

Ans

7. The diagonal z of the rectangle at the right is increasing at the rate of 1 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec (B) $\frac{1}{2}$ cm/sec (C) $\frac{1}{3}$ cm/sec (D) $\frac{1}{4}$ cm/sec (E) $\frac{1}{15}$ cm/sec

Ans

8. The position of a particle moving on the x-axis, starting at time $t = 0$, is given by $x(t) = (t-a)^3(t-b)$ where $0 < a < b$. Which of the following statements are true?

I. The particle is at a positive position on the x-axis at time $t = \frac{a+b}{2}$.

II. The particle is at rest at time $t = a$.

III. The particle is moving to the right at time $t = b$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

Ans

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9. Let $f(x) = \dfrac{\ln e^{x+1}}{2x}$ for $x > 0$. If g is the inverse of f, then $g'(1) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

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10. Let $F(x) = \cos(2x) + e^{-x}$. For what value of x on the interval $[0,3]$ will F have the same instantaneous rate of change as the average rate of change of F over the interval ?

(A) 1.542 (B) 1.610 (C) 1.678 (D) 1.746 (E) 1.814

Ans

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11. The region R is enclosed by the graphs of $y = \dfrac{x}{x^2 - 2}$, $y = 0$, $x = 2$, and $x = k$, where $k > 2$. If the area of R is 1 square unit, then the value of k is

(A) 2.727 (B) 4.096 (C) 10.450 (D) 14.213 (E) 22.256

Ans

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12. $\int_{-1}^{2} (|2x+1| + |2x-1|)\, dx =$

(A) 8 (B) 9.5 (C) 10 (D) 10.5 (E) 11

Ans

13. The rate at which water is being pumped into a tank is $r(t) = 20e^{0.02t}$, where t is in minutes and $r(t)$ in gallons per minute. Approximtely how many gallons of water are pumped into the tank during the first five minutes?

(A) 20 (B) 22 (C) 85 (D) 105 (E) 150

Ans

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14. If $F(x) = 3t \cdot \sin t - \cos t - \frac{3\pi}{2}$, then an equation of the line tangent to the graph of F at the point where $x = \frac{\pi}{2}$ is

(A) $y = 3x - \frac{\pi}{2}$

(B) $y = 3(x - \frac{\pi}{2})$

(C) $y = 4x$

(D) $y = 4x - \frac{\pi}{2}$

(E) $y = 4x - 2\pi$

Ans

15. The minimum distance from the origin to the curve $y = \frac{x^3 - 3x^2 - 4x + 12}{6}$ is

(A) 1.00

(B) 1.07

(C) 1.41

(D) 1.90

(E) 1.98

Ans

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16. The area of the region bounded by the graphs of $y = \ln(x+4)$ and $y = 0.5x^2$ is

(A) 3.03 units^2 (B) 3.09 units^2 (C) 3.15 units^2 (D) 3.21 units^2 (E) 3.27 units^2

Ans

17. If f is a continuous function that is defined for all real numbers with

$\int_1^3 f(x)\,dx = \frac{5}{2}$ and $\int_1^5 f(x)\,dx = 10$, then $\int_3^5 [2f(x)+6]\,dx =$

(A) 18 (B) 21 (C) 24 (D) 27 (E) 30

Ans

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EXAM V
CALCULUS BC
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE
PLEASE TURN OVER