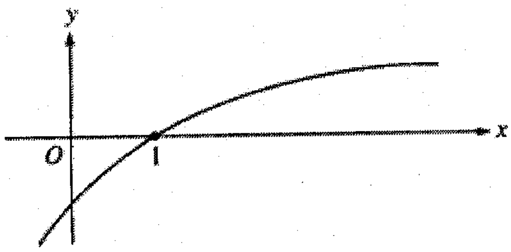
APCalcBC-HomeworkQuiz-#3

1.



$f(1)=0$
$f'(x) \approx 1$
$f''(1) < 0$ (concave down)
so $f''(1) < f(1) < f'(1)$

The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$
(B) $f(1)<f''(1)<f'(1)$
(C) $f'(1)<f(1)<f''(1)$
(D) $f''(1)<f(1)<f'(1)$
(E) $f''(1)<f'(1)<f(1)$

2.

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table above. If $f(x) = f(-x)$ what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

(A) $x = 0$ only
(B) $x = 1$ only
(C) $x = 0$ and $x = 1$
(D) $x = -1$ and $x = 1$
(E) There are no points of inflection on $-2 < x < 2$.

point of inflection occurs when sign of $f''(x)$ changes, so at $x=1$ and since $f(x)=f(-x)$ also at $x=-1$

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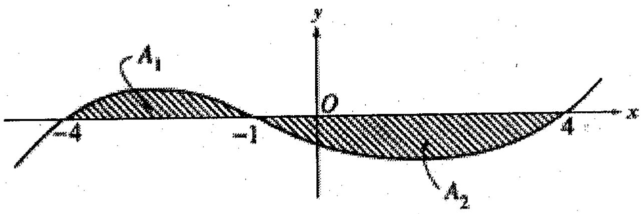
3.



The graph of *y=f(x)* is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = \int_{-4}^{-1} f(x)dx + \int_{-1}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx$$

$= A_1 + (-A_2) - 2(-A_2)$

$= A_1 - A_2 + 2A_2$

$= A_1 + A_2$

(A) A_1
(B) $A_1 - A_2$
(C) $2A_1 - A_2$
(D) $A_1 + A_2$
(E) $A_1 + 2A_2$

4. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

$\frac{dP}{dt} = kP \rightarrow$ solution: $P = P_0 e^{kt}$

$P = 1000 e^{kt}$

$1200 = 1000 e^{k(7)}$

$e^{7k} = 1.2$

$k = \frac{\ln(1.2)}{7}$

$P(t) = 1000 e^{\left[\frac{\ln(1.2)}{7}\right]t}$

$P(12) = 1366.90798$

days t	P
0	1000
7	1200

(A) 343
(B) 1,343
(C) 1,367
(D) 1,400
(E) 2,057

5. What is the slope of the line tangent to the polar curve $r = 2\cos\theta - 1$ at the point where $\theta = \pi$?

$x = r\cos\theta = (2\cos\theta - 1)\cos\theta$ $\quad y = r\sin\theta = (2\cos\theta - 1)\sin\theta$

$\frac{dx}{dt} = (2\cos\theta - 1)(-\sin\theta) + \cos\theta(-2\sin\theta)$ $\quad \frac{dy}{dt} = (2\cos\theta - 1)\cos\theta + \sin\theta(-2\sin\theta)$

$m = \left.\frac{dy/dt}{dx/dt}\right|_{\theta=\pi} = \frac{(2\cos\pi - 1)\cos\pi + \sin\pi(-2\sin\pi)}{(2\cos\pi - 1)(-\sin\pi) + \cos\pi(-2\sin\pi)} = \frac{(-3)(-1)+0}{(-0)-0}$

(A) -3
(B) 0
(C) 3
(D) undefined

6. $\int \frac{1}{x^2+4x+5}dx =$

$x^2 + 4x + 4 + 5 - 4$

$(x+2)^2 + 1$

$\int \frac{1}{(x+2)^2 + (1)^2} dx$ $\quad u = x+2$, $du = dx$

$\int \frac{1}{u^2+a^2} dx = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$

$\frac{1}{1}\arctan\left(\frac{x+2}{1}\right) + c$

(A) $arctan(x+2) + C$
(B) $arcsin(x+2) + C$
(C) $ln\left|x^2+4x+5\right| + C$
(D) $\frac{1}{\frac{1}{3}x^3+2x^2+5x} + C$

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7. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \left\langle 3, 2^{-t^2} \right\rangle$. If the particle is at the point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

(A) 2.304
(B) 3.107
(C) 4.209
(D) 5.310

$\int_0^1 3\,dt = x(1) - x(0)$
$3 = x(1) - 1$
$x(1) = 4$

$\int_0^1 2^{-t^2}\,dt = y(1) - y(0)$
(math 9)
$0.8100254544 = y(1) - \frac{1}{2}$
$y(1) = 1.3100254454$

$dist = \sqrt{4^2 + (1.31)^2}$
$= 4.209$

8. The path of a particle in the xy-plane is described by the parametric equations $x(t) = \sin t$ and $y(t) = e^t - t^2$. Which of the following gives the total distance traveled by the particle from $t = 1$ to $t = 2$?

(A) $\int_1^2 \sqrt{\sin^2 t + \left(e^t - t^2\right)^2}\,dt$

(B) $\int_1^2 \sqrt{\cos^2 t + \left(e^t - 2t\right)^2}\,dt$

(C) $\int_1^2 \sqrt{\cos t + \left(e^t - 2t\right)}\,dt$

(D) $\int_1^2 \left(\cos^2 t + \left(e^t - 2t\right)^2\right)dt$

dist traveled $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$= \int_1^2 \sqrt{(\cos t)^2 + (e^t - 2t)^2}\,dt$

9.

$$f(x) = \begin{cases} \frac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 5

$\frac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}} = k$ at $x=2$
$2(2)+1 = k$
$k = 5$

10. The graph of $y = e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606
(B) 2
(C) 2.242
(D) 2.961
(E) 3.747

$y' = e^{(\tan x)} \sec^2 x$
at $x = 0.60611193$

$y' = e^{\tan(0.606..)} \sec^2(0.606)$
2.9609

11. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

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$3x^2 + 2xy + y^2 = 1$

$\frac{d}{dx}[3x^2] + 2x\frac{d}{dx}[y] + y\frac{d}{dx}[2x] + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$

$6x + 2x\left(\frac{dy}{dx}\right) + y(2) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x+2y) = -6x - 2y$

$\frac{dy}{dx} = \frac{-6x-2y}{2x+2y} = \frac{-3x-y}{x+y} = -\frac{3x+y}{x+y}$

(A) $-\frac{3x+y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

12. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -1/2$, then $g'(-2) =$

(A) 2

(B) 1/2

(C) 1/5

(D) $-\frac{1}{5}$

(E) -2

then $f(g(x)) = x$

$\frac{d}{dx}[f(g(x))] = \frac{d}{dx}[x]$

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$, $g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(5)}$

$= \frac{1}{(-\frac{1}{2})}$

13.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

when $g'(x) < 0$

$-2 < x < 2$

14.

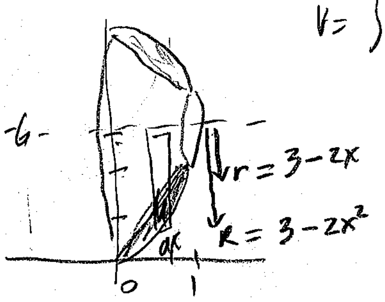
Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

$\begin{cases} y = 2x \\ y = 2x^2 \end{cases}$

$2x^2 = 2x$

$x^2 - x = 0$

$x(x-1) = 0$



$V = \int \pi R^2 dh - \int \pi r^2 dh = \int_0^1 \pi(3-2x^2)^2 dx - \int_0^1 \pi(3-2x)^2 dx$

$= \pi \int_0^1 [(3-2x^2)^2 - (3-2x)^2] dx$

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(A) $\pi \int_0^1 \left(\left(3-2x^2\right)^2 - (3-2x)^2 \right) \mathrm{dx}$

(B) $\pi \int_0^1 \left((3-2x)^2 - \left(3-2x^2\right)^2 \right) \mathrm{dx}$

(C) $\pi \int_0^1 \left(4x^2 - 4x^4\right) \mathrm{dx}$

(D) $\pi \int_0^2 \left(\left(3-\frac{y}{2}\right)^2 - \left(3-\sqrt{\frac{y}{2}}\right)^2 \right) \mathrm{dy}$

(E) $\pi \int_0^2 \left(\left(3-\sqrt{\frac{y}{2}}\right)^2 - \left(3-\frac{y}{2}\right)^2 \right) \mathrm{dy}$

15. What is the area of the region bounded by the graph of the polar curve $r = \sqrt{1 + \frac{3}{\pi}\theta}$ and the x-axis for $0 \le \theta \le \pi$? (No calculator)

(A) $\frac{7\pi}{9}$

(B) $\frac{5\pi}{4}$

(C) $\frac{14\pi}{9}$

(D) $\frac{5\pi}{2}$

$$A = \int_0^\pi \tfrac{1}{2} r^2 d\theta = \int_0^\pi \tfrac{1}{2}\left(\sqrt{1+\tfrac{3}{\pi}\theta}\right)^2 d\theta = \tfrac{1}{2}\int_0^\pi \left(1+\tfrac{3}{\pi}\theta\right) d\theta$$
$$= \tfrac{1}{2}\left[\theta + \tfrac{3}{2\pi}\theta^2\right]_0^\pi$$
$$= \tfrac{1}{2}\left[\left(\pi + \tfrac{3}{2\pi}\pi^2\right) - \left(0 + \tfrac{3}{2\pi}0^2\right)\right] = \tfrac{1}{2}\left(\pi + \tfrac{3\pi}{2}\right)$$
$$= \pi\left(\tfrac{1}{2}\right)\left(1+\tfrac{3}{2}\right) = \tfrac{5\pi}{4}$$

16. If $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$, then $f'(x) =$

(A) $\frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} + \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} + \cdots + \frac{x^{(2n+1)}}{(2n+1)n!} + \cdots$

(B) $x + \frac{3x^3}{2!} + \frac{5x^5}{3!} + \frac{7x^7}{4!} + \cdots + \frac{(2n-1)x^{(2n-1)}}{n!} + \cdots$

(C) $2 + 2x^2 + x^4 + \frac{x^6}{3} + \cdots + \frac{2x^{2(n-1)}}{(n-1)!} + \cdots$

(D) $2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{(2n-1)}}{n!} + \cdots$

$$f(x) = \frac{x^2}{1} + \frac{x^4}{2} + \frac{x^6}{6} + \frac{x^8}{24} + \cdots$$
$$f'(x) = 2x + 2x^3 + x^5 + \tfrac{1}{3}x^7 + \cdots$$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \ldots$ converges to which of the following?

(A) $\cos x$

(B) $\sin x$

(C) $\frac{\sin x}{x}$

(D) $e^x - e^{x^2}$

(E) $1 + e^x - e^{x^2}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$ (memorized form)

so $\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots$

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18. The series $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots + (-1)^n \frac{x^{2n}}{n!} + \cdots$ converges to which of the following?

(A) $cos(x^2) + sin(x^2)$

(B) $1 - xsinx$

(C) $cos\ x$

(D) e^{-x^2}

memorized form:

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$

so $e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \cdots$

$= 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$

19. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$

(B) $\frac{1}{2a^2}$

(C) $\frac{1}{6a^2}$

(D) 0

(E) nonexistent

$= \lim_{x \to a} \frac{\cancel{(x^2 - a^2)}}{\cancel{(x^2 - a^2)}(x^2 + a^2)} = \lim_{x \to a} \frac{1}{x^2 + a^2} = \frac{1}{a^2 + a^2} = \frac{1}{2a^2}$

20. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$?

(A) $\frac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exists at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exists at $(2, 1)$, and the tangent line at that point is horizontal.

$m = \frac{dy}{dx}$ (implicit diff)

$\frac{d}{dx}[x^2] + (-x)\frac{d}{dx}[y] + y\frac{d}{dx}[-x] + \frac{d}{dx}[y^2] = \frac{d}{dx}[3]$

$2x - x\frac{dy}{dx} + y(-1) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x + 2y) = y - 2x$

$\frac{dy}{dx} = \frac{y - 2x}{-x + 2y}\Big|_{(2,1)} = \frac{1 - 2(2)}{-2 + 2(1)} = \frac{-3}{0}$

vertic tang

21. Let f be a function that has derivatives of all orders for all real numbers, and let $P_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 1$, and $\left|f^{(n)}(x)\right| \leq \frac{n}{n+1}$, for $1 \leq n \leq 4$ and all values of x. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1) - P_3(1)| \leq k$?

(A) $\frac{4}{5}$

(B) $\frac{4}{5} \cdot \frac{1}{4!}$

(C) $\frac{4}{5} \cdot \frac{1}{3!}$

(D) $\frac{3}{4} \cdot \frac{1}{4!}$

(E) $\frac{3}{4} \cdot \frac{1}{3!}$

$|f(1) - P_3(1)| \leq K$

$\frac{\max|f^4(x)|}{4!}|x - 0|^4 \leq K$

$\frac{\left(\frac{4}{5}\right)}{4!}|1 - 0|^4 \leq K$

$\frac{4}{5} \cdot \frac{1}{4!} \leq K$

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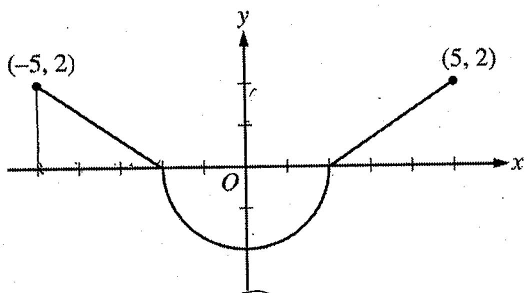
22. **To help restore a beach, sand is being added to the beach at a rate of $s(t) = 65 + 24\sin(0.3t)$ tons per hour, where t is measured in hours since 5:00 A.M. How many tons of sand are added to the beach over the 3-hour period from 7:00 A.M. to 10:00 A.M.?**

$\int_2^5 (65+24\sin(0.3t))\,dt = 255.36787$

(A) 255.368
(B) 225.271
(C) 85.123
(D) 10.388

23.



Graph of f'

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

$\int_{-5}^{2} f'(x) = f(2) - f(-5)$

$\frac{1}{2}(3)(2) - \frac{1}{2}\pi(2)^2 = 1 - f(-5)$

$3 - 2\pi = 1 - f(-5)$

$f(-5) = 1 - (3 - 2\pi) = -2 + 2\pi = 2\pi - 2$

(A) $2\pi - 2$
(B) $2\pi - 3$
(C) $2\pi - 5$
(D) $6 - 2\pi$
(E) $4 - 2\pi$

24. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

$m = \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$ ← vert. tan when...

$\frac{dx}{dt} = 0$

$3t^2 - 2t = 0$

$t(3t-2) = 0$

$0, \frac{2}{3}$

che...

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value

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25.

x	0	2	4	6
$f(x)$	4	k	8	12

The function f is continuous on the closed interval [0,6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2
(B) 6
(C) 7
(D) 10
(E) 14

interval	f(left)	f(right)	$A_{trapezoid} = \frac{1}{2}(f_{left}+f_{right})\Delta x$
[0,2]	4	k	$\frac{1}{2}(4+k)2$
[2,4]	k	8	$\frac{1}{2}(k+8)2$
[2,6]	8	12	$\frac{1}{2}(8+12)2$
			$A=52$

$(4+k)+(k+8)+(8+12)=52$

$2k+32=52$

$2k=20$

$k=10$