EXAM II
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

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1. $\int_0^1 xe^{x^2}\,dx$

(A) $\frac{1}{2}(e-1)$ (B) $2(e-1)$ (C) $2e$ (D) e (E) $\frac{1}{2}e$

Ans ☐

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2. If $f(x) = x^2 - 1$, then $\lim_{x\to 1} \frac{f(x+1)-f(2)}{x^2-1}$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) nonexistent

Ans ☐

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6. The rate at which a rumor spreads through a high school of 1000 students can be modeled by the differential equation $\frac{dH}{dt} = kH(1000 - H)$, where k is a positive constant and H is the number of students that have heard the rumor t hours after 8 am.

   (a) Explain the meaning of the expression $(1000 - H)$ in the differential equation.

   (b) How many students have heard the rumor when the rumor is spreading the fastest?

   (c) Suppose that 2 students know the rumor at 8 am and that 100 have heard it by 10 am. Write a formula for H as a function of t.

   (d) At what time of day has half of the student body heard the rumor?

3. If $\cos x = e^y$ and $0 < x < \pi$, then $\dfrac{dy}{dx}$ is

(A) $-\tan x$

(B) $-\cot x$

(C) $\tan x$

(D) $\cot x$

(E) $\csc x$

Ans

4. If $y = \text{Arcsin}(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1 - e^{4x}}}$

(B) $\dfrac{e^{2x}}{\sqrt{1 - e^{4x}}}$

(C) $\dfrac{2e^{2x}}{\sqrt{1 + e^{4x}}}$

(D) $\dfrac{e^{2x}}{1 - e^{4x}}$

(E) $\dfrac{2e^{2x}}{\sqrt{e^{4x} - 1}}$

Ans

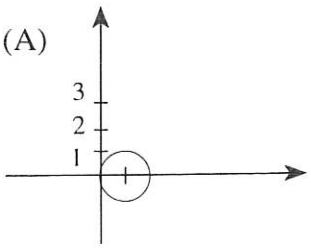
5. If $f(x) = \displaystyle\int_2^{2x} \frac{1}{\sqrt{t^3 + 1}}\, dt$, then $f'(1) =$

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\sqrt{2}$ (E) undefined

Ans

6. Which of the following represents the graph of the polar curve $r = 3\csc\theta$?

(A)



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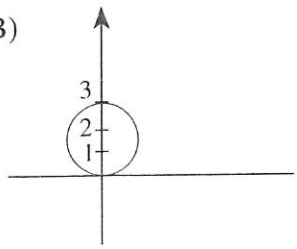

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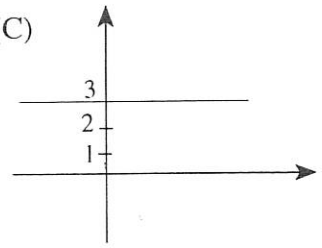

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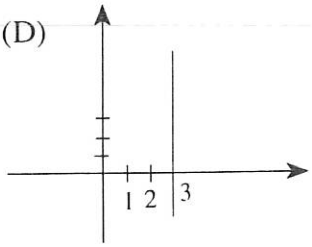
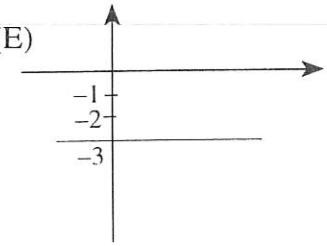


(E)



Ans

7. If $g(x) = \tan^2(e^x)$, then $g'(x) =$

(A) $2e^x \tan(e^x)\sec^2(e^x)$

(B) $2\tan(e^x)\sec^2(e^x)$

(C) $2\tan^2(e^x)\sec(e^x)$

(D) $e^x \sec^2(e^x)$

(E) $2e^x \tan(e^x)$

Ans

8. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Ans

9. If $e^{g(x)} = 2x + 1$, then $g'(x) =$

(A) $\frac{1}{2x + 1}$ (B) $\frac{2}{2x + 1}$ (C) $2(2x + 1)$ (D) e^{2x+1} (E) $\ln(2x + 1)$

Ans

10. F and G are two functions whose derivatives exist for all real x; $F'(x) < 0$ and $G'(x) > 0$ for all real x. Which of the following could be true about the graphs of $y = F(x)$ and $y = G(x)$?

I. they do not intersect II. they intersect once III. they intersect more than once

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

11. The length of the curve determined by the parametric equations $x = \sin t$ and $y = t$ from $t = 0$ to $t = \pi$ is

(A) $\int_0^{\pi} \sqrt{\cos^2 t + 1}\ dt$

(B) $\int_0^{\pi} \sqrt{\sin^2 t + 1}\ dt$

(C) $\int_0^{\pi} \sqrt{\cos t + 1}\ dt$

(D) $\int_0^{\pi} \sqrt{\sin t + 1}\ dt$

(E) $\int_0^{\pi} \sqrt{1 - \cos t}\ dt$

Ans

12. $\lim_{x \to 0} \dfrac{e^{3x} - 1}{\sin x} =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) nonexistent

Ans

13. The slope of the line tangent to the graph of $\ln(x + y) = x^2$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e – 1 (D) 2e – 1 (E) e – 2

Ans

14. At $x = 0$, which of the following is true of the function $f(x) = \sin x + e^{-x}$?

(A) f is increasing
(B) f is decreasing
(C) f is discontinuous
(D) f is concave up
(E) f is concave down

Ans

15. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}$ is

(A) 4
(B) 3
(C) 2
(D) 1
(E) 0

Ans

16. A particle moves along the curve $x^2y = 2$ at time $t > 0$. If $\frac{dy}{dt} = 8$ when $x = -1$, what is the value of $\frac{dx}{dt}$ at that time?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Ans

17. If $\int_a^b f(x)\,dx = 3$ and $\int_a^b g(x)\,dx = 2$, which of the following must be true?

I. $f(x) > g(x)$ for all $a \le x \le b$

II. $\int_a^b [f(x) + g(x)]\,dx = 1$

III. $\int_a^b [f(x) \cdot g(x)]\,dx = 6$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

18. Consider the curve in the xy-plane represented by $x = \frac{2}{t}$ and $y = \ln t$ for $t > 0$. The slope of the line tangent to the curve at the point where $x = 1$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Ans

19. If $\frac{dy}{dx} = xy^2$, then y could be

(A) $\frac{-1}{x^2 + 1}$

(B) $-\frac{1}{x^2} + 1$

(C) $\frac{-2}{x^2 + 1}$

(D) $3e^{x^2/2}$

(E) $3e^{x^2/2} + 1$

Ans

20. $\int \frac{x}{x + 2}\, dx$

(A) $x \ln|x+2| + C$

(B) $x+2 \ln|x+2| + C$

(C) $x-2 \ln|x+2| + C$

(D) $x - \ln|x+2| + C$

(E) $x - \text{Arc tan } x + C$

Ans

21. Let f be a function with $f(2) = 4$ and derivative $f'(x) = \sqrt{x^3 + 1}$. Using a tangent line approximation to the graph of f at $x = 2$, estimate $f(2.2)$.

(A) 4.0 (B) 4.2 (C) 4.4 (D) 4.6 (E) 4.8

Ans

22. A region in the plane is bounded by $y = \frac{1}{\sqrt{x}}$, the x-axis, the line $x = m$ and the line $x = 2m$ where $m > 0$. A solid is formed by revolving the region about the x-axis. The volume of this solid

(A) is independent of m

(B) increases as m increases

(C) decreases as m increases

(D) increases until $m = \frac{1}{2}$, then decreases

(E) is none of the above

Ans

23. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left\langle \sin(3t - \frac{\pi}{2}),\ 3t^2 \right\rangle$, then at time $t = \frac{\pi}{2}$ the velocity vector is

(A) $\langle -3, 3\pi \rangle$ (B) $\langle -1, 3\pi \rangle$ (C) $\langle -1, 2\pi \rangle$ (D) $\langle 3, 2\pi \rangle$ (E) $\langle 3, 3\pi \rangle$

Ans

24. The value of the derivative of $y = \dfrac{(x^2 - 3)^3}{(5x - 9)^2}$ at $x = 2$ is

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

Ans

25. F and G are differentiable functions such that $F(x) = \int_0^x G(t)\,dt$. If $F(a) = 3$ and $F(b) = 3$, where $0 < a < b$, which of the following must be true?

(A) $G(x) = 0$ for some x such that $a < x < b$

(B) $G(x) = 0$ for all x such that $a < x < b$

(C) $G(x) > 0$ for all x such that a < x < b

(D) $F(x) \geq 0$ for all x such that $a < x < b$

(E) $F(x) = 0$ for some x such that $a < x < b$

Ans

26. $\int_0^1 xe^{-x}\,dx =$

(A) 1 (B) $1-\frac{2}{e}$ (C) $\frac{2}{e}-1$ (D) $1+\frac{2}{e}$ (E) $-\frac{2}{e}$

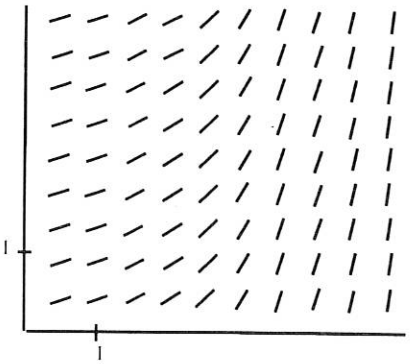
Ans

27. If the average rate of change of a function f over the interval from $x=2$ to $x=2+h$ is given by $7e^h - 4\cos(2h)$, then $f'(2) =$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Ans

28. The slope field shown in the figure at the right represents solutions to a certain differential equation. Which of the following could be a specific solution to that differential equation ?



(A) $y = e^{-x}$

(B) $y = \sin x$

(C) $y = \sqrt{x}$

(D) $y = \ln x$

(E) $y = e^{0.5x}$

Ans

EXAM II
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time--50 minutes
Number of questions--17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. The area of the region enclosed by the graph of the polar curve $r = 2 + 2\cos\theta$ is

(A) 4.712 (B) 9.424 (C) 18.849 (D) 37.699 (E) 75.398

Ans ☐

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2. Which of the following are true about the function f if its derivative is defined by

$$f'(x) = (x-1)^2(4-x) \ ?$$

I. f is decreasing for all $x < 4$.
II. f has a local maximum at $x = 1$.
III. f is concave up for all $1 < x < 3$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II and III

Ans ☐

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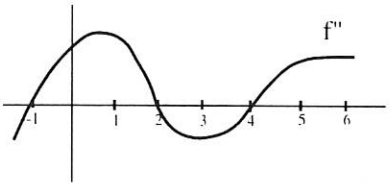
3. A tangent line drawn to the graph of $y = \dfrac{4x}{1 + x^3}$ at the point $(1, 2)$ forms a right triangle with the coordinate axes. The area of the triangle is

(A) 3.0
(B) 3.5
(C) 4.0
(D) 4.5
(E) 5.0

Ans

4. The graph of the second derivative f'' for a function f is shown below.



If f is increasing at $x = -1$, which of the following statements must be true?

I. $f'(2) = f'(4)$ II. $f'(4) > f'(-1)$ III. $f'(4) > 0$

(A) I only (B) II only (C II and III only (D) I and III only (E) I, II and III

Ans

5. Suppose a particle is moving along a coordinate line and its position at time t is given by $s(t) = \dfrac{9t^2}{t^2 + 2}$. For what value of t in the interval $[1, 4]$ is the instantaneous velocity equal to the average velocity?

(A) 2.00 (B) 2.11 (C) 2.22 (D) 2.33 (E) 2.44

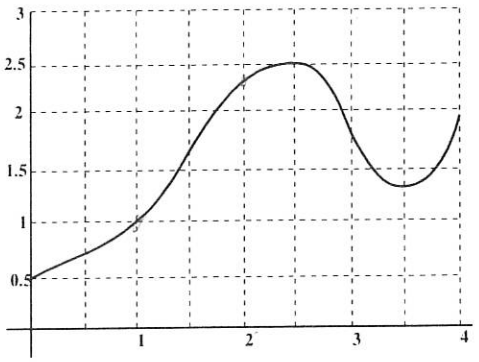
Ans

6. If $f(x) = \dfrac{e^x}{x+1}$ and $g(x) = \dfrac{x}{x+1}$, then $f'(x) = g'(x)$ at $x =$

(A) 0.563 (B) 0.565 (C) 0.567 (D) 0.569 (E) 0.571

Ans

7. A graph of the function f is shown at the right. Which of the following statements are true?



I. $f(1) > f'(3)$

II. $\displaystyle\int_1^2 f(x)\,dx > f'(3.5)$

III. $\displaystyle\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} > \frac{f(2.5) - f(2)}{2.5 - 2}$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II and III

Ans

8. Which of the following three improper integrals converge?

I. $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$ II. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ III. $\displaystyle\int_0^1 \frac{1}{x}\,dx$

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

Ans

9. Using the method of partial fractions to decompose $\frac{5x-1}{x^2-1}$, one of the fractions obtained is

(A) $\frac{2}{x+1}$ (B) $\frac{3}{x-1}$ (C) $\frac{2}{x-1}$ (D) $\frac{5}{x+1}$ (E) $\frac{5}{x-1}$

Ans

10. If the graph of $y = f(x)$ contains the point (0, 1), and if $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$, then $f(x)=$

(A) $\sqrt{2-\cos(x^2)}$

(B) $\sqrt{2} - \cos(x^2)$

(C) $2 - \cos(x^2)$

(D) $\cos(x^2)$

(E) $\sqrt{2-\cos x}$

Ans

11. If $g(x) = \int_0^{x^2} (t^2 + 7)^{2/3}\, dt$, then $g''(1) =$

(A) $\frac{2}{3}$ (B) 4 (C) $\frac{16}{3}$ (D) $\frac{32}{3}$ (E) 8

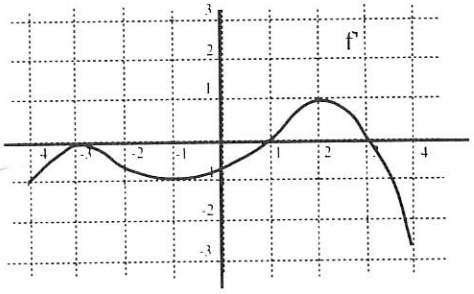
Ans

12. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-2x}{y}$ is a family of

(A) straight lines
(B) circles
(C) hyperbolas
(D) parabolas
(E) ellipses

Ans

13. The figure at the right shows the graph of f', the *derivative* of a function f The domain of f is the interval $-4 \le x \le 4$. Which of the following is are true about the graph of f?



graph of the derivative of f

I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents.
II. At the point where $x = 1$ there is a relative minimum point.
III. At the point where $x = -3$ there is an inflection point.

(A) None (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

14. The volume of the solid generated by revolving the first quadrant region bounded by the curve $y = e^{x/2}$ and the lines $x = \ln 3$ and $y = 1$ about the x-axis is

(A) 2.80 (B) 2.83 (C) 2.86 (D) 2.89 (E) 2.92

Ans

15. Consider $f(x) = 12 - x^2$ for $0 \le x \le 2\sqrt{3}$. Let $A(t)$ be the area of the triangle formed by the coordinate axes and the tangent to the graph of f at the point $(t, 12 - t^2)$. For what value of t is $A(t)$ a minimum?

(A) 1.8
(B) 1.9
(C) 2.0
(D) 2.1
(E) 2.2

Ans

16. If the function f is differentiable on the interval $[a, b]$ and $a < c < b$, which of the following statements are true?

I. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

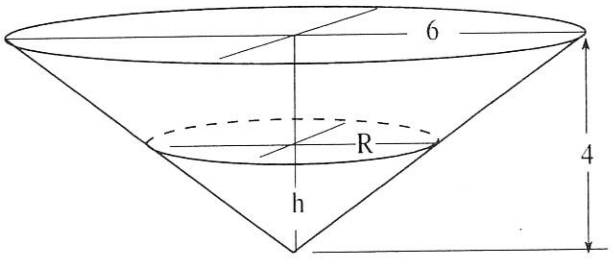
II. There exists a number d in (a, b) such that $f'(d) = \dfrac{f(b) - f(a)}{b - a}$

III. $\lim_{x \to c} f(x) = f(c)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

17. The conical reservoir shown at the right has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of



$V = \frac{1}{3}\pi R^2 h$

(A) 0.177 ft/min

(B) 0.354 ft/min

(C) 0.531 ft/min

(D) 0.708 ft/min

(E) 0.885 ft/min

Ans