EXAM IV
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. The value of $\int_0^\infty e^{-x}dx$ is

(A) -1 (B) 0 (C) $\frac{1}{e}$ (D) 1 (E) nonexistent

Ans

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2. The graph of $f(x) = x\sin x$ defined on $0 < x < \pi$ has an inflection point whenever

(A) $\tan x = -\frac{2}{x}$

(B) $\tan x = \frac{2}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

Ans

3. The area of the region in the first quadrant bounded by the curve $y = e^{-x}$ and the line $x = \ln 2$ is equal to

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) $\frac{3}{2}$

Ans

4. $\lim_{x \to 3} \left[\frac{\ln\left(\frac{x-1}{2}\right)}{3-x} \right]$

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) 1

Ans

5. There is a point between $P(-1, 1)$ and $Q(7, 3)$ on the graph of $y = \sqrt{x+2}$ such that the line tangent to the graph at that point is parallel to the line through P and Q. The coordinates of this point are

(A) $(1, \sqrt{3})$

(B) $(2, 2)$

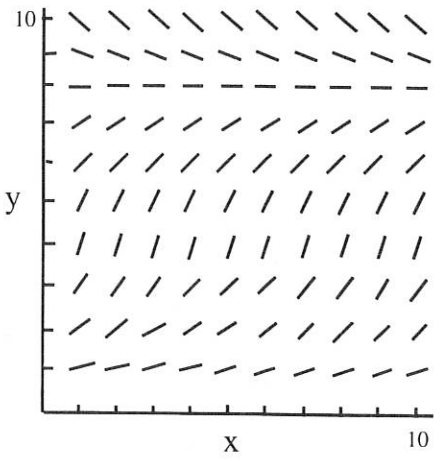
(C) $(3, \sqrt{5})$

(D) $(4, \sqrt{6})$

(E) none of the above

Ans

6. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure at the right. Which of the following statements are true?



I. The value of $\frac{dy}{dx}$ at the point $(3, 3)$ is approximately 1.

II. As y approaches 8 the rate of change of y approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of x.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

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7. $\lim_{h\to 0}\left(\frac{\tan(x+h)-\tan x}{h}\right)=$

(A) $\sec x$ (B) $-\sec x$ (C) $\sec^2 x$ (D) $-\sec^2 x$ (E) does not exist

Ans

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8. $\int \frac{x^2+2x+9}{x^2+9}\,dx =$

(A) $x+\frac{1}{8}\text{Arctan}\frac{x}{3}+C$

(B) $x+\frac{1}{4}\text{Arctan}\frac{x}{3}+C$

(C) $x+\frac{1}{2}\ln(x^2+9)+C$

(D) $1+\ln(x^2+9)+C$

(E) $x+\ln(x^2+9)+C$

Ans

9. If $y = \ln(\cos x)$ and $0 < x < \frac{\pi}{2}$, what is $\frac{d^2y}{dx^2}$ in terms of x?

(A) $\tan x$

(B) $-\tan x$

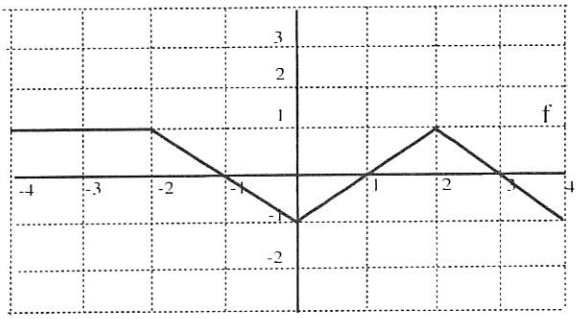
(C) $\sec^2 x$

(D) $-\sec^2 x$

(E) $-\csc^2 x$

Ans

10. The graph of f is shown at the right. Which of the following statements are true?



I. $f'(3) > f'(1)$

II. $\int_0^2 f(x)\,dx > f'(3.5)$

III. $\int_1^0 f(x)\,dx = \int_2^3 f(x)\,dx$

A) I only B) II only C) I and II only D) II and III only E) I, II and III

Ans

11. The base of a solid is the region in the first quadrant bounded by the curve $y = \sqrt{\sin x}$ for $0 \le x \le \pi$. If each cross section of the solid perpendicular to the x-axis is a square, the volume of the solid is

(A) 0 units^3 (B) 1 units^3 (C) 2 units^3 (D) 3 units^3 (E) 4 units^3

Ans

12. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)(t-6)^3$. The particle is moving to the left for

(A) $t > 3$

(B) $2 < t < 6$

(C) $3 < t < 6$

(D) $0 \leq t < 3$

(E) $t > 6$

Ans

13. A function f is defined for all real numbers and has the following property: $f(a+b) - f(b) = 3a^2b + 2b^2$. $f'(x)$ is

(A) 0

(B) 1

(C) $3x^2$

(D) $3x^2 + b$

(E) nonexistent

Ans

14. Consider the curve $5x - xy + y^2 = 7$. The slope of the line tangent to the curve at the point (1, 2) is

(A) –2

(B) –1

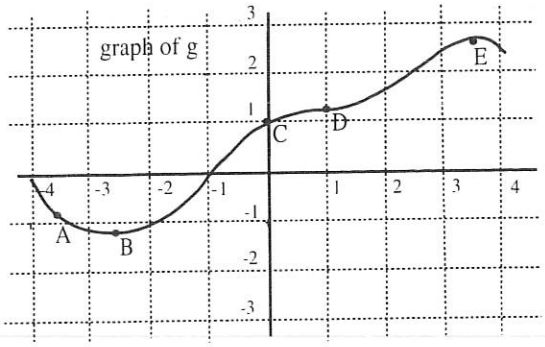
(C) 0

(D) 1

(E) 2

Ans

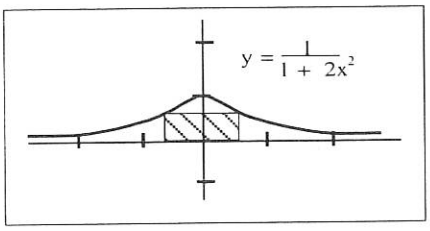
15. At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) < 0$?



(A) A (B) B (C) C (D) D (E) E

Ans

16. The height of the rectangle with the largest area that can be inscribed under the graph of $y = \dfrac{1}{1 + 2x^2}$ is



(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

(E) none of these

Ans

17. $\displaystyle\int_0^3 \sqrt{x^2 - 2x + 1}\ dx$ is

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 3

Ans

18. The function f is defined by $f(x) = x^3 + 1$. If f^{-1} is the inverse function of f and $h(x) = f^{-1}(x)$, then $h'(2)$ is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 1

(E) nonexistent

Ans

19. If $f(x) = \begin{cases} x^2 & \text{for } 0 \le x \le 2 \\ 6 - x & \text{for } x > 2 \end{cases}$, then $\int_0^4 f(x)\,dx$ is approximately

(A) $21\frac{1}{3}$

(B) $18\frac{2}{3}$

(C) 16

(D) $8\frac{2}{3}$

(E) $4\frac{1}{3}$

Ans

20. An equation of the line normal to the graph of $f(x) = \dfrac{x}{x-2}$ at $(1, -1)$ is

(A) $2x - y - 3 = 0$

(B) $2x + y + 1 = 0$

(C) $x - 2y + 3 = 0$

(D) $x + 2y + 1 = 0$

(E) $x - 2y - 3 = 0$

Ans

21. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1 + t,\ t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, then the position of the particle at $t = 2$ is

(A) $\langle 1, 12 \rangle$

(B) $\langle 4, 4 \rangle$

(C) $\langle 5, 9 \rangle$

(D) $\langle 9, 4 \rangle$

(E) $\langle 5, 0 \rangle$

Ans

22. Let f be defined by $f(x) = x^{2/3}(2x - 5)$. f is decreasing on the interval

(A) $x < -\dfrac{5}{2}$ (B) $-\dfrac{5}{2} < x < 0$ (C) $x > 1$ (D) $0 < x < \dfrac{5}{8}$ (E) $0 < x < 1$

Ans

23. What is $\lim_{x\to 0} \frac{1-e^{3x}}{\ln(1-x)}$?

(A) –1

(B) –3

(C) 1

(D) 3

(E) The limit does not exist.

Ans

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24. If f is continuous at $x = 2$, and if $f(x) = \begin{cases} \frac{\sqrt{x+2}-\sqrt{2x}}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$

then $k =$

(A) $-\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$

Ans

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25. The approximate value of $y = \sqrt{x^2+3}$ at $x = 1.04$, obtained from the tangent to the graph at $x = 1$, is

(A) 2.01

(B) 2.02

(C) 2.03

(D) 2.04

(E) 2.05

Ans

26. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(0) = 2$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) 3 (B) $\frac{7}{2}$ (C) $\frac{15}{4}$ (D) $\frac{19}{4}$ (E) $\frac{21}{4}$

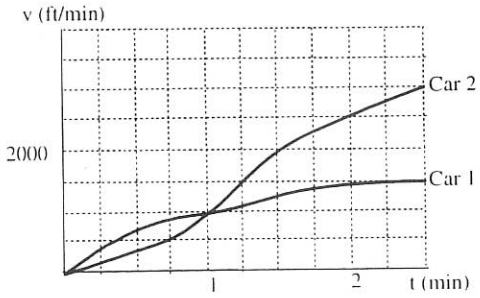
Ans

27. If $\frac{dy}{dx} = -10y$ and if $y = 50$ when $x = 0$, then $y =$

(A) $50\cos 10x$ (B) $50e^{10x}$ (C) $50e^{-10x}$ (D) $50 - 10x$ (E) $50 - 5x^2$

Ans

28. Two cars start from rest at a traffic light and accelerate for several minutes. The figure at the right shows their velocities as a function of time. Which of the following statements are true?



I. Car I is ahead at one minute.

II. Car II is ahead at two minutes.

III. Car I and Car II are accelerating at the same rate at $t = 1$.

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

Ans

EXAM IV
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem. Calculators may be used on this part of the examination.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distanced traveled in the first 10 seconds is

(A) 150 ft (B) 250 ft (C) 350 ft (D) 450 ft (E) 550 ft

Ans ☐

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2. If $f(x) = \dfrac{\ln x^2 - x \ln x}{x - 2}$ when $x \neq 2$, and f is continuous for all positive real numbers, then $f(2)$ is

(A) -1 (B) -2 (C) $-\dfrac{e}{4}$ (D) $-\ln 2$ (E) undefined

Ans ☐

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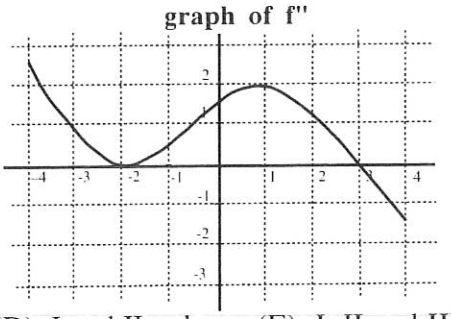
3. The graph of the **second derivative** of a function f is shown below. Which of the following are true about the original function f ?



I. The graph of f has an inflection point at $x = -2$.

II. The graph of f is concave down on the interval $(0, 4)$.

III. If $f'(0) = 0$, then f is increasing at $x = 2$.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II and III

Ans

4. If $y = 1$ when $x = 0$, solve $\frac{dy}{dx} = y(3 - y)$.

(A) $y(3-y) = 3e^{3x} - 1$

(B) $y(3-y) = 2e^{3x}$

(C) $y(3-y) = e^{3x} + 1$

(D) $y(3-y) = 2$

(E) $y(3-y) = e^{-3x} + 1$

Ans

5. $\int x^3 \ln x \, dx =$

(A) $\frac{x^4}{4}(4\ln x - 1) + C$

(B) $\frac{x^4}{16}(4\ln x - 1) + C$

(C) $\frac{x^2}{4}(\ln x - 1) + C$

(D) $3x^2(\ln x - \frac{1}{2}) + C$

(E) $x^2(3\ln x + 1) + C$

Ans

6. The position of a particle moving in the xy-plane is given by

$$x = t^2 + 2t, \quad y = 2t^2 - 6t.$$

What is the length of the velocity vector at $t = 2$?

(A) $2\sqrt{10}$

(B) $4\sqrt{10}$

(C) $6\sqrt{10}$

(D) $8\sqrt{10}$

(E) $10\sqrt{10}$

Ans

7. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\frac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinate $(1, 2)$ is equal to

(A) 1.565 (B) 2.236 (C) 3.354 (D) 6.708 (E) 7.500

Ans

8. If $F(x) = \int_0^x \frac{\sin t}{1 + \cos t}\, dt$, then $F''\left(\frac{\pi}{3}\right)$ is

(A) $-\sqrt{3}$

(B) $-\frac{1}{2}$

(C) $4 - 2\sqrt{3}$

(D) $2 - \sqrt{3}$

(E) $\frac{2}{3}$

Ans

9. If the substitution $u = \sqrt{x+1}$ is made in the integrand of $\int_0^3 \frac{1}{x\sqrt{x+1}}\,dx$, the resulting integral is

(A) $\int_1^2 \frac{1}{u^2-1}\,du$ (B) $\int_1^2 \frac{2}{u^2-1}\,du$ (C) $\int_0^3 \frac{1}{(u-1)(u+1)}\,du$

(D) $2\int_1^2 \frac{1}{u(u^2-1)}\,du$ (E) $2\int_0^3 \frac{1}{u^2-u}\,du$

Ans

10. The functions f and g are defined on the closed interval [0, b] by $f(x) = \cos(2x)$ and $g(x) = e^x - 1$. They will have the same average value if b is

(A) 0.848 (B) 0.852 (C) 0.854 (D) 0.858 (E) 0.862

Ans

11. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$ is

(A) ∞ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6

Ans

12. Which of the following functions has a derivative at $x = 0$?

I. $y = \arcsin(x^2 - 1) - x$

II. $y = x \cdot |x|$

III. $y = \sqrt{x^4}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

13. The slope of the tangent line to the curve $y(\cos x) + e^y = 5$ at the point where $x = \frac{\pi}{2}$ is

(A) 0 (B) 5 (C) $\frac{\ln 5}{5}$ (D) $\frac{(5 + \ln 5)}{5}$ (E) none of these

Ans

14. The following table lists the known values of a function f.

x	1	2	3	4	5
$f(x)$	0	1.1	1.4	1.2	1.5

If the Trapezoid Rule is used to approximate $\int_1^5 f(x)\,dx$ the result is

(A) 4.1

(B) 4.3

(C) 4.5

(D) 4.7

(E) 4.9

Ans

15. Which of the following statements are true about the function f, if its derivative f' is defined by $f'(x) = x(x-a)^3,\ a > 0$.

I. The graph of f is increasing at $x = 2a$.

II. The function f has a local maximum at $x = 0$.

III. The graph of f has an inflection point at $x = a$.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, III

Ans

16. The approximate total area of the region enclosed by the polar graph of $r = \sin(2\theta)$ is

(A) 0.393 units^2

(B) 0.785 units^2

(C) 1.178 units^2

(D) 1.571 units^2

(E) 1.873 units^2

Ans

17. The average rate of change of the differentiable function f from $(3, f(3))$ to $(x, f(x))$ is given by $\dfrac{x^2 - x - 6}{x - 3}$. The value of $f'(3)$ is

(A) 0

(B) 1

(C) 3

(D) 5

(E) undefined

Ans

EXAM IV
CALCULUS BC
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER