EXAM I
CALCULUS BC
SECTION I PART A
MULTIPLE-CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

1. $\int_0^5 \frac{dx}{\sqrt{3x+1}} =$

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) 2

(E) 6

Ans

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2. Which of the following is continuous at $x = 0$?

I. $f(x) = |x|$

II. $f(x) = e^x$

III. $f(x) = \ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) none

Ans

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3. For what value of c will $x^2 + \frac{c}{x}$ have a relative minimum at $x = -1$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

Ans

4. Which of the following could be a solution to the differential equation represented by the slope field at the right?

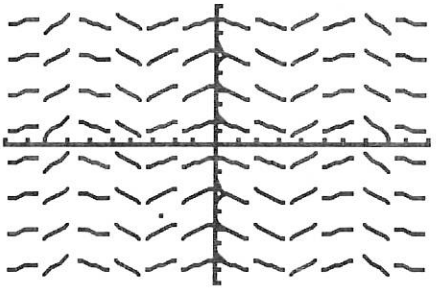
(A) $y = x^2$

(B) $y = \sin x$

(C) $y = \cos x$

(D) $y = e^x$

(E) $y = \ln x$



Ans

5. The volume of the solid generated by rotating about the x-axis the region enclosed between the curve $y = 3x^2$ and the line $y = 6x$ is given by

(A) $\pi\int_0^3 (6x - 3x^2)^2\, dx$

(B) $\pi\int_0^2 (6x - 3x^2)^2\, dx$

(C) $\pi\int_0^2 (9x^4 - 36x^2)\, dx$

(D) $\pi\int_0^2 (36x^2 - 9x^4)\, dx$

(E) $\pi\int_0^2 (6x - 3x^2)\, dx$

Ans

6. The asymptotes of the graph of the parametric equations $x = \frac{1}{t-1}$, $y = \frac{2}{t}$ are:

(A) $x = 1,\ y = 0$
(B) $y = 2$ only
(C) $x = -1,\ y = 2$
(D) $x = -1$ only
(E) $x = 0,\ y = -1$

Ans

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7. $\int_1^{\infty} \frac{3x^2}{(1 + x^3)^2}\, dx =$

(A) $-\frac{1}{2}$
(B) 0
(C) $\frac{1}{2}$
(D) 1
(E) nonexistent

Ans

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8. The radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n \cdot 3^n}$ is

(A) 3
(B) 2
(C) 1
(D) 0
(E) ∞

Ans

9. There is a point between P(1, 0) and Q(e, 1) on the graph of $y = \ln x$ such that the tangent to the graph at that point is parallel to the line through points P and Q. The x-coordinate of this point is

(A) e – 1

(B) e

(C) –1

(D) $\frac{1}{e - 1}$

(E) $\frac{1}{e + 1}$

Ans

10. If $4x^2 + 2xy + 3y = 9$, then the value of $\frac{dy}{dx}$ at the point $(2, -1)$ is

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) –2

(E) none of these

Ans

11. If $\frac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval [0, 4] is

(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

Ans

12. A particle moves along the x-axis and its position for time $t \geq 0$ is $x(t) = \cos(2t) + \sec t$ When $t = \pi$, the acceleration of the particle is

(A) -6

(B) -5

(C) -4

(D) -3

(E) none of these

Ans

13. The region bounded by the x-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = p$. If the area of the region for $0 \leq x \leq p$ exceeds the area of the region for $p \leq x \leq \pi$ by one square unit, then $p =$

(A) $\arccos \frac{1}{4}$

(B) $\arccos \frac{1}{3}$

(C) $\frac{\pi}{4}$

(D $\frac{\pi}{3}$

(E) $\frac{2\pi}{3}$

Ans

14. The graph of $y = 5x^4 + 3x^5$ has a point of inflection at

(A) (0, 0) only

(B) (1, 8) only

(C) (−1, 2) only

(D) (0, 0) and (1, 8)

(E) (0,0) and (−1, 2)

Ans

15. If $h(x) = [f(x)]^2 + f(x)g(x)$, $f'(x) = g(x)$ and $g'(x) = -f(x)$, then $h'(x) =$

(A) $f(x)g(x)$
(B) $2f(x) - f(x)g(x)$
(C) $[f(x) + g(x)]^2$
(D) $[f(x) - g(x)]^2$
(E) $[g(x)]^2 + 2g(x)f(x) - [f(x)]^2$

Ans

16. Which of the following integrals gives the length of the graph of $y = \text{Arcsin}\dfrac{x}{2}$ between $x = a$ and $x = b$, where $0 < a < b < \dfrac{\pi}{2}$?

(A) $\displaystyle\int_a^b \sqrt{1 - \frac{1}{\sqrt{4 - x^2}}}\, dx$

(B) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{\sqrt{4 - x^2}}}\, dx$

(C) $\displaystyle\int_a^b \sqrt{1 - \frac{1}{4 - x^2}}\, dx$

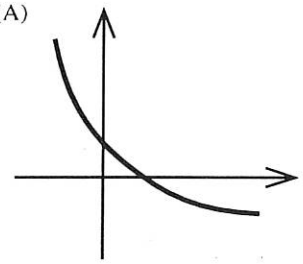
(D) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{4 - x^2}}\, dx$

(E) $\displaystyle\int_a^b \left[1 + \frac{1}{4 - x^2}\right] dx$

Ans

17. If y is a function of x such that $\frac{dy}{dx} > 0$ for all x and $\frac{d^2y}{dx^2} < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

(A)



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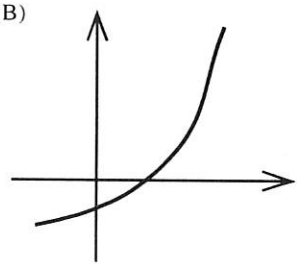

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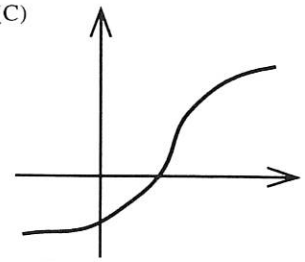

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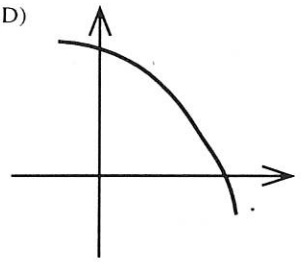
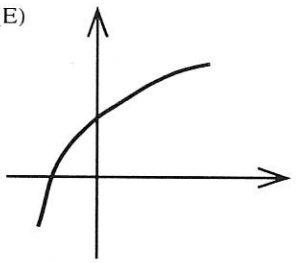


(E)



Ans

18. $\int \frac{1}{x^2 + x}\, dx =$

(A) $\frac{1}{2} \arctan\left(x + \frac{1}{2}\right) + C$

(B) $\ln\left|x^2 + x\right| + C$

(C) $\ln\left|\frac{x + 1}{x}\right| + C$

(D) $\ln\left|\frac{x}{x + 1}\right| + C$

(E) none of these

Ans

19. If $f(x) = x^2e^{-2x}$, then the graph of f is increasing for all x such that

(A) $0 < x < 1$ (B) $0 < x < \frac{1}{2}$ (C) $0 < x < 2$ (D) $x < 0$ (E) $x > 0$

Ans

20. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about $x = 0$ for which of the following functions?

(A) $\sin x$

(B) $\cos x$

(C) e^x

(D) e^{-x}

(E) $\ln(1 + x)$

Ans

21. Evaluate $\int_{-\pi/4}^{e-1} f(x)\,dx$ if $f(x) = \begin{cases} \sec^2 x \text{ for } x \le 0 \\ \frac{1}{x+1} \text{ for } x > 0 \end{cases}$

(A) 0

(B) 1

(C) 2

(D) e

(E) π

Ans

22. $\lim_{x \to 0} \left(1 + \frac{x}{2}\right)^{\cot x} =$

(A) $\frac{e}{2}$

(B) e

(C) $\sqrt{e}$

(D) $2e$

(E) e^2

Ans

23. The area of the closed region bounded by the polar graph of $r = \sqrt{1+\cos\theta}$ is given by

(A) $\int_0^{2\pi} \sqrt{1+\cos\theta}\ d\theta$

(B) $\int_0^{\pi} \sqrt{1+\cos\theta}\ d\theta$

(C) $2\int_0^{2\pi} (1+\cos\theta)\ d\theta$

(D) $\int_0^{\pi} (1+\cos\theta)\ d\theta$

(E) $2\int_0^{\pi} \sqrt{1+\cos\theta}\ d\theta$

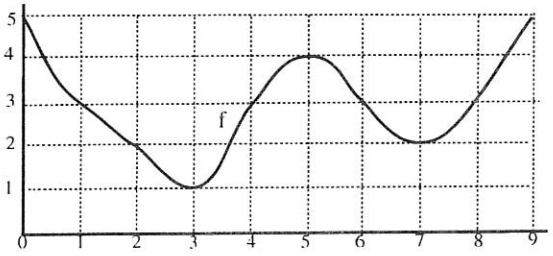
Ans

24. $\lim_{h\to 0}\left[\dfrac{(3+h)^5 - 3^5}{9h}\right]$ is

(A) 0
(B) 1
(C) 45
(D) 405
(E) nonexistent

Ans

25. Consider the function f whose graph is shown at the right. Use the Trapezoid Rule with $n = 4$ to estimate the value of $\int_1^9 f(x)\,dx$.



(A) 21
(B) 22
(C) 23
(D) 24
(E) 25

Ans

26. $\int x\sqrt{1 - x^2}\,dx =$

(A) $\dfrac{(1-x^2)^{3/2}}{3} + C$ (B) $-(1-x^2)^{3/2} + C$ (C) $\dfrac{x^2(1-x^2)^{3/2}}{3} + C$

(D) $-\dfrac{x^2(1-x^2)^{3/2}}{3} + C$ (E) $-\dfrac{(1-x^2)^{3/2}}{3} + C$

Ans

27. Suppose a continuous function f and its derivative f' have values as given in the following table. Given that $f(1) = 2$, use Euler's method to approximate the value of $f(2)$.

x	1.0	1.5	2.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	2.0		

(A) 2.1 (B) 2.3 (C) 2.5 (D) 2.7 (E) 2.9

Ans

28. A particle is moving in the *xy*-plane and its position at time t is given by $x = \cos\left(\frac{\pi}{3}t\right)$ and $y = 2\sin\left(\frac{\pi}{3}t\right)$. When $t = 3$, the speed of the particle is

(A) $-\frac{2\pi}{3}$ (B) 0 (C) $\frac{2\pi}{3}$ (D) π (E) $\frac{\sqrt{13}}{3}\pi$

Ans

EXAM I
CALCULUS BC
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers or x for which f(x) is a real number.

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1. The slope of the curve $y = x^2 - e^{-x}$ at its point of inflection is

(A) $-\ln 2$ (B) $-\ln 4$ (C) $2 - \ln 4$ (D) $2 + \ln 4$ (E) $\frac{e^2}{2}$

Ans ☐

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2. If the graph of the parabola $y = 2x^2 + x + k$ is tangent to the line $3x + y = 1$, then $k =$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Ans ☐

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3. If a function f is defined by $f(x) = \int_0^x \frac{1}{1+t^4}\,dt$, which of the following statements are true?

I. $f(x) = \frac{1}{2}$

II. the graph of f is concave down at $x = 3$.

III. $f(x) + f(-x) = 0$ for all real numbers x.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

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4. Consider the function f defined on the domain $-0.5 \le x \le 0.5$ with $f(0) = 1$, and $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \sec^2(3x)$. Evaluate: $\int_0^{0.5} f(x)\,dx$.

(A) 0.294
(B) 0.794
(C) 1.294
(D) 1.794
(E) 4.700

Ans

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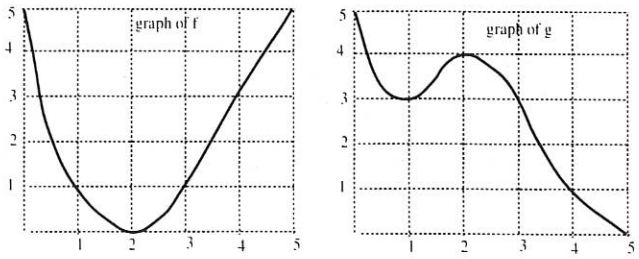
5. The graphs of functions f and g are shown at the right. If $h(x) = f[g(x)]$, which of the following statements are true about the function h?



I. $h(2) = 5$.

II. h is increasing at $x = 4$.

III. The graph of h has a horizontal tangent at $x = 1$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

6. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \cos(t + \sqrt{t})$. The total distance traveled by the particle from $t = 0$ to $t = 4$ is

(A) 2.26 (B) 2.30 (C) 2.34 (D) 2.38 (E) 2.42

Ans

7. The area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is approximately

(A)10.80 (B) 10.97 (C) 11.14 (D) 11.31 (E) 11.48

Ans

8. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n + 1}$ II. $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{3^n}$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, III

Ans

9. If $x = t^2$ and $y = \ln(t^2 + 1)$, then at $t = 1$, $\dfrac{d^2y}{dx^2}$ is

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) $\frac{1}{4}$

Ans

10. When a wholesale produce market has x crates of lettuce available on a given day, it charges p dollars per crate as determined by the supply equation $px - 20p - 6x + 40 = 0$. If the daily supply is decreasing at the rate of 8 crates per day, at what rate is the price changing when the supply is 100 crates?

(A) not changing

(B) increasing at \$0.10 per day

(C) decreasing at \$0.10 per day

(D) increasing at \$1.00 per day

(E) decreasing at \$1.00 per day

Ans

11. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 2\text{Arc}\tan x$ and below by the graph of $y = x$. What is the volume of the solid generated when R is rotated about the x-axis?

(A) 1.21 (B) 2.28 (C) 2.69 (D) 6.66 (E) 7.15

Ans

12. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = e^t$ and $y = e^{-t}$, then the speed of the particle at time $t = 1$ is

(A) 2.693 (B) 2.743 (C) 3.086 (D) 3.844 (E) 7.542

Ans

13. If the derivative of the function f is $f'(x) = 3(x + 2)(x+1)^2(x-3)^3$, then f has a local minimum at $x =$

(A) -2 only (B) -1 only (C) 3 only (D) -2 and 3 (E) -1 and 3

Ans

14. What is the x-coordinate of the point on the curve $y = e^x$ that is closest to the origin?

(A) -0.452 (B) -0.426 (C) -0.400 (D) -0.374 (E) -0.372

Ans

15. Let R be the region in the first quadrant enclosed by the graphs of $y = x\cos x$, $x = 0$, and $x = k$ for $0 < k < \frac{\pi}{2}$. The area of R, in terms of k, is

(A) $k\sin k + \cos k - 1$
(B) $-k\cos k + \sin k$
(C) $-k\sin k + \cos k - 1$
(D) $k\sin k - \cos k + 1$
(E) $-k\sin k - \cos k + 1$

Ans

16. If $\int f(x) \cdot \cos x \, dx = f(x) \cdot \sin x - \int 6x^2 \cdot \sin x \, dx$, then $f(x)$ could be

(A) $-2x^3$

(B) $2x^3$

(C) $-3x^2$

(D) $3x^2$

(E) $x \sin x$

Ans

17. Which of the following statements are true?

I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand sums.

II. If the function f is continuous on the interval $[a, b]$ and $\int_a^b f(x) \, dx = 0$, then f must have at least one zero between a and b.

III. $f'(x) > 0$ for all x in an interval, then the function f is concave up in that interval.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) none

Ans