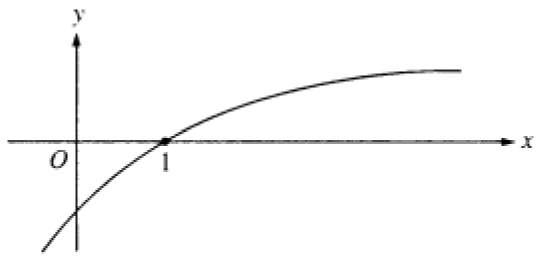
APCalcBC-HomeworkQuiz-#3

1.



The graph of a twice-differentiable function *f* is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$

(B) $f(1)<f''(1)<f'(1)$

(C) $f'(1)<f(1)<f''(1)$

(D) $f''(1)<f(1)<f'(1)$

(E) $f''(1)<f'(1)<f(1)$

2.

	$0<x<1$	$1<x<2$
f(x)	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

Let *f* be a function that is twice differentiable on $-2<x<2$ and satisfies the conditions in the table above. If $f(x)=f(-x)$ what are the *x*-coordinates of the points of inflection of the graph of *f* on $-2<x<2$?

(A) $x=0$ only

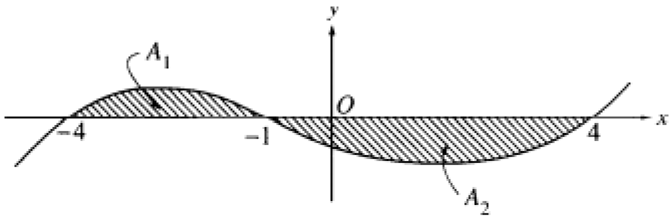
(B) $x=1$ only

(C) $x=0$ and $x=1$

(D) $x=-1$ and $x=1$

(E) There are no points of inflection on $-2<x<2$.

APCalcBC-HomeworkQuiz-#3

3. 

The graph of *y=f(x)* is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$$

(A) A_1
(B) A_1-A_2
(C) $2A_1$-A_2
(D) A_1+A_2
(E) A_1+$2A_2$

4. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343
(B) 1,343
(C) 1,367
(D) 1,400
(E) 2,057

5. What is the slope of the line tangent to the polar curve $r = 2\cos\theta - 1$ at the point where $\theta = \pi$?

(A) -3
(B) 0
(C) 3
(D) undefined

6. $$\int \frac{1}{x^2+4x+5}dx =$$

(A) $arctan(x+2) + C$
(B) $arcsin(x+2) + C$
(C) $ln\left|x^2+4x+5\right| + C$
(D) $\frac{1}{\frac{1}{3}x^3+2x^2+5x} + C$

APCalcBC-HomeworkQuiz-#3

7. At time $t \geq 0$, a particle moving in the *xy*-plane has velocity vector given by $v(t) = \left\langle 3, 2^{-t^2} \right\rangle$. If the particle is at the point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

(A) 2.304

(B) 3.107

(C) 4.209

(D) 5.310

8. The path of a particle in the xy-plane is described by the parametric equations $x(t) = \sin t$ and $y(t) = e^t - t^2$. Which of the following gives the total distance traveled by the particle from $t = 1$ to $t = 2$?

(A) $\int_1^2 \sqrt{\sin^2 t + \left(e^t - t^2\right)^2} dt$

(B) $\int_1^2 \sqrt{\cos^2 t + \left(e^t - 2t\right)^2} dt$

(C) $\int_1^2 \sqrt{\cos t + \left(e^t - 2t\right)} dt$

(D) $\int_1^2 \left(\cos^2 t + \left(e^t - 2t\right)^2\right) dt$

9.

$$f(x) = \begin{cases} \frac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

Let *f* be the function defined above. For what value of *k* is *f* continuous at $x = 2$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 5

10. The graph of $y = e^{\tan x} - 2$ crosses the *x*-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606

(B) 2

(C) 2.242

(D) 2.961

(E) 3.747

11. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

APCalcBC-HomeworkQuiz-#3

(A) $-\frac{3x+y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

12. Let *f* and *g* be functions that are differentiable everywhere. If *g* is the inverse function of *f* and if $g(-2) = 5$ and $f'(5) = -1/2$, then $g'(-2) =$

(A) 2

(B) 1/2

(C) 1/5

(D) $-\frac{1}{5}$

(E) -2

13.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

The derivative *g'* of a function *g* is continuous and has exactly two zeros. Selected values of *g'* are given in the table above. If the domain of *g* is the set of all real numbers, then *g* is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

14.

Let *S* be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when *S* is revolved about the line $y = 3$?

APCalcBC-HomeworkQuiz-#3

(A) $\pi \int_0^1 \left(\left(3 - 2x^2\right)^2 - (3 - 2x)^2 \right) \text{dx}$

(B) $\pi \int_0^1 \left((3 - 2x)^2 - \left(3 - 2x^2\right)^2 \right) \text{dx}$

(C) $\pi \int_0^1 \left(4x^2 - 4x^4\right) \text{dx}$

(D) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2 \right) \text{dy}$

(E) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2 \right) \text{dy}$

15. What is the area of the region bounded by the graph of the polar curve $r = \sqrt{1 + \frac{3}{\pi}\theta}$ and the x-axis for $0 \le \theta \le \pi$?

(A) $\frac{7\pi}{9}$

(B) $\frac{5\pi}{4}$

(C) $\frac{14\pi}{9}$

(D) $\frac{5\pi}{2}$

16. If $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$, then $f'(x) =$

(A) $\frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} + \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} + \cdots + \frac{x^{(2n+1)}}{(2n+1)n!} + \cdots$

(B) $x + \frac{3x^3}{2!} + \frac{5x^5}{3!} + \frac{7x^7}{4!} + \cdots + \frac{(2n-1)x^{(2n-1)}}{n!} + \cdots$

(C) $2 + 2x^2 + x^4 + \frac{x^6}{3} + \cdots + \frac{2x^{2(n-1)}}{(n-1)!} + \cdots$

(D) $2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{(2n-1)}}{n!} + \cdots$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \ldots$ converges to which of the following?

(A) $\cos x$

(B) $\sin x$

(C) $\frac{\sin x}{x}$

(D) $e^x - e^{x^2}$

(E) $1 + e^x - e^{x^2}$

APCalcBC-HomeworkQuiz-#3

18. The series $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots + (-1)^n \frac{x^{2n}}{n!} + \cdots$ converges to which of the following?

(A) $cos(x^2)\ +\ sin(x^2)$

(B) $1\ -\ xsinx$

(C) $cos\ x$

(D) e^{-x^2}

19. If $a \neq 0$, then $\lim\limits_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$

(B) $\frac{1}{2a^2}$

(C) $\frac{1}{6a^2}$

(D) 0

(E) nonexistent

20. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2,\ 1)$?

(A) $\frac{dy}{dx}$ exists at $(2,\ 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2,\ 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2,\ 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exists at $(2,\ 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exists at $(2,\ 1)$, and the tangent line at that point is horizontal.

21. Let f be a function that has derivatives of all orders for all real numbers, and let $P_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 1$, and $\left|f^{(n)}(x)\right| \leq \frac{n}{n+1}$, for $1 \leq n \leq 4$ and all values of x. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1) - P_3(1)| \leq k$?

(A) $\frac{4}{5}$

(B) $\frac{4}{5} \cdot \frac{1}{4!}$

(C) $\frac{4}{5} \cdot \frac{1}{3!}$

(D) $\frac{3}{4} \cdot \frac{1}{4!}$

(E) $\frac{3}{4} \cdot \frac{1}{3!}$

APCalcBC-HomeworkQuiz-#3

22. **To help restore a beach, sand is being added to the beach at a rate of $s(t) = 65 + 24\sin(0.3t)$ tons per hour, where t is measured in hours since 5:00 A.M. How many tons of sand are added to the beach over the 3-hour period from 7:00 A.M. to 10:00 A.M.?**

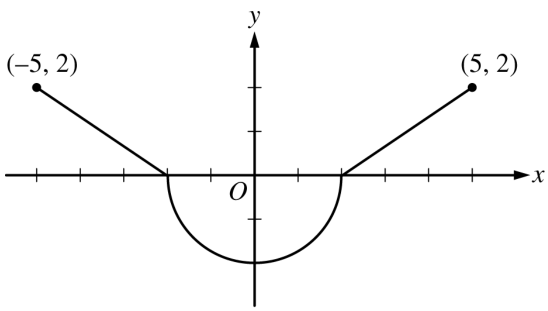
(A) 255.368

(B) 225.271

(C) 85.123

(D) 10.388

23.



Graph of f'

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

24. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only

(B) 1 only

(C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1

(E) No value

APCalcBC-HomeworkQuiz-#3

25.

x	0	2	4	6
$f(x)$	4	k	8	12

The function f is continuous on the closed interval [0,6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

(A) 2
(B) 6
(C) 7
(D) 10
(E) 14