APCalcBC-HomeworkQuiz-#1

1. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x) + f''(x)$

(B) $f'(x) + (f''(x))^2$

(C) $(f'(x) + f''(x))^2$

(D) $(f'(x))^2 + f''(x)$

(E) $2f'(x) + f''(x)$

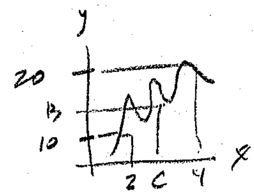
$g(x) = e^{f(x)}$

$g'(x) = \underline{e^{f(x)}} \cdot \underline{f'(x)}$

$g''(x) = e^{f(x)} f''(x) + f'(x) e^{f(x)} f'(x)$

$= [f''(x) + [f'(x)]^2] e^{f(x)}$

2. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?



(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

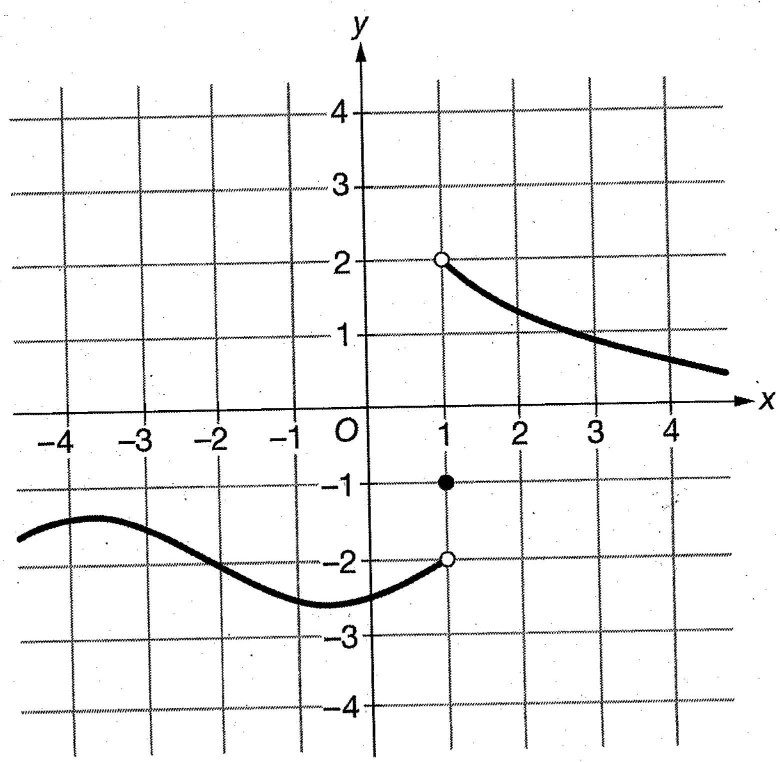
(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

3.



Graph of f

The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1^+} f(x)$ is

APCalcBC-HomeworkQuiz-#1

(A) -2

(B) -1

(C) 2

(D) nonexistent

4. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of 4/9 meter per second, at what rate, in meters per second, is the person walking?

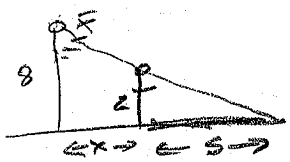
(A) 4/27

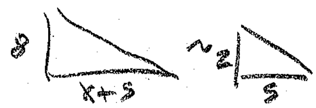
(B) 4/9

(C) 3/4

(D) 4/3

(E) 16/9





$\frac{ds}{dt} = \frac{4}{9}$

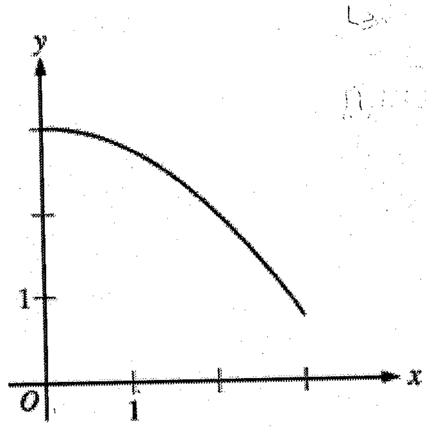
$\frac{x+s}{s} = \frac{8}{2} = 4$

$x+s = 4s$

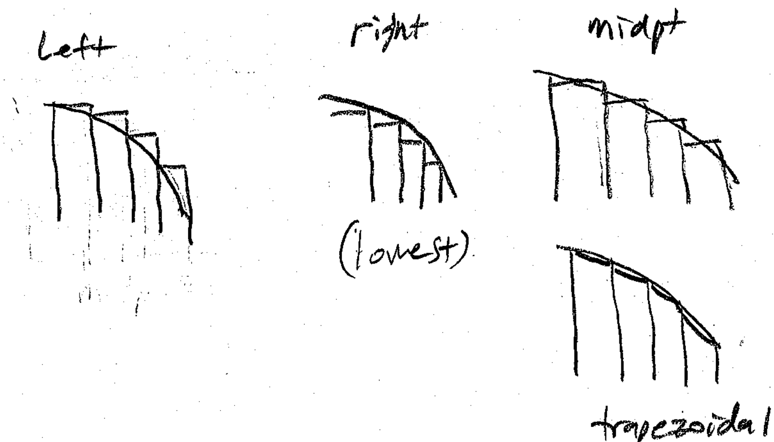
$x = 3s$

$\frac{dx}{dt} = 3\frac{ds}{dt} = 3\left(\frac{4}{9}\right) = \frac{12}{9} = \frac{4}{3}$

5.



Graph of f



The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

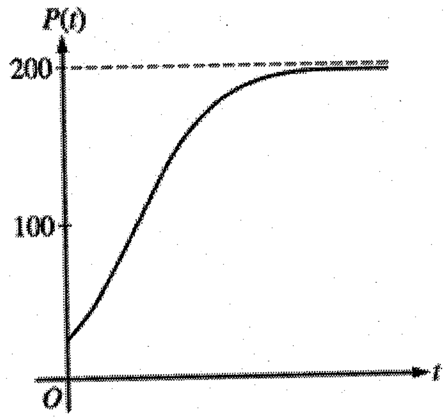
(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

APCalcBC-HomeworkQuiz-#1

6.



$L = 200$

$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right) = kP - \frac{k}{L}P^2$

↑ must be negative...

Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$ (not this)

$\frac{dP}{dt} = kP - \frac{k}{L}P^2$

$\frac{k}{L} = .001$; $L = 200$

so $\frac{k}{200} = .001$, $\underline{k = 0.2}$

$\frac{dP}{dt} = 0.2P - .001P^2$

7. $\lim\limits_{x \to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ is

$\frac{1-1-0}{0-0}$ $\frac{0}{0}$

L'Hop $= \lim\limits_{x \to 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{1 + 0 - 2}{0 - 2} = \frac{-1}{-2} = \frac{1}{2}$

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

8. A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = \sin\left(t^2\right)$. Which of the following is an expression for $\frac{d^2y}{dx^2}$ in terms of t ?

(A) $-\sin\left(t^2\right)$

(B) $-2t\sin\left(t^2\right)$

(C) $\cos\left(t^2\right) - 2t^2\sin\left(t^2\right)$

(D) $2\cos\left(t^2\right) - 4t^2\sin\left(t^2\right)$

$\frac{dy}{dx} = \frac{\cos(t^2)2t}{2t} = \cos(t^2)$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-\sin(t^2)2t}{2t} = -\sin(t^2)$

9. Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

$f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$ $f(x)$ min when $f'(x)=0$

APCalcBC-HomeworkQuiz-#1

$f'(x) = \frac{d}{dx}\left[\int_{-2}^{x^2-3x} e^{t^2}\,dt\right] = e^{(x^2-3x)^2}(2x-3) = 0$

$x = 3/2$

(A) For no value of x

(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) 2

(E) 3

10. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

$P_3 = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$f(x) = \ln(1-x)$ $f(0) = \ln 1 = 0$

$f'(x) = \frac{1}{1-x}(-1)$ $f'(0) = \frac{-1}{1-0} = -1$

$f''(x) = \frac{(1-x)(0) - (-1)(-1)}{(1-x)^2} = \frac{-1}{(1-x)^2}$ $f''(0) = \frac{-1}{1} = -1$

$f'''(x) = \frac{\cancel{(1-x)^2(0)} - (-1)(2(1-x)'(-1))}{(1-x)^4}$

$= \frac{-2(1-x)}{(1-x)^4} = \frac{-2}{(1-x)^3}$ $f'''(0) = \frac{-2}{1} = -2$

$P_3 = 0 + (-1)x + \frac{(-1)}{2}x^2 + \frac{2}{6}x^3$

$= -x - \frac{1}{2}x^2 + \frac{1}{3}x^3$