EXAM III
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. A particle moves on the x-axis in such a way that its position at time t, $t > 0$, is given by $x(t) = (\ln x)^2$. At what value of t does the velocity of the particle attain its maximum?

(A) 1 (B) $e^{1/2}$ (C) e (D) $e^{3/2}$ (E) e^2

Ans

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2. Which of the following is equal to $\int_0^{\pi} \cos x\, dx$?

(A) $\int_0^{\pi} \sin x\, dx$ (B) $\int_{-\pi/2}^{\pi/2} \cos x\, dx$ (C) $\int_{-\pi/2}^{\pi/2} \sin x\, dx$

(D) $\int_{\pi}^{2\pi} \sin x\, dx$ (E) $\int_{\pi/2}^{3\pi/2} \cos x\, dx$

Ans

6. Let f be the function defined by $f(x) = \ln(x+1)$.

   (a) Find $f^{(n)}(0)$ for $n = 1$ to $n = 3$, where $f^{(n)}$ is the n^{th} derivative of f.

   (b) Write the first three nonzero terms and the general term for the Taylor series expansion of $f(x)$ about $x = 0$.

   (c) Determine the radius of convergence for the series in part (b). Show your reasoning.

   (d) Use the series in part (b) to evaluate $\int_0^{0.5} f(x)\,dx$ with an error no greater than 0.001.

3. A solution to $\frac{dy}{dx} = \frac{1}{xy}$ that goes through the point $(1, 1)$ is

(A) $\frac{1}{x^2}$

(B) $\sqrt{2 \ln x} + 1$

(C) $\sqrt{2 \ln x + 1}$

(D) $\sqrt{\ln x + 1}$

(E) e^{x-1}

Ans

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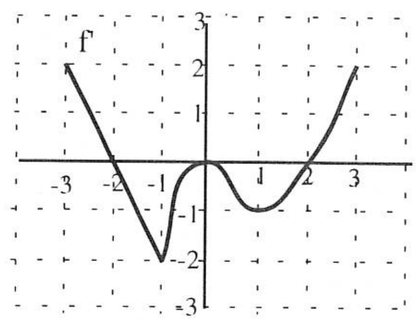
4. At the right is the graph of $y = f'(x)$, the derivative of $y = f(x)$. The domain of f is the interval $-3 \le x \le 3$. Which of the following are true about the graph of f?



I. f is increasing on $-3 < x < -2$.

II. f is concave down on $-3 < x < -1$.

III. The maximum value of $f(x)$ on $-3 < x < 2$ is $f(-3)$.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

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5. $\int_{e}^{+\infty} \frac{1}{x(\ln x)^2} \, dx$

(A) $\frac{1}{e}$ (B) $-\frac{1}{e}$ (C) e (D) 1 (E) divergent

Ans

6. If $x = t^2$ and $y = (t^2 + 1)^2$, then at $t = 3$, $\frac{dy}{dx}$ is

(A) 0

(B) $\frac{5}{3}$

(C) 6

(D) 20

(E) undefined

Ans

7. Suppose a population of bears grows according to the logistic differential equation

$$\frac{dP}{dt} = 2P - 0.01P^2$$

where P is the number of bears at time t in years. Which of the following statements are true?

I. The growth rate of the bear population is greatest at $P = 100$.

II. If $P > 200$, the population of bears is decreasing.

III. $\lim_{t \to \infty} P(t) = 200$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Ans

8. The substitution of $x = \sin\theta$ in the integrand of $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$, results in

(A) $\int_0^{1/2} \frac{\sin^2\theta}{\cos\theta}\,d\theta$ (B) $\int_0^{1/2} \sin^2\theta\,d\theta$ (C) $\int_0^{\pi/6} \sin^2\theta\,d\theta$

(D) $\int_0^{\pi/3} \sin^2\theta\,d\theta$ (E) $\int_0^{1/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

Ans

9. Let f and g be functions whose derivatives exist for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} f'(x) = 6$ and $\lim_{x\to 0} g'(x) = 2$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) 1

(C) 3

(D) $\frac{f'(x)}{g'(x)}$

(E) nonexistent

Ans

10. The slope of the tangent to the graph of $y = \text{Arc}\tan\frac{x}{2}$ at $(2, \frac{\pi}{4})$ is

(A) $\frac{1}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

Ans

11. If $\frac{dy}{dx} = \frac{3\sin x}{\sec^2 x}$, then $y =$

(A) $\ln|\cos x| + C$

(B) $\sec x + C$

(C) $\cos^3 x + C$

(D) $-3\cos^3 x + C$

(E) $-\cos^3 x + C$

Ans

12. Consider the set of all right circular cylinders for which the sum of the height and the diameter is 18 inches. What is the radius of the cylinder with the maximum volume?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Ans

13. The total area of the region enclosed by the polar graph of $r = 1 + \sin\theta$ is

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

(E) $\frac{5\pi}{2}$

Ans

14. The acceleration of a particle moving along the x-axis at any time $t \geq 0$ is given by $a(t) = 1 + e^{-t}$. At $t = 0$ the velocity of the particle is -2 and its position is 3. The position of the particle at any time t is

(A) $\dfrac{t^2}{2} - t + e^{t} + 2$

(B) $\dfrac{t^2}{2} - 3t + e^{-t} + 2$

(C) $\dfrac{t^2}{2} - t - e^{-t} + 2$

(D) $\dfrac{t^2}{2} - 3t - e^{-t} + 2$

(E) $t^2 - t + e^{-t} + 2$

Ans

15. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

Ans

16. If $v = \sin(u^2 - 1)$ and $u = \sqrt{x^2 + 1}$, then $\dfrac{dv}{dx}$ is

(A) $\dfrac{\cos(x^2)}{2\sqrt{x^2+1}}$

(B) $\dfrac{x\cos(x^2)}{2\sqrt{x^2+1}}$

(C) $\dfrac{x\cos(x^2-1)}{\sqrt{x^2+1}}$

(D) $2x\cos(x^2)$

(E) $\cos(x^2)$

Ans

17. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x\to c} f(x)$ exists

(B) $\lim_{x\to c} f(x) = f(c)$

(C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

(D) $f(c)$ is defined

(E) $f'(c)$ exists

Ans

18. The area of the region in the first quadrant under the curve $y = \dfrac{1}{\sqrt{1-x^2}}$ bounded on the left by $x = \frac{1}{2}$ and on the right by $x = 1$ is

(A) ∞

(B) π

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{\pi}{3}$

(E) none of these

Ans

19. The function f is defined by $f(x) = 3x^2 - x^3 + h$. For which values of h will f have three distinct zeros ?

(A) all $h > 4$
(B) $0 < h < 4$
(C) all $h < 0$
(D) $-4 < h < 0$
(E) all $h < -4$

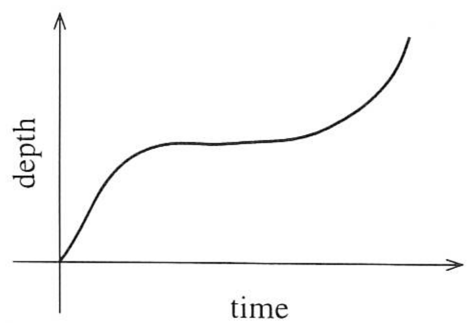
Ans

20. An isosceles triangle has one vertex at the origin and the other two at the points where a line parallel to and above the x-axis intersects the curve $y = 12 - x^2$. The maximum area of the triangle is

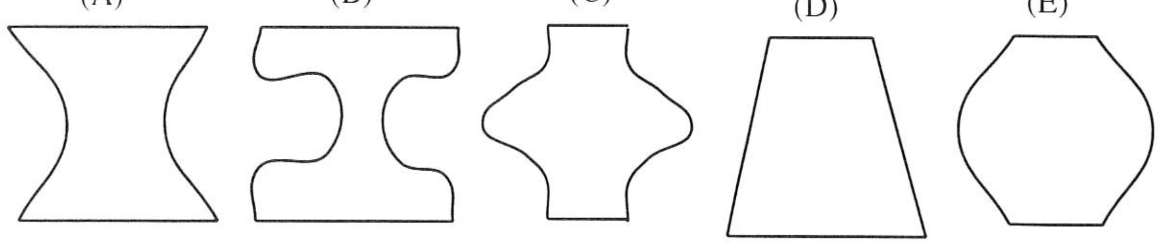
(A) 40 (B) 32 (C) 24 (D) 16 (E) 8

Ans

21. Every cross section perpendicular to the axis of a container is a circle. Water is flowing into the container at a constant rate. A graph of the depth of the water as a function of time is shown at the right. Which of the following best describes the profile of the container?



(A) (B) (C) (D) (E)



Ans

22. The sales of a small company are expected to grow at a rate given by $\frac{dS}{dt} = 300t + t^{1/2} + t^{3/2}$, where $S(t)$ is the sales in dollars in t days. The accumulated sales through 4 days is approximately

(A) \$2400
(B) \$2406
(C) \$2412
(D) \$2418
(E) \$2424

Ans

23. If $F(x) = \int_{\pi/2}^{x} 4t \sin\left(\frac{t}{3}\right) dt$, then an equation of the line tangent to $y = F(x)$ at the point where $x = \frac{\pi}{2}$ is

(A) $2x - \pi y - \pi = 0$
(B) $2x - 2y - \pi = 0$
(C) $2\pi x - 2y - \pi^2 = 0$
(D) $\pi x - 2y - \pi^2 = 0$
(E) $\pi x - y - \pi = 0$

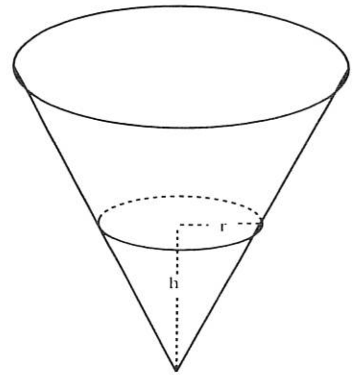
Ans

24. If $\int_{0}^{k} \frac{\sec^2 x}{1 + \tan x}\, dx = \ln 2$, then the value of k is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

Ans

25. A conical tank is being filled with water at the rate of 16 ft^3/min. The rate of change of the depth of the water is 4 times the rate of change of the radius of the water surface. At the moment when the depth is 8 ft and the radius of the surface is 2 ft, the area of the surface is changing at the rate of



(A) $\frac{1}{\pi}$ ft^2/min

(B) 1 ft^2/min

(C) 4 ft^2/min

(D) 4π ft^2/min

(E) 16π ft^2/min

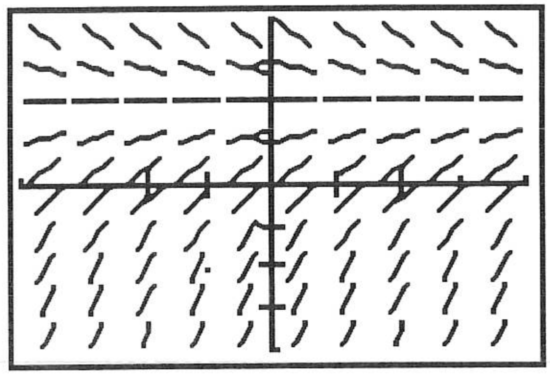
Ans

26. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+1}$ and $y(0) = 0$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{9}{10}$

Ans

27. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given at the right. Which of the following could be a solution ?



(A) $y = 2 + \ln x$

(B) $y = 2 - \ln x$

(C) $y = 2 - e^{x}$

(D) $y = 2 - e^{-x}$

(E) $y = 2 + e^{2x}$

Ans

28. $\int x\, e^{2x}\, dx =$

(A) $\frac{1}{4}\, e^{2x}(2x - 1) + C$

(B) $\frac{1}{2} e^{2x}(2x - 1) + C$

(C) $\frac{1}{4} e^{2x}(4x - 1) + C$

(D) $\frac{1}{2} e^{2x}(x - 1) + C$

(E) $\frac{1}{4}\, e^{2x}(x - 1) + C$

Ans

EXAM III
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem. Calculators may be used on this part of the examination.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. Which of the following is (are) true about a particle that starts at $t = 0$ and moves along a number line if its position at time t is given by $s(t) = (t-2)^3(t-6)$?

I. The particle is moving to the right for $t > 5$.
II. The particle is at rest at $t = 2$ and $t = 6$.
III. The particle changes direction at $t = 2$.

(A) I only (B) II only (C) III only (D) I and III only (E) none

Ans

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2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\,dt$ over the interval $[1, 3]$ is

(A) 0.19 (B) 0.23 (C) 0.27 (D) 0.31 (E) 0.35

Ans

3. $\int \frac{1}{\sqrt{x}\,(1-\sqrt{x})}\,dx =$

(A) $\frac{1}{2}\ln\left|1-\sqrt{x}\right|+C$

(B) $2\ln\left|1-\sqrt{x}\right|+C$

(C) $4\sqrt{1-\sqrt{x}}\ +C$

(D) $-2\ln\left|1-\sqrt{x}\right|+C$

(E) none of these

Ans

4. Let R be the region in the first quadrant that is enclosed by the graph of $f(x)=\ln(x+1)$, the x-axis and the line $x=\mathrm{e}$. What is the volume of the solid generated when R is rotated about the line $y=-1$?

(A) 5.037 (B) 6.545 (C) 10.073 (D) 20.146 (E) 28.686

Ans

5. $\lim_{h\to 0} \dfrac{\int_1^{1+h}\sqrt{x^3+8}\ dx}{h}$ is

(A) 0

(B) 1

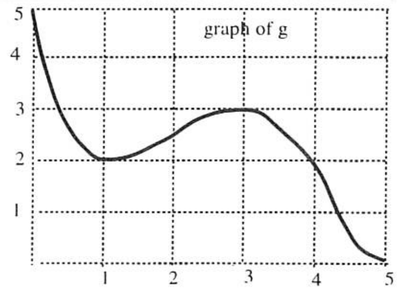
(C) 3

(D) $2\sqrt{2}$

(E) nonexistent

Ans

6. A graph of the function g is shown in the figure. If the function h is defined by $h(x) = g(x^2)$, use the graph to estimate $h'(2)$.



(A) –8 (B) –4 (C) –2 (D) 2 (E) 4

Ans

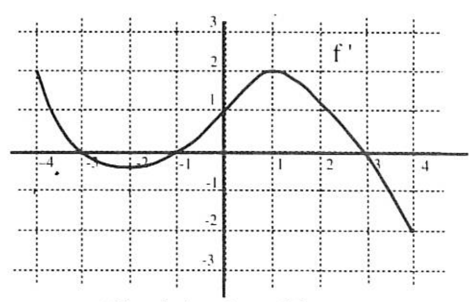
7. $\int_0^{\infty} xe^{-x^2}\,dx$ is

(A) –1 (B) 0 (C) 1 (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

Ans

8. The graph of the **derivative** of a function f is shown to the right. Which of the following are true about the original function f?

I. f is increasing on the interval (–2, 1).
II. f is continuous at $x = 0$.
III. f has an inflection point at $x = -2$.



The derivative of f

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

9. A curve is defined parametrically by $x = e^t$ and $y = 2e^{-t}$. An equation of the tangent line to the curve at $t = \ln 2$ is

(A) $x - 2y + 3 = 0$

(B) $x + 2y - 4 = 0$

(C) $x + 2y - 5 = 0$

(D) $x - 2y - 4 = 0$

(E) $2x + y - 5 = 0$

Ans

10. If $x^2 - y^2 = 25$ then $\dfrac{d^2y}{dx^2} =$

(A) $-\dfrac{x}{y}$ (B) $\dfrac{5}{y^2}$ (C) $-\dfrac{x^2}{y^3}$ (D) $-\dfrac{25}{y^3}$ (E) $\dfrac{4}{y^3}$

Ans

11. Which of the following series are convergent?

I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} + \ldots$

II. $1 - \frac{1}{2} + \frac{1}{3} - \ldots + \frac{(-1)^n}{n} + \ldots$

III. $2 + 1 + \frac{8}{9} + \ldots + \frac{2^n}{n^2} + \ldots$

(A) I only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II and III

Ans

12. If $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \frac{x^2+1}{x^2}$, then $g(x)$ could be equal to

(A) x^{-3} (B) $-2x^{-3}$ (C) $\frac{x^2-1}{x}$ (D) $x - x^2$ (E) $1 + x^{-2}$

Ans

13. Two particles move along the x-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

Ans

14. A rectangle with one side on the x-axis has its upper vertices on the graph of the parabola $y = 4 - x^2$. The maximum area of such a rectangle is

(A) 1.155 (B) 1.855 (C) 3.709 (D) 6.158 (E) 12.316

Ans

15. The radius of convergence of the series $x + \frac{2x^2}{2^2} + \frac{6x^3}{3^3} + \cdots + \frac{n!x^n}{n^n} + \cdots$ is

(A) ∞ (B) e^2 (C) e (D) $\frac{e}{2}$ (E) 0

Ans

16. When using the method of partial fractions to decompose $\dfrac{8x-4}{x^2+2x-3}$, one of the fractions obtained is

(A) $\dfrac{1}{x+3}$ (B) $\dfrac{7}{x-1}$ (C) $\dfrac{7}{x+3}$ (D) $\dfrac{1}{x-3}$ (E) $\dfrac{7}{x+1}$

Ans

17. A particle moves on the xy-plane so that at time t, $0 \le t \le 5$, its acceleration vector is $\left\langle \sin t,\ e^{-t} \right\rangle$. If the particle is at rest when $t = 0$, what is the maximum speed it obtains?

(A) 2.10 (B) 2.22 (C) 2.34 (D) 2.46 (E) 2.58

Ans

EXAM III
CALCULUS BC
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE
PLEASE TURN OVER