Unit 7 Part 1 Review Solutions

① $\int \sin\theta \, d\theta$

$\boxed{-\cos\theta + C}$

② $\int e^x \, dx$

$\boxed{e^x + c}$

③ $\int 2x^3 \, dx$

$2\int x^3 \, dx$

$2 \, \frac{x^4}{4} = \boxed{\frac{1}{2}x^4 + c}$

④ $\int 2x^{-1} \, dx$

$2\int \frac{1}{x} \, dx$

$\boxed{2\ln|x| + c}$

⑤ $\int 3\sec^2 x \, dx$

$3\int \sec^2 x \, dx$

$\boxed{3\tan x + C}$

⑥ $\int \csc x \cot x \, dx$

$\boxed{-\csc x + C}$

⑦ $\int \sec x \tan x \, dx$

$\boxed{\sec x + c}$

⑧ $\int \frac{1-\sin^2 x}{\cos x} \, dx$

identity: $\sin^2 x + \cos^2 x = 1$

$1 - \sin^2 x = \cos^2 x$

$\int \frac{\cos^2 x}{\cos x} \, dx$

$\int \cos x \, dx$

$\boxed{\sin x + C}$

⑨ $\int \frac{\sec x}{(\tan^2 x + 1)} \, dx$

identity: $\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$\int \frac{\sec x}{\sec^2 x} \, dx$

$\int \frac{1}{\sec x} \, dx = \int \cos x \, dx$

$= \boxed{\sin x + C}$

⑩ $\int \frac{2}{(x-10)^2 + 36} \, dx$

$\int \frac{2}{(x-10)^2 + (6)^2} \, dx$

$u = x - 10$

$\frac{du}{dx} = 1$

$du = dx$

memorized:

$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + c$

$2\int \frac{1}{u^2 + (6)^2} \, du$

$= \boxed{2\left(\frac{1}{6}\right)\arctan\left(\frac{x-10}{6}\right) + c}$

⑪ $\int \frac{5}{x^2 - 12x + 38} \, dx$

complete the square:

$x^2 - 12x + 36 + 38 - 36$

$(x-6)^2 + 2$

$\int \frac{5}{(x-6)^2 + 2} \, dx$

$5\int \frac{1}{(x-6)^2 + (\sqrt{2})^2} \, dx$

$u = x - 6$

$\frac{du}{dx} = 1$

$du = dx$

$5\int \frac{1}{u^2 + (\sqrt{2})^2} \, du$

$\boxed{5\left(\frac{1}{\sqrt{2}}\right)\arctan\left(\frac{x-6}{\sqrt{2}}\right) + c}$

(12) $\int 7x^3(3x^4+6)^5\,dx$

$u = 3x^4+6$

$\frac{du}{dx} = 12x^3, \quad \frac{1}{12}du = x^3dx$

$7\int u^5 \underline{x^3dx}$

$7\int u^5(\frac{1}{12}du)$

$\frac{7}{12}\int u^5\,du$

$\frac{7}{12}\frac{u^6}{6} + C$

$\boxed{\frac{7}{72}(3x^4+6)^6 + C}$

(13) $\int 7e^{5x}dx$

$u = 5x$

$\frac{du}{dx} = 5, \quad \frac{1}{5}du = dx$

$7\int e^u \frac{1}{5}du$

$\frac{7}{5}\int e^u du$

$\frac{7}{5}e^u + C$

$\boxed{\frac{7}{5}e^{5x} + C}$

(14) $\int (4x+6)e^{(x^2+3x)}dx$

$u = x^2+3x$

$\frac{du}{dx} = 2x+3$

$du = (2x+3)dx$

$\int e^u(4x+6)dx$

$\int e^u\, 2\underline{(2x+3)dx}$

$2\int e^u du$

$2e^u + C$

$\boxed{2e^{(x^2+3x)} + C}$

(15) $\int x\sin(2x)\,dx$

$u = x \qquad dv = \sin(2x)dx$ need usub (use w)

$\frac{du}{dx} = 1 \qquad \int dv = \underline{\int \sin(2x)dx}$

$w = 2x$

$du = dx \qquad v = -\frac{1}{2}\cos(2x)$

$\frac{dw}{dx} = 2$

$uv - \int v\,du$

$dw = 2dx$

$dx = \frac{1}{2}dw$

$(x)(-\frac{1}{2}\cos(2x)) - \int(-\frac{1}{2}\cos(2x))dx$

$-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)dx$ (similarly)

$-\frac{1}{2}x\cos(2x) + \frac{1}{2}(\frac{1}{2}\sin(2x)) + C$

$\boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

16. $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$

$\frac{du}{dx} = 2x \qquad \int dv = \int \sin x dx$

$du = 2x dx \qquad v = -\cos x$

$uv - \int v du$

$(x^2)(-\cos x) - \int (-\cos x)(2x dx)$

$-x^2 \cos x + 2\boxed{\int x \cos x dx}$

requires by parts again

$\left[\int x \cos x dx\right] \quad u = x \qquad dv = \cos x dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int \cos x dx$

$du = dx \qquad v = \sin x$

$uv - \int v du$

$(x)(\sin x) - \int (\sin x) dx$

$x \sin x - (-\cos x)$

$[x \sin x + \cos x]$

with 1st result...

$-x^2 \cos x + 2[x \sin x + \cos x]$

$\boxed{-x^2 \cos x + 2x \sin x + 2\cos x + C}$

17. $\int e^{4x} \cos(2x) dx$

$u = \cos(2x) \qquad dv = e^{4x} dx$

$\frac{du}{dx} = -2\sin(2x) \qquad \int dv = \int e^{4x} dx$

$du = -2\sin(2x) dx \qquad v = \frac{1}{4} e^{4x}$

$uv - \int v du$

$(\cos(2x))(\frac{1}{4} e^{4x}) - \int \frac{1}{4} e^{4x}(-2\sin(2x)) dx$

$\frac{1}{4} e^{4x} \cos(2x) + \frac{1}{2}\boxed{\int e^{4x} \sin(2x) dx}$

requires by parts again

$u = \sin(2x) \qquad dv = e^{4x} dx$

$\frac{du}{dx} = 2\cos(2x) \qquad \int du = \int e^{4x} dx$

$du = 2\cos(2x) dx \qquad v = \frac{1}{4} e^{4x}$

$uv - \int v du$

$(\sin(2x))(\frac{1}{4} e^{4x}) - \int \frac{1}{4} e^{4x} 2\cos(2x) dx$

$\frac{1}{4} e^{4x} \sin(2x) - \frac{1}{2} \underline{\int e^{4x} \cos(2x) dx}$

this is the original integral

so...

$\int e^{4x} \cos(2x) dx = \frac{1}{4} e^{4x} \cos(2x) + \frac{1}{2}\left[\frac{1}{4} e^{4x} \sin(2x) - \frac{1}{2}\int e^{4x} \cos(2x)\right.$

$\underline{\int e^{4x} \cos(2x) dx} = \frac{1}{4} e^{4x} \cos(2x) + \frac{1}{8} e^{4x} \sin(2x) - \underline{\frac{1}{4} e^{4x} \cos(2x) dx}$

Combine like terms

$\int e^{4x} \cos(2x) dx + \frac{1}{4}\int e^{4x} \cos(2x) dx = \frac{1}{4} e^{4x} \cos(2x) + \frac{1}{8} e^{4x} \sin(2x)$

$\underbrace{\left(1 + \frac{1}{4}\right)}_{5/4} \int e^{4x} \cos(2x) dx = \frac{1}{4} e^{4x} \cos(2x) + \frac{1}{8} e^{4x} \sin(2x)$

$\boxed{\int e^{4x} \cos(2x) dx = \frac{4}{5}\left[\frac{1}{4} e^{4x} \cos(2x) + \frac{1}{8} e^{4x} \sin(2x)\right]}$

(18) $\int x \ln x \, dx$

$u = \ln x \quad dv = x\,dx$

$\frac{du}{dx} = \frac{1}{x} \quad \int dv = \int x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$uv - \int v\,du$

$(\ln x)(\frac{1}{2}x^2) - \int (\frac{1}{2}x^2)\frac{1}{x}dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$

$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

(19) $\int x^2 e^x dx$

$u = x^2 \quad dv = e^x dx$

$\frac{du}{dx} = 2x \quad \int dv = \int e^x dx$

$du = 2x\,dx \quad v = e^x$

$uv - \int v\,du$

$(x^2)(e^x) - \int e^x 2x\,dx$

$x^2 e^x - 2\left[\int x e^x dx\right]$

requires by parts again

$u = x \quad dv = e^x dx$

$\frac{du}{dx} = 1 \quad \int dv = \int e^x dx$

$du = dx \quad v = e^x$

$uv - \int v\,du$

$(x)(e^x) - \int e^x du$

$x^2 e^x - 2\left[x e^x - e^x\right]$

$\boxed{x^2 e^x - 2xe^x + 2e^x + C}$

(20) $\int \sin^3 x \cos x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\int u^3 du$

$\frac{u^4}{4} + C$

$\boxed{\frac{1}{4}\sin^4 x + C}$

(22) $\int \sin^2 x\,dx$

power reducing identity:

$\sin^2 x = \frac{1}{2}[1 - \cos 2x]$

$\int \frac{1}{2}[1 - \cos 2x]dx$

$\int \frac{1}{2}dx - \frac{1}{2}\int \cos(2x)dx \quad u = 2x, \ \frac{du}{dx} = 2, \ du = 2dx, \ dx = \frac{1}{2}du$

$\int \frac{1}{2}dx - \frac{1}{4}\int \cos u\,du$

$\frac{1}{2}x - \frac{1}{4}\sin u + C$

$\boxed{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$

(21) $\int \sin^3 x \cos^5 x\,dx$

$\int \sin^2 x \cos^5 x \underbrace{\sin x\,dx}_{\text{(for du)}}$

identity: $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\int (1 - \cos^2 x)\cos^5 x \sin x\,dx$

$\int (\cos^5 x - \cos^7 x)\sin x\,dx$

$\int \cos^5 x \sin x\,dx - \int \cos^7 x \sin x\,dx$

$u = \cos x, \ du = -\sin x\,dy$

$-\int u^5 du - (-1)\int u^7 du$

$-\frac{u^6}{6} + \frac{u^8}{8} + C,$ $\boxed{-\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C}$

(23) $\int \cos^2 x\,dx$

power reducing identity:

$\cos^2 x = \frac{1}{2}[1+\cos 2x]$

$\int \frac{1}{2}(1+\cos 2x)\,dx$

$\int \frac{1}{2}dx + \frac{1}{2}\int \cos(2x)\,dx$ $\quad u = 2x$, $du = 2dx$

$\int \frac{1}{2}dx + \frac{1}{4}\int \cos u\,du$

$\frac{1}{2}x + \frac{1}{4}(\sin u) + C$

$\boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$

(24) $\int \sec x\,dx$

must be memorized:

$= \boxed{\ln|\sec x + \tan x| + C}$

(25) $\int \csc x\,dx$

must be memorized:

$= \boxed{\ln|\csc x - \cot x| + C}$

(26) $\int \cot x\,dx$

identity: $\cot x = \frac{\cos x}{\sin x}$

$\int \frac{\cos x}{\sin x}\,dx$

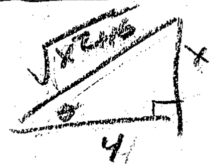
$u = \sin x$ $\quad \int \frac{1}{u}\,du$

$\frac{du}{dx} = \cos x$ $\quad \ln|u| + C$

$du = \cos x\,dx$

$\boxed{\ln|\sin x| + C}$

(27) $\int \frac{dx}{\sqrt{x^2+16}}$



$\int \frac{4\sec^2\theta\,d\theta}{4\sec\theta}$ $\quad \tan\theta = \frac{x}{4}$ $\quad \cos\theta = \frac{4}{\sqrt{x^2+16}}$

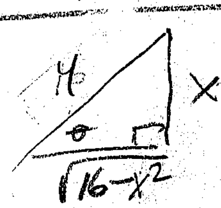
$x = 4\tan\theta$ $\quad \sec\theta = \frac{\sqrt{x^2+16}}{4}$

$\int \sec\theta\,d\theta$ $\quad \frac{dx}{d\theta} = 4\sec^2\theta$ $\quad \sqrt{x^2+16} = 4\sec\theta$

$\ln|\sec x + \tan x| + C$ $\quad dx = 4\sec^2\theta\,d\theta$

$\boxed{\ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C}$

(28) $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$



$\sin\theta = \frac{x}{4}$ $\quad \cos\theta = \frac{\sqrt{16-x^2}}{4}$

$x = 4\sin\theta$ $\quad \sqrt{16-x^2} = 4\cos\theta$

$\frac{dx}{d\theta} = 4\cos\theta$

$dx = 4\cos\theta\,d\theta$

$\int \frac{(4\sin\theta)^3}{4\cos\theta}\,4\cos\theta\,d\theta$

$4^3\int \sin^3\theta\,d\theta$

identity: $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$64\int \sin^2\theta\sin\theta\,d\theta = 64\int(1-\cos^2\theta)\sin\theta\,d\theta = 64\int\sin\theta\,d\theta - 64\int\cos^2\theta\sin\theta\,d\theta$ $\quad u = \cos\theta \quad du = \sin\theta\,d\theta$

one last step!

$= 64\int\sin\theta\,d\theta - 64\int u^2\,du$

$= 64(-\cos\theta) - 64(\frac{1}{3}u^3) + C$

$= -64\cos\theta - \frac{64}{3}\cos^3\theta + C$ $\quad$ but $\cos\theta = \frac{\sqrt{16-x^2}}{4}$

$\boxed{-64\left[\frac{\sqrt{16-x^2}}{4}\right] - \frac{64}{3}\left(\frac{\sqrt{16-x^2}}{4}\right)^3 + C}$

(29) $\int \frac{dx}{(x-4)(x+5)}$

$\frac{1}{(x-4)(x+5)} = \frac{A}{x-4} + \frac{B}{x+5}$

$\frac{A(x+5)}{(x-4)(x+5)} + \frac{B(x-4)}{(x+5)(x-4)} = \frac{1}{(x-4)(x+5)}$

$Ax + 5A + Bx - 4B = 1$

$(A+B)x + (5A-4B) = (0)x + 1$ (match)

$\begin{cases} A+B=0 \\ 5A-4B=1 \end{cases}$ Solve system $B=-A$

$5A-4(-A)=1$

$9A=1,\ A=\frac{1}{9},\ B=\frac{-1}{9}$

$\frac{1}{9}\int \frac{1}{x-4}dx - \frac{1}{9}\int \frac{1}{x+5}dx$

$u=x-4$

$\frac{du}{dx}=1$

$du=dx$

(similarly)

(can omit the u-sub)

$\frac{1}{9}\ln|u|$

$\boxed{\frac{1}{9}\ln|x-4| - \frac{1}{9}\ln|x+5| + C}$

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(30) $\int \frac{dx}{x^2-5x-14}$

factor: $x^2-5x-14$

$(x-7)(x+2)$

$\int \frac{dx}{(x-7)(x-2)}$

$\frac{1}{(x-7)(x+2)} = \frac{A}{x-7} + \frac{B}{x+2}$

$A(x+2) + B(x-7) = 1$

$(A+B)x + (2A-7B) = (0)x + 1$

$\begin{cases} A+B=0 \\ 2A-7B=1 \end{cases}$

$B=-A$

$2A-7(-A)=1$

$9A=1,\ A=\frac{1}{9},\ B=\frac{-1}{9}$

$\frac{1}{9}\int \frac{1}{x-7}dx - \frac{1}{9}\int \frac{1}{x+2}dx$

$\boxed{\frac{1}{9}\ln|x-7| - \frac{1}{9}\ln|x+2| + C}$

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(31) $\int 5\csc^2 x\,dx$

$5\int \csc^2 x\,dx$

memorized shortcut

$5(-\cot x) + C$

$\boxed{-5\cot x + C}$

(32) $\int xe^x dx$ (by parts)

$u=x \quad dv=e^x dx$

$\frac{du}{dx}=1 \quad \int dv = \int e^x dx$

$du=dx \quad v=e^x$

$uv - \int v\,du$

$(x)(e^x) - \int e^x dx$

$\boxed{xe^x - e^x + C}$

(33) $\int 2x^3\cos(x^2)\,dx$ hmm... maybe a u-sub first?

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\int \cos u \, 2x^3\,dx$

$\int \cos u \, \underline{x^2 \, 2x\,dx}$

$\int \cos u \,(u)\,du$

$\int u\cos u\,du$ now, by parts, let's change the letter first

$\int y\cos y\,dy$

now: $u = y$ $\quad dv = \cos y\,dy$

$\frac{du}{dy} = 1$ $\quad \int dv = \int \cos y\,dy$

$du = dy$ $\quad v = \sin y$

$uv - \int v\,du$

$(y)(\sin y) - \int \sin y\,dy$

$y\sin y - (-\cos y)$

y is the original $u = x^2$

so $\boxed{x^2\sin(x^2) + \cos(x^2) + C}$

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(34) $\int \frac{3}{\sqrt{9-(x+5)^2}}\,dx$ is the memorized form $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$

with $u = x+5$ and $a = 3$

$\frac{du}{dx} = 1$

so $du = dx$

$= \boxed{3\arcsin\left(\frac{x+5}{3}\right) + C}$

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(35) $\int 3x\ln(x^2)\,dx$ x^2 is inside, and it's derivative $2x$, the x part is in the integral, so u-substitution

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$\int \ln u \, 3x\,dx$

$3\int \ln u \, \frac{1}{2}du$

$\frac{3}{2}\int \ln u\,du$ no shortcut for $\ln u$ ... but we can do by parts

again, let's first change the letter:

$\frac{3}{2}\int \ln y\,dy$ $\quad u = \ln y \quad dv = dy$

$\frac{du}{dy} = \frac{1}{y} \quad \int dv = \int dy$

$du = \frac{1}{y}dy \quad v = y$

$uv - \int v\,du$

$(\ln y)(y) - \int y \frac{1}{y}\,dy$

$y\ln y - \int 1\,dy$

$y\ln y - y + C$

and y is the original $u = x^2$

$\boxed{\frac{3}{2}(x^2\ln(x^2) - x^2) + C}$

(36) $\int \csc^4 x \cot^3 x\,dx$ deriv. of $\tan x = \sec^2 x$

of $\cot x = -\csc^2 x$ Keep one for du

$\int \csc^2 x \cot^3 x \underbrace{\csc^2 x\,dx}_{\text{for du}}$

identity: $\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\int (1+\cot^2 x)\cot^3 x \csc^2 x\,dx$

$\int (\cot^3 x + \cot^5 x)\csc^2 x\,dx$

$\int \cot^3 x \csc^2 x\,dx + \int \cot^5 x \csc^2 x\,dx$

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x$

$du = -\csc^2 x\,dx$

$-\int u^3\,du - \int u^5\,du$

$-\frac{1}{4}u^4 - \frac{1}{6}u^6 + C = \boxed{-\frac{1}{4}\cot^4 x - \frac{1}{6}\cot^6 x + C}$

(37) $\int \tan x\,dx$ identity: $\tan x = \frac{\sin x}{\cos x}$

$\int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$-\int \frac{1}{u}\,du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

(38) $\int \cos x(1+\sin^2 x)\,dx$ u-sub

$u = 1+\sin^2 x$ whoops, not really works

$\frac{du}{dx} = 2(\sin x)$

how about $u = \sin x$

$\frac{du}{dx} = \cos x$:)

$du = \cos x\,dx$

$\int (1+u^2)\,du$

$\int 1\,du + \int u^2\,du$

$u + \frac{1}{3}u^3 + C$

$\boxed{\sin x + \frac{1}{3}\sin^3 x + C}$

(39) $\int \frac{\sin x+\sec x}{\tan x}\,dx$

split by common denominator

$\int \frac{\sin x}{\tan x}dx + \int \frac{\sec x}{\tan x}dx$ →

trig identities:

$\int \frac{\sin x}{1}\frac{\cos x}{\sin x}dx + \int \frac{1}{\cos x}\frac{\cos x}{\sin x}dx$

$\int \cos x\,dx + \int \csc x\,dx$

memorized shortcuts:

$\boxed{\sin x + \ln|\csc x - \cot x| + C}$

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(40) $\int \frac{2t}{(t-3)^2}\,dt$

u-sub:

$u=t-3$ also need: $t=u+3$

$\frac{du}{dt}=1$

$du=dt$

$\int u^{-2}\,2t\,dt$

$\int u^{-2}\,2(u+3)\,du$

$2\int u^{-2}(u+3)\,du$

$2\int (u^{-2}u+3u^{-2})\,du$

$2\int (u^{-1}+3u^{-2})\,du$

$2\int \frac{1}{u}du + 6\int u^{-2}du$

$2\ln|u| + 6\frac{u^{-1}}{-1} + C$

$\boxed{2\ln|t-3| - \frac{6}{t-3} + C}$