Unit 7 Part 2 Review Solutions

(1) $\int e^{6x}\cos(2x)\,dx$ (table #99)

$$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu)+C$$

$u=x$ $a=6$

$du=dx$ $b=2$

$$\boxed{\frac{e^{6x}}{36+4}\left(6\cos(2x)+2\sin(2x)\right)+C}$$

---

(2) $\int\sqrt{16-9x^2}\,dx$ (closest to table #30)

$$\int\sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+C$$

$u^2=9x^2$ $a=4$

$u=3x$

$\frac{du}{dx}=3$

$du=3dx$

$\frac{1}{3}du=dx$

$$\int\sqrt{(4)^2-u^2}\,\frac{1}{3}du = \frac{1}{3}\int\sqrt{4^2-u^2}\,du$$

$$\boxed{\frac{1}{3}\left[\frac{3x}{2}\sqrt{16-9x^2}+\frac{16}{2}\sin^{-1}\left(\frac{3x}{4}\right)\right]+C}$$

---

(3) $\int xe^{4x}\,dx$ (table #96)

$$\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au}+C$$

$u=x$ $a=4$

$du=dx$

$$\boxed{\frac{1}{16}(4x-1)e^{4x}+C}$$

---

(4) $\int xe^{3x^2}\cos(3x^2)\,dx$

1st, u-sub: $u=3x^2$

$\frac{du}{dx}=6x$

$du=6x\,dx$

$x\,dx=\frac{1}{6}du$

$$\int e^u\cos(u)\,\frac{1}{6}du$$

$$\frac{1}{6}\int e^u\cos(u)\,du$$ (table #99)

$$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu+b\sin bu)$$

$u=u$

$a=1$

$b=1$

$$\frac{1}{6}\left[\frac{e^u}{1+1}\left(1\cos(u)+(1)\sin(u)\right)\right]+C$$

original $u=3x^2\ldots$

$$\boxed{\frac{1}{6}\left[\frac{e^{3x^2}}{2}\left(\cos(3x^2)+\sin(3x^2)\right)\right]+C}$$

(5) $\int t^2 \sec^4(t^3)\,dt \rightarrow \int \sec^4(u)\, \underline{t^2 dt}$

1st, u-sub: $u=t^3$

$\frac{du}{dt}=3t^2$

$du=3t^2 dt$

$t^2 dt=\frac{1}{3}du$

$\int \sec^4(u)\,\frac{1}{3}du$

$\frac{1}{3}\int \sec^4(u)\,du$ now, (table #77)

$\int \sec^n u\,du=\frac{1}{n-1}\tan u \sec^{n-2}u+\frac{n-2}{n-1}\int \sec^{n-2}u\,du$

$(n=4)$ $u=t^3$

$\frac{1}{3}\left[\frac{1}{3}\tan(t^3)\sec^2(t^3)+\frac{2}{3}\underbrace{\int \sec^2 u\,du}\right]$

this also recuires table #77, w/ $n=2$

$\left[\frac{1}{1}\tan u \cancel{\sec^0(u)}^{1}+\left(\frac{0}{1}\right)^{=0}\cancel{\int \sec^0(u)du}\right]$

$\boxed{\frac{1}{3}\left[\frac{1}{3}\tan(t^3)\sec^2(t^3)+\frac{2}{3}[\tan(t^3)]\right]+C}$

---

(6) $\int x\sqrt{9x^4+25}\,dx$ (closest is table #21)

$\int\sqrt{a^2+u^2}\,du=\frac{u}{2}\sqrt{a^2+u^2}+\frac{a^2}{2}\ln(u+\sqrt{a^2+u^2})+C$

$u^2=9x^4$

$u=3x^2$

$\frac{du}{dx}=6x$

$du=6x\,dx$

$x\,dx=\frac{1}{6}du$

$\int\sqrt{u^2+5^2}\,\frac{1}{6}du$

$\frac{1}{6}\int\sqrt{u^2+5^2}\,du$

$u=3x^2,\ a=5$

$\boxed{\frac{1}{6}\left[\frac{3x^2}{2}\sqrt{9x^4+25}+\frac{25}{2}\ln(3x^2+\sqrt{9x^4+25})\right]+C}$

---

(7) $\int \sin^4(5t)\,dt$ (table #73)

$u=5t$

$\frac{du}{dt}=5$

$du=5dt$

$\frac{1}{5}du=dt$

$\int \sin^n u\,du=-\frac{1}{n}\sin^{n-1}u\cos u+\frac{n-1}{n}\int \sin^{n-2}u\,du$

$n=4,\ u=5t$

$\frac{1}{5}\left[-\frac{1}{4}\sin^3(5t)\cos(5t)+\frac{3}{4}\int \sin^2 u\,du\right]$

for this, table #63:

$\int \sin^2 u\,du=\frac{1}{2}u-\frac{1}{4}\sin 2u$

$\boxed{\frac{1}{5}\left[-\frac{1}{4}\sin^3(5t)\cos(5t)+\frac{3}{4}\left(\frac{1}{2}(5t)-\frac{1}{4}\sin(10t)\right)\right]+C}$

⑧ $\int x^3 \ln x \, dx$ (table #101)

$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

$u = x \quad n = 3$

$$\boxed{\frac{x^4}{16}\left[4\ln u - 1\right] + C}$$

---

⑨ $\int \sin(3x)\cos(4x)\, dx$ (table #81)

$$\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$a=3,\ b=4,\ u=x$

$$\boxed{-\frac{\cos(-x)}{2(-1)} - \frac{\cos(7x)}{2(7)} + C}$$

---

⑩ $\int x\sqrt{\underbrace{x^4}_{u^2}-4}\, dx \rightarrow \int \sqrt{u^2-2^2}\ \tfrac{1}{2}du$

$u^2 = x^4$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \tfrac{1}{2}du$

$\frac{1}{2}\int \sqrt{u^2-2^2}\, du$ (table #39)

$$\int \sqrt{u^2-a^2}\, du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$u = x^2,\ a = 2$

$$\boxed{\frac{1}{2}\left[\frac{x^2}{2}\sqrt{x^4-4} - \frac{4}{2}\ln\left|x^2+\sqrt{x^4-4}\right|\right] + C}$$

---

⑪ $\int x\tan^3(5x^2)\, dx \rightarrow \int \tan^3 u \ \tfrac{1}{10}du$

$u = 5x^2$

$\frac{du}{dx} = 10x$

$du = 10x\,dx$

$x\,dx = \tfrac{1}{10}du$

$\frac{1}{10}\int \tan^3 u \, du$ (table #69)

$$\int \tan^3 u \, du = \tfrac{1}{2}\tan^2 u + \ln|\cos u| + C$$

$$\boxed{\frac{1}{10}\left[\tfrac{1}{2}\tan^2(5x^2) + \ln\left|\cos(5x^2)\right|\right] + C}$$

(12) $\int \frac{\sqrt{2+9x^2}}{x^2}dx \to \int \frac{\sqrt{(\sqrt{2})^2+u^2}}{(\frac{u}{3})^2}\,\frac{1}{3}du \to \int \frac{\sqrt{(\sqrt{2})^2+u^2}}{u^2}\left(\frac{\frac{1}{3}}{(\frac{1}{3})^2}\right)du$

$u=3x \to x=\frac{u}{3}$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

$= 3\int \frac{\sqrt{(\sqrt{2})^2+u^2}}{u^2}du$

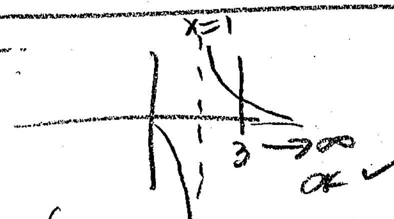
(table #24) $\int \frac{\sqrt{a^2+u^2}}{u^2}du = \frac{-\sqrt{a^2+u^2}}{u} + \ln\left(u+\sqrt{a^2+u^2}\right)+C$

$a=\sqrt{2},\ u=3x$

$$\boxed{3\left[\frac{-\sqrt{2+9x^2}}{3x}+\ln\left(3x+\sqrt{2+9x^2}\right)\right]+C}$$

---

(13) $\int_3^{\infty} \frac{1}{x(\ln x)^5}dx$ graph 1st.



$u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$\int \frac{1}{x(\ln x)^5}dx = \int u^{-5}du$

$\frac{u^{-4}}{-4} \to \left[-\frac{1}{(\ln x)^4}\right]_3^{\infty}$

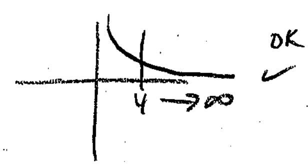
$= \lim_{b\to\infty}\left[-\frac{1}{(\ln b)^4}\right] - \left(-\frac{1}{(\ln 3)^4}\right)$

$-\frac{1}{\infty} + \frac{1}{(\ln 3)^4}$

$0 + \frac{1}{(\ln 3)^4} = \boxed{\frac{1}{(\ln 3)^4}}$

---

(14) $\int_4^{\infty} \frac{x}{x^{7/2}}dx = \int_4^{\infty} x^{-5/2}dx$ graph



$\frac{x^{-3/2}}{-3/2} = \left[-\frac{2}{3}x^{-3/2}\right]_4^{\infty}$

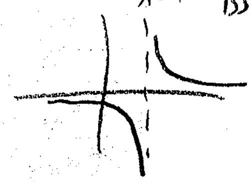
$\lim_{b\to\infty}\left[-\frac{2}{3}b^{-3/2}\right] - \left(-\frac{2}{3}(4)^{-3/2}\right)$

$\frac{-2}{3(\infty)^{3/2}} + \frac{2}{3}4^{-3/2}$

$0 + \frac{2}{3}(4)^{-3/2} = \boxed{\frac{2}{3}(4)^{-3/2} \simeq 0.083333}$

Issue at $x=1$ (split here)

(15) $\int_0^5 \frac{1}{(x-1)^{1/5}}dx$ graph:



$= \int_0^1 \frac{1}{(x-1)^{1/5}}dx + \int_1^5 \frac{1}{(x-1)^{1/5}}dx$

$\int \frac{1}{(x-1)^{1/5}}dx$ $u = x-1$, $\frac{du}{dx} = 1$, $du = dx$

$\int u^{-1/5}du = \frac{5}{4}u^{4/5} = \left[\frac{5}{4}(x-1)^{4/5}\right]$

$\left[\frac{5}{4}(x-1)^{4/5}\right]_0^{1} + \left[\frac{5}{4}(x-1)^{4/5}\right]_{1}^{5}$

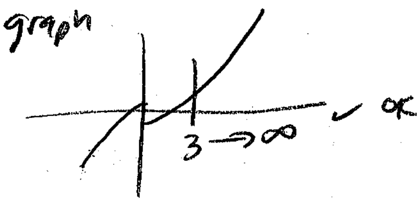
$\lim_{b\to 1^-}\left(\frac{5}{4}(b-1)^{4/5}\right) - \frac{5}{4}(0-1)^{4/5} + \frac{5}{4}(5-1)^{4/5} - \lim_{b\to 1^+}\left(\frac{5}{4}(b-1)^{4/5}\right)$

$0 - \frac{5}{4}(1) + \frac{5}{4}(4)^{4/5} - 0 = \boxed{\frac{5}{4}\left[4^{4/5}-1\right] \approx 2.53929}$

---

(16) $\int_3^\infty x\ln(x^2)dx$ $u = x^2$, $\frac{du}{dx} = 2x$, $du = 2xdx$, $xdx = \frac{1}{2}du$

graph



$\frac{1}{2}\int \ln u\, du$

requires by parts:

change letter: $\frac{1}{2}\int \ln y\, dy$

now, by parts: $u = \ln y$ $dv = dy$

$\frac{du}{dy} = \frac{1}{y}$ $\int dv = \int dy$

$du = \frac{1}{y}dy$ $v = y$

$uv - \int v\,du = (\ln y)(y) - \int y\left(\frac{1}{y}dy\right) = y\ln y - \int 1 dy$

$= \left[y\ln y - y\right]$

back to original letter → $\left[u\ln u - u\right]_3^\infty$

$\lim_{b\to\infty}\left[b\ln b - b\right] - (3\ln 3 - 3)$

$\lim_{b\to\infty}\left[b(\ln b - 1)\right]$

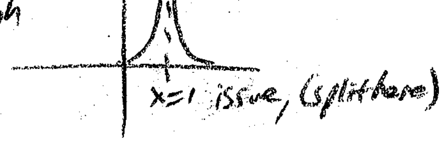
$\infty(\infty - 1)$

∞ − a number

$\boxed{\text{diverges}}$

(17) $\int_0^\infty \frac{x^2}{(1-x^3)^2}dx$

graph



$$= \int_0^1 \frac{x^2}{(1-x^3)^2}dx + \int_1^\infty \frac{x^2}{(1-x^3)^2}dx$$

$\int \frac{x^2}{(1-x^3)^2}dx$ $\quad u = 1-x^3$

$\frac{du}{dx} = -3x^2$

$du = -3x^2 dx$

$x^2 dx = -\frac{1}{3}du$

$-\frac{1}{3}\int u^{-2}du$

$-\frac{1}{3}\frac{u^{-1}}{-1} = \frac{1}{3u} = \left[\frac{1}{3(1-x^3)}\right]$

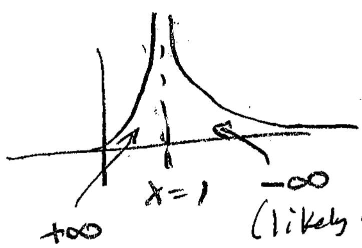
$$\left[\frac{1}{3(1-x^3)}\right]_0^{1} + \left[\frac{1}{3(1-x^3)}\right]_1^{\infty}$$

$$\lim_{b\to 1^-}\left(\frac{1}{3(1-b^3)}\right) - \frac{1}{3(1-0^3)} + \lim_{b\to\infty}\left(\frac{1}{3(1-b^3)}\right) - \lim_{b\to 1^+}\left(\frac{1}{3(1-b^3)}\right)$$

$\frac{1}{0}\ \frac{(+)}{(+)}$ $\qquad \frac{1}{-\infty}$ $\qquad \frac{1}{0}\ \frac{(+)}{(-)}$

$+\infty \quad - \quad \frac{1}{3} \quad + \quad 0 \quad -\infty$

we can just say this $\boxed{\text{diverges}}$



(we can't say +∞ and −∞ cancel)

(likely a larger negative area)