2012

(no Calculator)

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

$(\sin x)^3$

1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$ $3(\sin x)^2 \cdot \cos x$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ $\boxed{\text{(E)}}$ $3\sin^2 x \cos x$

---

2. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

$\boxed{\text{(A)}}$ $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

rest = $\frac{dx}{dt} = 0$ & $\frac{dy}{dt} = 0$

$3t^2 - 6t = 0$ $12 - 6t = 0$

$3t(t-2) = 0$ $6(2-t) = 0$

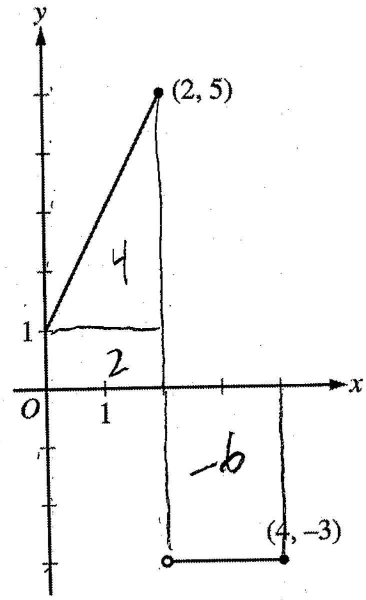
$t = 0, t = 2$ $t = 2$

at $t = 2$: $x = (2)^3 - 3(2)^2 = 8 - 12 = -4$

$y = 12(2) - 3(2)^2 = 24 - 12 = 12$

$(-4, 12)$

GO ON TO THE NEXT PAGE.



Graph of f

3. The graph of f is shown above for $0 \leq x \leq 4$. What is the value of $\int_0^4 f(x)\,dx$? = area

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

$2+4-6=0$

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

$\boxed{\text{(A)}}$ $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right) dx$

$\int_a^b \sqrt{1+[f'(x)]^2}\, dx$ $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$\int_1^2 \sqrt{1+\left(\frac{1}{x^2}\right)}\, dx$

---

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ $\boxed{\text{(C)}}$ $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

$\sum_{n=0}^{\infty} \left(-\frac{3}{4}\right)^n$ is geometric w/ $r = \frac{-3}{4}$ $|r| < 0$ converges to sum of $\frac{a}{1-r}$ $a = \left(-\frac{3}{4}\right)^0 = 1$

$\frac{1}{1-\left(-\frac{3}{4}\right)} = \frac{1}{1+\frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$

GO ON TO THE NEXT PAGE.

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2 - 3\right)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$

(B) $\int_{-2}^{13} u^5 \, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

$u = x^2 - 3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$\frac{1}{2}\int_{-2}^{13} u^5 \, du$

$x = -1$

$u = (-1)^2 - 3$

$= -2$

$x = 4$

$u = (4)^2 - 3$

$= 16 - 3$

$= 13$

---

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1 - x^2}}$

(B) $\frac{xy}{\sqrt{1 - x^2}}$

(C) $\frac{y}{1 + x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1 + x^2}$

implicit differentiation:

$\frac{d}{dx}[\arcsin x] = \frac{d}{dx}[\ln y]$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$

GO ON TO THE NEXT PAGE.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

interval	x_i	$f(x_i)$	$\cdot\ \Delta x$	= area
[4,7]	7	6.2	$\cdot$ 3	= 18.6
[7,12]	12	5.9	$\cdot$ 5	= 29.5
[12,15]	15	5.6	$\cdot$ 3	= 16.8

64.9 + 50 starting $= \boxed{114.9}$

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

ratio test

$\lim_{n\to\infty} \left| \frac{8^{n+1}}{(n+1)!} \cdot \frac{n!}{8^n} \right|$

$= \lim_{n\to\infty} \left| \frac{8 \cdot 8^n \cdot n!}{(n+1) n! \, 8^n} \right|$

$= \lim_{n\to\infty} \left| \frac{8}{n+1} \right| = 0 < 1$

Converges

II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

ratio test

$\lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1) n! \, n^{100}}{(n+1)^{100} n!} \right|$

$= \lim_{n\to\infty} \frac{(n+1) n^{100}}{(n+1)^{100}}$

$= \lim_{n\to\infty} \frac{n \cdot n^{100} + n^{100}}{(n+1)^{100}}$

$= \lim_{n\to\infty} \frac{n^{101} + n^{100}}{n^{100} + \dots}$

$\to \infty$

diverges

III. $\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)(n+3)}$ (ratio test = 1)

limit compare w/ $\frac{n}{n^3}$, $\frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{n+1}{n(n+2)(n+3)}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2(n+1)}{n(n+2)(n+3)}$

$= \lim_{n\to\infty} \frac{n^3+n^2}{n^3+\dots} = 1$ finite, positive

now $\sum_{n=1}^{\infty} \frac{1}{n^2}$ p-series w/ $p=2$ converges

so converges

Unauthorized copying or reuse of any part of this page is illegal.

GO ON TO THE NEXT PAGE.

10. $\int_1^4 t^{-3/2}\,dt =$ $\dfrac{t^{-1/2}}{-1/2} = \left[-2\dfrac{1}{\sqrt{t}}\right]_1^4 = \dfrac{-2}{\sqrt{4}} - \dfrac{-2}{\sqrt{1}} = \dfrac{-2}{2} + \dfrac{2}{1} = -1 + 2 = 1$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ $\boxed{\text{(E)}}$ 1

---

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true? MATH, num

$\boxed{\text{(A)}}$ f is continuous but not differentiable at $x = 2$.

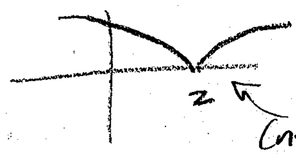
(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x\to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

graph in calc. $| \, | =$ abs



continuity: ① $f(2) = \sqrt{|2-2|} = \sqrt{0} = 0$ ✓

② $\lim_{x\to 2} f(x)$

$\lim_{x\to 2^-} \sqrt{|x-2|}$ $\lim_{x\to 2^+} \sqrt{|x-2|}$ $= 0$ ✓

0 0

③ $f(2) = \lim_{x\to 2} f(x)$

$0 = 0$ ✓

GO ON TO THE NEXT PAGE.

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

$\boxed{\text{(C)}}$ $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

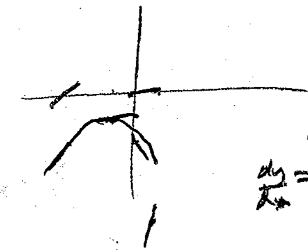
(E) $(-1, -1)$ is a point of inflection of the graph of f.

$(-1,-1)$

$\frac{dy}{dx} = (-1)^2 + (-1)$
$= 1 - 1$
$= 0$

$(1,-5)$

$\frac{dy}{dx} = (1)^2 + (-5)^2$
$= 26$



$(-2,-2)$ $\frac{dy}{dx} = (-2)^2 + (-2)$ $= 4-2$ $= 2$

$(0,-2)$ $\frac{dy}{dx} = (0)^2 + (-2)$ $= -2$

$(-2,0)$ $\frac{dy}{dx} = (-2)^2 + 0$ 4

$(0,-)$ $\frac{dy}{dx} = -$

$\frac{d^2y}{dx^2} = 2x + \frac{dy}{dx} = 2x + x^2 + y$

at $(-1,-1)$ $\frac{d^2y}{dx^2} = 2(-1) + (-1)^2 + (-1)$
$= -2 + 1 - 1$
$= -2$ not inflection pt.

---

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 $\boxed{\text{(C)}}$ $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

ratio test $\lim_{n\to\infty} \left| \frac{(x-4)^{2n+2}}{3^{n+1}} \cdot \frac{3^n}{(x-4)^{2n}} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-4)^2 (x-4)^{2n} \cdot 3^n}{3 \cdot 3^n (x-4)^{2n}} \right|$

$= \lim_{n\to\infty} \left| \frac{1}{3} \right| \left| (x-4)^2 \right|$

$\frac{1}{3}(x-4)^2 < 1$

$(x-4)^2 < 3$

$x - 4 < \sqrt{3}$

GO ON TO THE NEXT PAGE.

14. Let k be a positive constant. Which of the following is a logistic differential equation?

(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

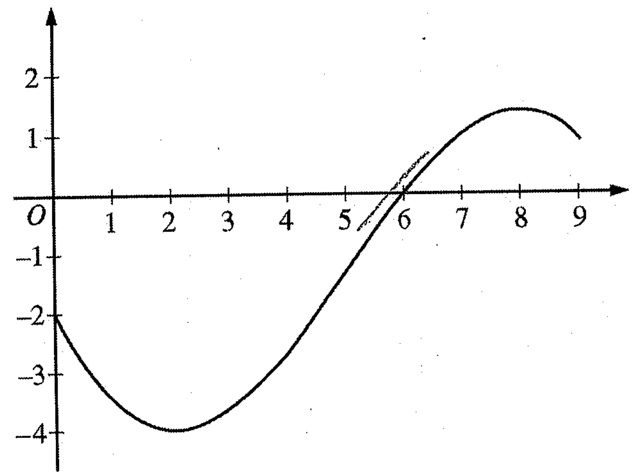
(C) $\frac{dy}{dt} = kt(1-t)$

(D) $\frac{dy}{dt} = ky(1-t)$

$\boxed{\text{(E)}}$ $\frac{dy}{dt} = ky(1-y)$

$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

GO ON TO THE NEXT PAGE.



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

$\boxed{\text{(A)}}$ $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

$h(6) = \int_0^6 f(t)dt$ = area under curve $(-)$

$h'(6) = \frac{d}{dx}\left[\int_0^x f(t)dt\right] = f(x) = f(6) = 0$

$h''(6) = f'(6) \approx 1$ from graph

$h(6) < h'(6) < h''(6)$

$(-) \quad 0 \quad \approx 1$

GO ON TO THE NEXT PAGE.

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

$h = 0.5$

(A) $-\frac{5}{4}$ (B) 1 $\boxed{\text{(C)}}$ $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

(x, y)	$y_{new} = y_{old} + h\frac{dy}{dx}$
$(1, 3)$	$y = 3 + (0.5)((1) - (3)) = 3 + \frac{1}{2}(-2) = 3 - 1 = 2$
$(1.5, 2)$	$y = 2 + \frac{1}{2}(\frac{3}{2} - 2) = 2 + \frac{1}{2}(-\frac{1}{2}) = 2 - \frac{1}{4} = \frac{8}{4} - \frac{1}{4} = \frac{7}{4}$
$(2, \frac{7}{4})$	

---

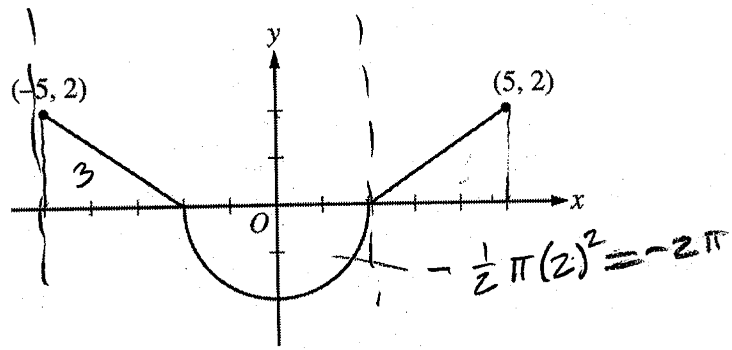
17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ $\boxed{\text{(C)}}$ $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$

not $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ← equals this with 1 x divided out

not $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$ so $\frac{\sin x}{x}$

GO ON TO THE NEXT PAGE.



Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

$\int_{-5}^{2} f'(x)\,dx = f(2) - f(-5)$

$3 - 2\pi = 1 - f(-5)$

$f(-5) = 1 - 3 + 2\pi = -2 + 2\pi$

$= 2\pi - 2$

GO ON TO THE NEXT PAGE.

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

$\boxed{\text{(C)}}$ $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2} = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$

$f'(x) = \frac{1}{2} = \frac{2}{(x+2)^2}$

$(x+2)^2 = 4$

$x+2 = \pm 2$

$x = -2 \pm 2$

$x = 0, x = -4$

$f(0) = 0 \quad f(-4) = \frac{-4}{-4+2} = \frac{-4}{-2} = 2$

$(0,0) \quad (-4,2)$

---

20. $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

x^2+3x+2

$(x+1)(x+2)$

(A) $\ln(8)$ (B) $\ln\left(\frac{27}{2}\right)$ $\boxed{\text{(C)}}$ $\ln(18)$ (D) $\ln(288)$ (E) divergent

$\frac{5x+8}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$

$A(x+2) + B(x+1) = 5x+8$

$(A+B)x + (2A+B) = (5)x + (8)$

$\begin{cases} A+B=5 \\ 2A+B=8 \end{cases}$

$-A-B=-5$

$2A+B=8$

$A=3$

$3+B=5$

$B=2$

$3\int_0^1 \frac{1}{x+1}dx + 2\int_0^1 \frac{1}{x+2}dx$

$\left[3\ln|x+1| + 2\ln|x+2|\right]_0^1$

$3\ln 2 + 2\ln 3 - 3\cancel{\ln 1}^{0} - 2\ln 2$

$2\ln 3 + \ln 2$

$\ln(3^2) + \ln 2$

$\ln 9 + \ln 2$

$\ln(9 \cdot 2)$

$\ln(18)$

GO ON TO THE NEXT PAGE.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

$\lim_{x\to\infty} y = 5$

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

~~(A)~~ $y = \dfrac{\sin(5x)}{x}$ ~~(B)~~ $y = 5x$ ~~(C)~~ $y = \dfrac{1}{x-5}$ ~~(D)~~ $y = \dfrac{5x}{1-x}$ $\boxed{\text{(E)}}$ $y = \dfrac{20x^2 - x}{1 + 4x^2}$

no H/A

HA $y=0$

HA $y = \frac{5}{-1}$

$y=-5$

$\frac{20x^2}{4x^2}$

$\boxed{y=5}$

---

22. The power series $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

$\boxed{\text{(D)}}$ The series converges at $x = 2$.

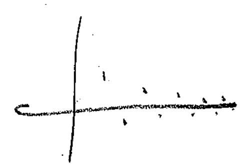
(E) The series converges at $x = 6$.

$\sum_{n=0}^{\infty} a_n (5-3)^n$

$= \sum_{n=0}^{\infty} a_n (2)^n$ converges

$\boxed{D}$ $\sum_{n=0}^{\infty} a_n (2-3)^n$

$= \sum_{n=0}^{\infty} a_n (-1)^n$



Theorem: If $\sum |a_n|$ converges then $\sum a_n$ also converges

(test for absolute convergence)

GO ON TO THE NEXT PAGE.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

---

24. Let f be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \int 4x^3 \cos x\,dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ $\boxed{\text{(E)}}$ x^4 by parts

$\int x^4 \sin x\,dx$ $u = x^4$ $dv = \sin x\,dx$

$\frac{du}{dx} = 4x^3$ $\int dv = \int \sin x\,dx$

$du = 4x^3\,dx$ $v = -\cos x$

$uv - \int v\,du$

$-x^4 \cos x + \int 4x^3 \cos x\,dx$

GO ON TO THE NEXT PAGE.

25. $\int_1^{\infty} xe^{-x^2}\,dx$ is

(A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent

$\lim_{b\to\infty} \int_1^b xe^{-x^2}dx$

$u = -x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$x\,dx = -\frac{1}{2}du$

$\int e^u(-\frac{1}{2}du)$

$-\frac{1}{2}\int e^u du$

$-\frac{1}{2}e^u$

$\lim_{b\to\infty}\left(-\frac{1}{2}e^{(-x^2)}\right]_1^b$

$-\frac{1}{2}\lim_{b\to\infty} e^{-b^2} - \left[-\frac{1}{2}e^{-(1)^2}\right]$

$0 + \frac{1}{2}\frac{1}{e}$

$\frac{1}{2e}$

GO ON TO THE NEXT PAGE.

26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 $\boxed{\text{(B)}}$ $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

$x = r\cos\theta$ $y = r\sin\theta$

$x = (1+2\sin\theta)\cos\theta$ $y = (1+2\sin\theta)\sin\theta$

$x = \cos x + 2\sin\theta\cos\theta$ $y = \sin\theta + 2(\sin\theta)^2$

$\frac{dx}{d\theta} = -\sin\theta + 2\sin\theta(-\sin\theta) + \cos\theta(2\cos\theta)$ $\frac{dy}{d\theta} = \cos\theta + 4\sin\theta\cos\theta$

$= -\sin\theta - 2\sin^2\theta + 2\cos^2\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta + 4\sin\theta\cos\theta}{-\sin\theta - 2\sin^2\theta + 2\cos^2\theta}$

at $\theta = 0$:

$\frac{dy}{dx} = \frac{\cos(0) + 4(\sin(0))\cos(0)}{-\sin(0) - 2(\sin(0))^2 + 2(\cos(0))^2}$

$= \frac{1 + 4(0)(1)}{-0 - 2(0)^2 + 2(1)^2}$

$= \frac{1}{2}$

GO ON TO THE NEXT PAGE.

27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

$\boxed{\text{(C)}}$ $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$

p-series

converges

when

$2p > 1$

$p > \frac{1}{2}$

$\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$

geometric

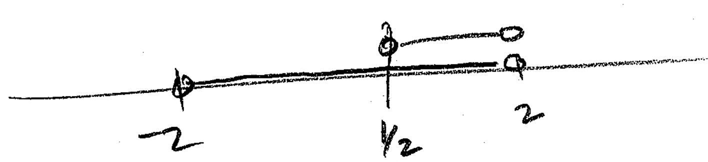
converges

when

$\left|\frac{p}{2}\right| < 1$

$|p| < 2$

$-2 < p < 2$



$\frac{1}{2} < p < 2$

GO ON TO THE NEXT PAGE.

28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) The limit does not exist.

$$\lim_{x \to 1} \frac{\int_1^x g(t)dt}{g(x) - 6}$$

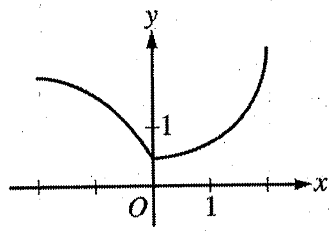
$$\frac{\int_1^1 g(t)dt}{g(1) - 6} = \frac{0}{6-6} = \frac{0}{0}$$

by L'Hopital's $\lim_{x \to 1} \frac{\frac{d}{dx}\int_1^x g(t)dt}{\frac{d}{dx}[g(x)-6]} = \frac{g(x)}{g'(x)} = \frac{g(1)}{g'(1)} = \frac{6}{3} = 2$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



Graph of f

76. The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

---

77. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

(A) $(0.831, 7.384)$ only

(B) $(-\infty, 0.831)$ and $(7.384, \infty)$

(C) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(D) $(-0.816, 1.430)$ and $(8.613, \infty)$

(E) $(-\infty, \infty)$

$f'(x) = e^x$

$g'(x) = 4x^3$

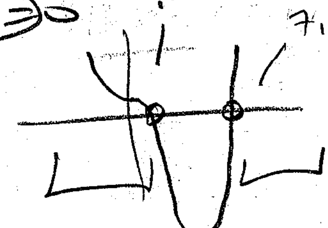
$f'(x) = g'(x)$

$e^x = 4x^3$

$e^x - 4x^3 = 0$

graph this

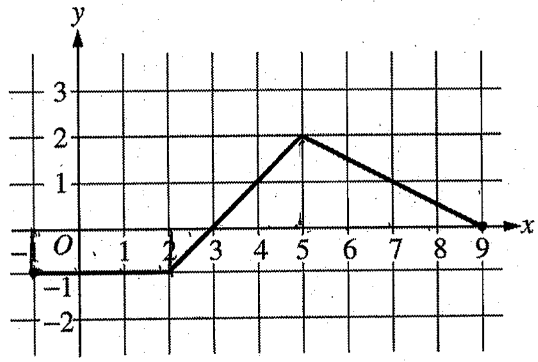
.831 7.384



$f'(x) - g'(x)$

$(-\infty, .831) \cup (7.384, \infty)$

GO ON TO THE NEXT PAGE.



Graph of f

78. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9} (3f(x) + 2)\, dx$?

(A) 7.5 (B) 9.5 $\boxed{\text{(C)}}$ 27.5 (D) 47 (E) 48.5

$= 3\int_{-1}^{9} f(x)\,dx + \int_{-1}^{9} 2\,dx$

$= 3\,\text{area} + 2(9-(-1)) = 3\left(-3 - \frac{1}{2} + 2 + 4\right) + 2(10)$

$= 27.5$

---

79. Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

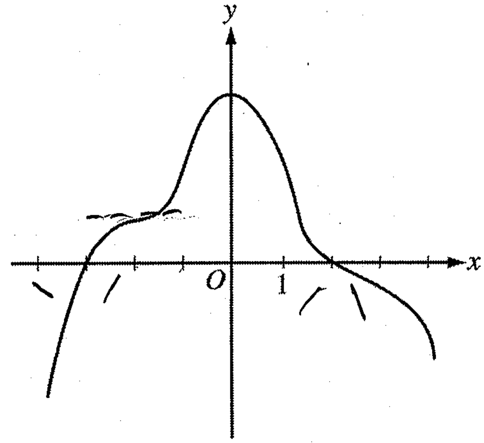
(D) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

$\boxed{\text{(E)}}$ $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

$P_3(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!}$

$= 2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

GO ON TO THE NEXT PAGE.



Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$. true

II. The graph of f has a point of inflection at $x = -2$. false (f' increasing though, staying concave up)

III. The graph of f is concave down for $0 < x < 4$. f' decreasing true

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

GO ON TO THE NEXT PAGE.

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

81. Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table above. If $\boxed{f(x) = f(-x),}$ what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

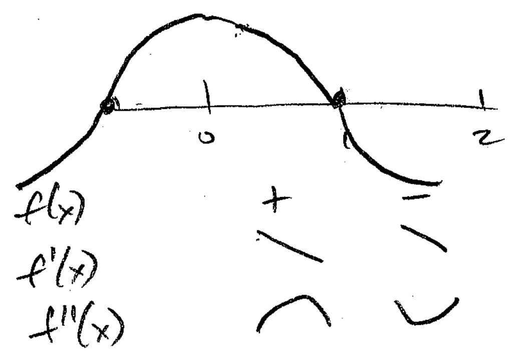
(A) $x = 0$ only

(B) $x = 1$ only

(C) $x = 0$ and $x = 1$

(D) $x = -1$ and $x = 1$

(E) There are no points of inflection on $-2 < x < 2$.



82. What is the average value of $y = \sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{2}$?

(A) -0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2}\sqrt{\cos x}\,dx \Rightarrow \frac{2}{\pi}(1.19814064) = 0.76276$$

GO ON TO THE NEXT PAGE.

83. If the function f is continuous at $x = 3$, which of the following must be true?

(A) $f(3) < \lim_{x\to 3} f(x)$

(B) $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$

$\boxed{\text{(C)}}$ $f(3) = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

(D) The derivative of f at $x = 3$ exists.

(E) The derivative of f is positive for $x < 3$ and negative for $x > 3$.

---

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4-2x^2+1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

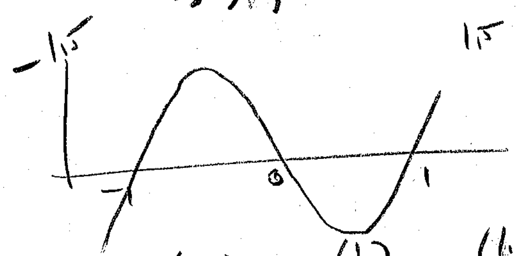
(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

$\boxed{\text{(D)}}$ $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

$f''(x) = e^{(x^4-2x^2+1)} \cdot (4x^3-4x)$

by graph $f''(x)=0$;



(-1.1) $(-\frac{1}{2})$ $(\frac{1}{2})$ (1.4)

$-$ $+$ $-$ $+$

$(-1.5,-1) \cup (0,1)$

GO ON TO THE NEXT PAGE.

85. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(s) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, what is the rate at which its fuel consumption is changing?

(A) 0.215 mpg per hour

$\boxed{\text{(B)}}$ 4.299 mpg per hour

(C) 19.793 mpg per hour

(D) 25.793 mpg per hour

(E) 515.855 mpg per hour

$$\frac{dF}{ds} = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)} \cdot \left(\frac{1}{20} - \frac{2}{2400}s\right)$$

$s = 50$ mph

$$\frac{dF}{ds} = 6e^{\left(\frac{50}{20} - \frac{50^2}{2400}\right)} \cdot \left(\frac{1}{20} - \frac{2}{2400}(50)\right)$$

$$= .2149394453 \ \frac{\text{mpg}}{\text{mph}}$$

$$\frac{ds}{dt} = \frac{20 \text{ mph}}{\text{hr}}$$

$$.2149394453 \ \frac{\text{mpg}}{\text{mph}} \cdot \frac{20 \text{ mph}}{\text{hr}} = 4.29879 \text{ mpg/hr}$$

$$\frac{dF}{ds} \cdot \frac{ds}{dt} = \frac{dF}{dt}$$

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)\,dt = 0$, which of the following could be a table of values for the function f ?

$f'(x) > 0$ = increasing

$\int_4^7 f(t)\,dt \Rightarrow$ mean area $= 0$

(Riemann Sums)

(A)

x	$f(x)$
4	−4
5	−3
7	0

(B)

x	$f(x)$
4	−4
5	−2
7	5

only B has both $f(x)>0$, $f(x)<0$ so that area could sum to zero.

~~(C)~~

x	$f(x)$
4	−4
5	6
7	3

not increasin

~~(D)~~

x	$f(x)$
4	0
5	0
7	0

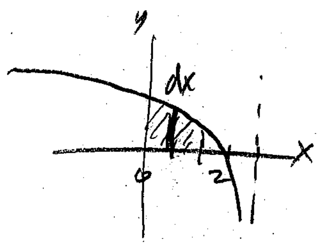
(E)

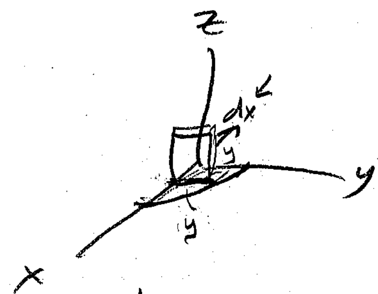
x	$f(x)$
4	0
5	4
7	6

GO ON TO THE NEXT PAGE.

87. Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3 - x)$, for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.442 $\boxed{\text{(B)}}$ 1.029 (C) 1.296 (D) 3.233 (E) 4.071





$A_{cross} = y^2$

$V = \int_a^b A_{cross}\,dx$

$= \int_0^2 y^2\,dx$

$= \int_0^2 (\ln(3-x))^2\,dx$

math 9

$= 1.02917$

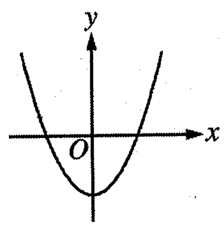
GO ON TO THE NEXT PAGE.

88. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

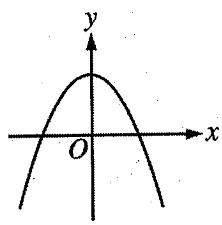
$f'(x)$ increasing $\Rightarrow f''(x) > 0$ for $x < 0$
$f'(x)$ decreasing $\Rightarrow f''(x) < 0$ for $x > 0$

$f''(x)$ + | 0 | −

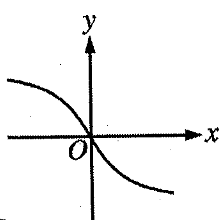
(A)



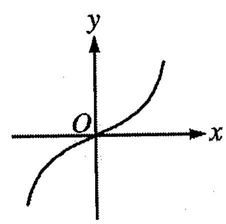
(B)



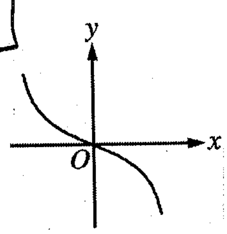
(C)



(D)



$\boxed{\text{(E)}}$



89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 $\boxed{\text{(E)}}$ 11.710

$v(t) = \int a(t)dt$ $\int_0^3 \frac{t+3}{\sqrt{t^3+1}} dt = v(3) - v(0)$

math 9

$6.710054 = v(3) - 5$

$v(3) = 6.710054 + 5 = 11.71005$

---

90. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} na_n$ diverges.

$\boxed{\text{(E)}}$ $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

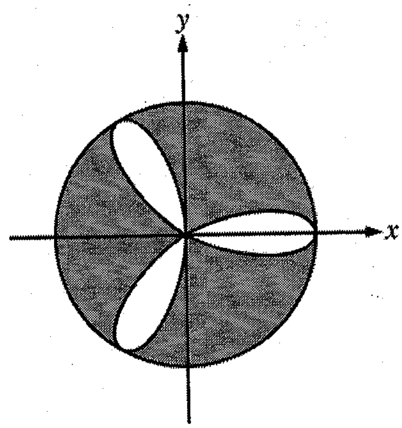
direct comparison test

$0 < b_n \leq a_n$

$\frac{a_n}{n} < a_n$

so if $\sum_{n=1}^{\infty} a_n$ converges

$\sum_{n=1}^{\infty} \frac{a_n}{n}$ also converges



91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

$r = 2\cos(3\theta) = 0$

$\cos(3\theta) = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}$

one petal area $= \frac{1}{2}\int_{\pi/6}^{\pi/2} r^2 \, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{\pi/2} (2\cos(3\theta))^2 \, d\theta$

math 9

(π)

hole ÷ $= 1.04719551 \times 3 = 3.141592653$

circle area $= \pi r^2 = \pi (2)^2 = 4\pi$

Shaded $= 4\pi - \pi = 3\pi = \boxed{9.425}$

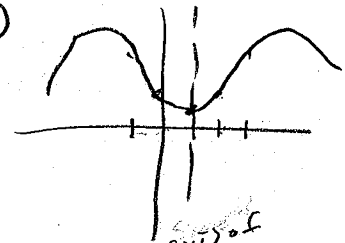
GO ON TO THE NEXT PAGE.

92. The function h is differentiable, and for all values of x, $h(x) = h(2-x)$. Which of the following statements must be true?

I. $\int_0^2 h(x)\,dx > 0$ False (h(x) could be negative)

II. $h'(1) = 0$ True (can't be cusp)

III. $h'(0) = h'(2) = 1$ False (could be any slope)



axis of symmetry

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

   (d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) = \dfrac{W(15)-W(9)}{15-9} = \dfrac{67.9-61.8}{15-9} = \boxed{1.0167\ °F/min}$

The water temperature is increasing at a rate of approximately 1.0167 °F/min at t = 12 minutes

(b) $\int_0^{20} W'(t)dt = W(20) - W(0) = 71.0 - 55.0 = \boxed{16\ °F}$

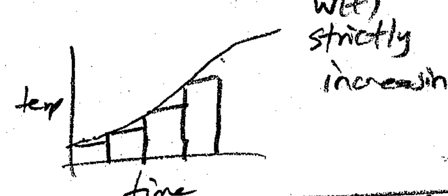
The water has warmed by 16 °F from t = 0 to t = 20 minutes.

(c)

interval	x_i	$f(x_i)\cdot\Delta x$ = area
[0,4]	0	55.0 · 4 = 220
[4,9]	4	57.1 · 5 = 285.5
[9,15]	9	61.8 · 6 = 370.8
[15,20]	15	67.9 · 5 = 339.5
		1215.8

$\int_0^{20} W(t)dt \approx 1215.8$

so $\frac{1}{20}\int_0^{20} W(t)dt \approx \frac{1215.8}{20} = \boxed{60.79\ °F}$



Because we used left end heights with a strictly increasing function, this approximation will underestimate the avg temp

(d) $\int_{20}^{25} W'(t)dt = W(25) - W(20)$

$\int_{20}^{25} 0.4\sqrt{t}\cos(0.06t)dt = W(25) - 71.0$ (math 9) , $2.0431547 = W(25) - 71.0$

$W(25) = 71 + 2.0431547 = \boxed{73.043\ °F}$

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{\sqrt{(2)+2}}{e^2} = \frac{2}{e^2} > 0$ $\boxed{\text{Because } \frac{dx}{dt} > 0 \text{, the particle is moving to the right at } t=2}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)}\right|_{t=2} = \frac{(\sin(2))^2}{\frac{\sqrt{(2)+2}}{e^2}} = \boxed{3.055}$

(b) $\int_2^4 x'(t)\,dt = x(4) - x(2)$ $\quad \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = x(4) - 1$, $x(4) = 1 + 0.252954$

$x(4) = \boxed{1.253}$

(c) speed $= |\vec{v}(t)| = \left.\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right|_{t=4} = \sqrt{\left(\frac{\sqrt{4+2}}{e^4}\right)^2 + (\sin(4))^4} = \boxed{0.575}$

$\vec{v}(t) = \left\langle \frac{(t+2)^{1/2}}{e^t}, (\sin t)^2 \right\rangle$ $\quad \vec{a}(t) = \left\langle \frac{e^t\left(\frac{1}{2}(t+2)^{-1/2}\right) - (t+2)^{1/2}e^t}{e^{2t}}, 2\sin t\cos t \right\rangle$

at $t=4$: $\vec{a}(t) = \left\langle \frac{e^4\frac{1}{2}(4+2)^{-1/2} - (4+2)^{1/2}e^4}{e^8}, 2\sin(4)\cos(4) \right\rangle$

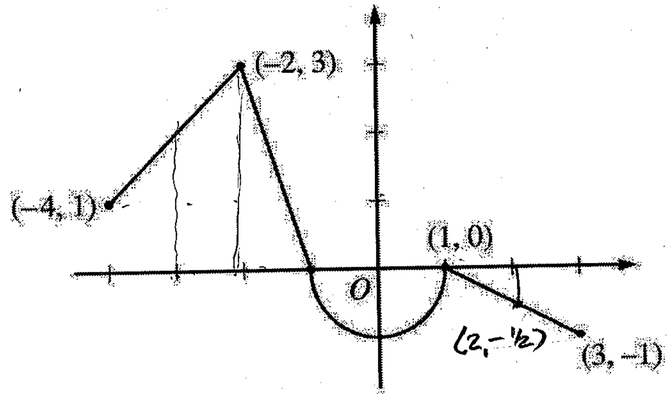
$= \boxed{\langle -0.041, 0.989 \rangle}$

(d) dist travelled $= \int_2^4 |\vec{v}(t)|\,dt = \int_2^4 \sqrt{\frac{(t+2)}{e^{2t}} + \sin^4 t}\,dt$

$\approx \boxed{0.651}$

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t)dt = \boxed{-\frac{1}{4}}$ $g(-2) = \int_1^{-2} f(t)dt = -\int_{-2}^{1} f(t)dt = -\left[\frac{3}{2} - \frac{1}{2}\pi(1)^2\right] = \boxed{-\frac{3}{2} + \frac{\pi}{2}}$

(b) $g'(-3) = \frac{d}{dx}\int_1^x f(t)dt = f(x) - f(1) = f(-3) - f(1) = 2 - 0 = \boxed{2}$

$g''(-3) = f'(-3)$ = slope from graph of f at $x = -3$ $= \boxed{1}$

(c) $g(x)$ has horiz. ~~tangent~~ when $g'(x) = 0$, $g'(x) = \frac{d}{dx}\int_1^x f(t)dt = f(x) - f(1)$
so when $f(x) - f(1) = 0$ or $f(x) = f(1) = 0$ This occurs at $\boxed{x = -1}$ and $\boxed{x = 1}$
at $\underline{x = -1}$: $f(x) = g'(x)$ changes sign from + to − so $g(x)$ has $\boxed{\text{relative maximum at } x = -1}$

at $\underline{x = 1}$: $f(x) = g'(x)$ does not change sign, so $g(x)$ $\boxed{\text{has neither a rel. max or min at } x = 1}$

(d) $g(x)$ has point of inflection when $g''(x)$ changes sign. $g'(x) = f(x)$, so this occurs when $f'(x)$ changes sign which is when $f(x)$ changes from increasing to decreasing or vice-versa. This occurs at $\boxed{x = -2,\ x = 0,\ \text{and } x = 1}$

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

(a) $\boxed{(y-15) = 8(x-1)}$ $\quad y-15 = 8(1.4-1)$, $y = 8(1.4-1)+15 \approx \boxed{18.2}$

(b)

interval	x_i	$f(x_i)$	Δx	= area
[1, 1.2]	1.1	10	0.2	= 2
[1.2, 1.4]	1.3	13	0.2	= 2.6
				4.6

$\int_1^{1.4} f'(t)\,dx \approx 4.6 = f(1.4) - f(1)$

$f(1.4) = f(1) + 4.6 = 15 + 4.6 = \boxed{19.6}$

(c)

(x, y)	$y_{new} = y_{old} + h(f'(x))$
(1, 15)	$y_{new} = 15 + 0.2(8) = 16.6$
(1.2, 16.6)	$y_{new} = 16.6 + 0.2(12) = 19$

$f(1.4) \approx \boxed{19.0}$

(d) $P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$= 15 + 8(x-1) + \frac{20}{2}(x-1)^2$

$P_2(1.4) = 15 + 8(1.4-1) + 10(1.4-1)^2$

$= \boxed{19.8}$

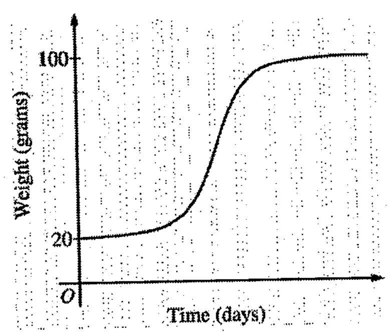
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = 12$ $\qquad$ $\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = 6$

Because $\frac{dB}{dt}$ is greater w/ $B=40$ than w/ $B=70$ the bird is gaining weigh faster when it weighs $\boxed{\text{40 grams}}$

(b) by the Chain Rule: $\frac{d^2B}{dt^2} = \frac{d}{dB}\left[\frac{1}{5}(100-B)\right] \cdot \frac{dB}{dt} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\left(\frac{1}{5}(100-B)\right) = \frac{-1}{25}(100-B)$

since $20 \leq B \leq 100$, $\frac{d^2B}{dt^2} < 0$ (concave down),

but in the graph shown, the curve is concave up at first.

(c) $\frac{dB}{dt} = \frac{1}{5}(100-B)$

$\frac{1}{100-B}\, dB = \frac{1}{5}dt$

$\int \frac{1}{100-B}\, dB = \int \frac{1}{5}dt$

$u = 100-B$ $\qquad -\int \frac{1}{u}\,du = \int \frac{1}{5}dt$

$\frac{du}{dB} = -1$ $\qquad -\ln|100-B| = \frac{1}{5}t + C$

$du = -dB$ $\qquad B(0) = 20$

$-\ln|100-20| = \frac{1}{5}(0) + C$

$C = -\ln|80| = -\ln(80)$

$-\ln|100-B| = \frac{1}{5}t - \ln(80)$

$\ln|100-B| = \ln 80 - \frac{1}{5}t$

$e \qquad e$

$|100-B| = e^{\ln 80 - \frac{1}{5}t} = e^{\ln 80} \cdot e^{-1/5 t} = 80e^{-1/5 t}$

$\boxed{B = 100 - 80e^{-1/5 t}}$

6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \dots + \frac{x^7}{9}$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

(a) $\lim_{n\to\infty} \left| \frac{x^{2n+3}}{2n+5} \cdot \frac{2n+3}{x^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{x^2 x^{2n+1}(2n+3)}{(2n+5)\, x^{2n+1}} \right| = \lim_{n\to\infty} \frac{2n+3}{2n+5} |x^2| = (1)|x^2|$

converges when $x^2 < 1$ $\quad -1 < x < 1$

endpoints: $x = -1$: $\sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{2n+1}}{2n+3}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^1(-1)^{2n}}{2n+3}$

$= (-1)\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+3}$

alternating series test

$\lim_{n\to\infty} \frac{1}{2n+3} = 0$ ✓

$a_{n+1} \overset{?}{<} a_n$ $\quad$ converges

$\frac{1}{2n+5} < \frac{1}{2n+3}$ ✓

$x = 1$: $\sum_{n=0}^{\infty} (-1)^n \frac{(1)^{2n+1}}{2n+3}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+3}$

alternating series test

← same as this

converges

so interval of convergence is

$\boxed{-1 \le x \le 1}$

(b) error is $\le$ |first disregarded term|, error $< \left|\frac{x^5}{7}\right| = \left|\frac{(\frac{1}{2}-0)^5}{7}\right| = \boxed{\frac{1}{224} < \frac{1}{200}}$ which is

(c) $g'(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)x^{2n}}{2n+3} = \boxed{\frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \dots (-1)^n \frac{(2n+1)x^{2n}}{2n+3} + \dots}$