A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

chain rule:

$2(x^3+1)(3x^2)$

$6x^2(x^3+1)$

---

2. $\int_0^1 e^{-4x}\,dx =$ $u = -4x$, $du = -4\,dx$ $-\frac{1}{4}\int e^u\,du = \left[-\frac{1}{4}e^{-4x}\right]_0^1 = -\frac{1}{4}e^{-4} - \left[-\frac{1}{4}e^0\right]$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$

$-\frac{1}{4}e^{-4} + \frac{1}{4}$

GO ON TO THE NEXT PAGE.

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim\limits_{x \to 2} f(x) = \infty$

$\boxed{\text{(E)}}$ $\lim\limits_{x \to \infty} f(x) = 2$

$\lim\limits_{x \to \infty} f(x) = 2$

---

4. If $y = \dfrac{2x+3}{3x+2}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{12x+13}{(3x+2)^2}$ (B) $\dfrac{12x-13}{(3x+2)^2}$ (C) $\dfrac{5}{(3x+2)^2}$ $\boxed{\text{(D)}}$ $\dfrac{-5}{(3x+2)^2}$ (E) $\dfrac{2}{3}$

quotient rule: $\dfrac{(3x+2)\frac{d}{dx}[2x+3] - (2x+3)\frac{d}{dx}[3x+2]}{(3x+2)^2}$

$\dfrac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$

$\dfrac{6x+4-6x-9}{(3x+2)^2} = \dfrac{-5}{(3x+2)^2}$

GO ON TO THE NEXT PAGE.

5. $\int_0^{\frac{\pi}{4}} \sin x \, dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ $\boxed{\text{(D)}}$ $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

$\left[-\cos x\right]_0^{\pi/4} = -\cos\left(\frac{\pi}{4}\right) - \left(-\cos(0)\right)$

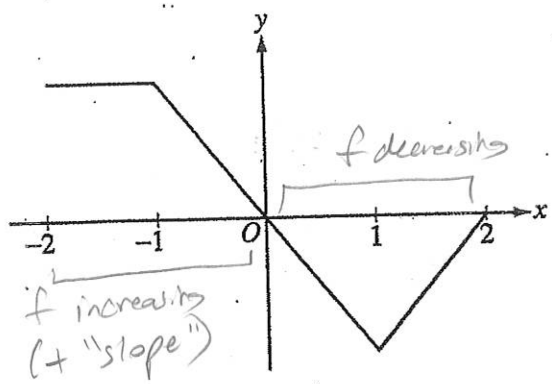
$-\frac{\sqrt{2}}{2} + 1$

---

6. $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$ numerator & denom. degree the same, only first terms matter

(A) 4 (B) 1 $\boxed{\text{(C)}\ \frac{1}{4}}$ (D) 0 (E) –1

$\approx \lim_{x\to\infty} \frac{x^3}{4x^3} = \lim_{x\to\infty} \frac{1}{4} = \frac{1}{4}$



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.

u-substitution:

$u = x^3 \quad du = 3x^2 dx$

$x^2 dx = \frac{1}{3} du$

8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

$\boxed{\text{(B)}}$ $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$\frac{1}{3}\int \cos u \, du$

$\frac{1}{3}\sin u + C$

$\frac{1}{3}\sin(x^3) + C$

---

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

$\boxed{\text{(A)}}$ $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

chain rule:

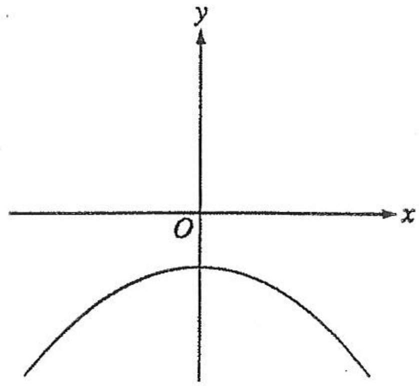
$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot \frac{d}{dx}\left[x+4+e^{-3x}\right]$

$\left.\frac{1}{x+4+e^{-3x}}(1-3e^{-3x})\right|_{x=0} = \frac{1}{(0)+4+e^{(0)}}(1-3e^{0}) = \frac{-2}{5}$

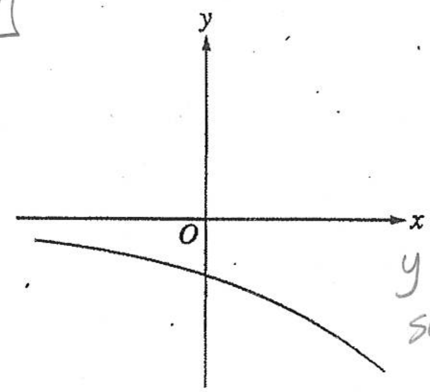
---

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

(A)

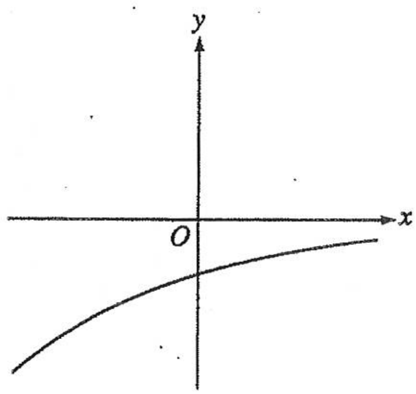


(B)

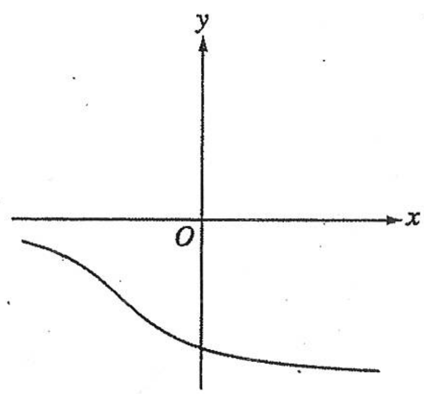


y is −
slope is −
concavity is −

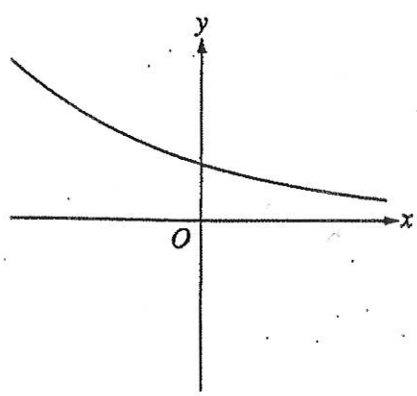
(C)



(D)



(E)



11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ $\boxed{\text{(C)}}$ $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

$\frac{du}{dx} = 2$

$du = 2dx$

$dx = \frac{1}{2}du$

$x = 0,\ u = 2(0)+1 = 1$

$x = 2,\ u = 2(2)+1 = 5$

$\frac{1}{2}\int_1^5 \sqrt{u}\,du$

---

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

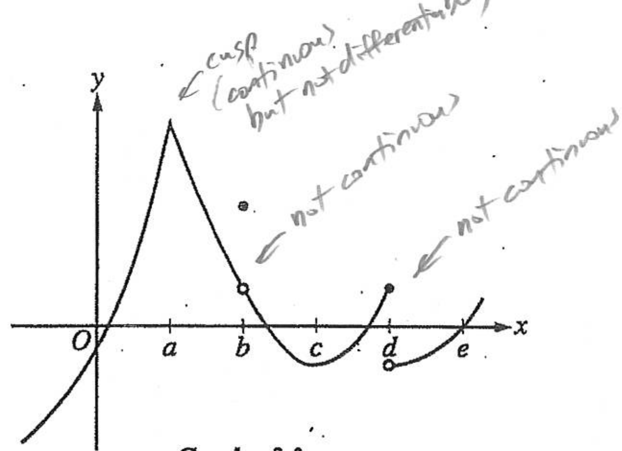
$\boxed{\text{(E)}}$ $\frac{dV}{dt} = k\sqrt{V}$

$\frac{dv}{dt} = k\sqrt{v}$

rate of change of volume

proportional to

square root of volume



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

---

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

product rule:

$y' = x^2 \frac{d}{dx}[\sin(2x)] + \sin(2x) \frac{d}{dx}[x^2]$

$= x^2 \cos(2x) \cdot 2 + \sin(2x)(2x)$

$= 2x^2 \cos(2x) + 2x \sin(2x)$

$= 2x(x \cos(2x) + \sin(2x))$

---

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

$\boxed{\text{(D)}}$ $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

critical values when $f'(x)=0$ or DNE

$x^2 - \frac{2}{x} = 0$ DNE when $x=0$

$x^3 - 2 = 0$

$x^3 = 2$

$x = \sqrt[3]{2}$

$-\infty$ 0 $\sqrt[3]{2}$ ∞

test -1: $(-1)^2 - \frac{2}{(-1)}$, $1 + 2$, $+$

test $\frac{1}{10}$: $(\frac{1}{10})^2 - \frac{2}{(\frac{1}{10})}$, $\frac{1}{100} - 20$, $-$

test 10: $(10)^2 - \frac{2}{10}$, $100 - \frac{2}{10}$, $+$

decreasing $(0, \sqrt[3]{2})$

---

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 $\boxed{\text{(C) } 3}$ (D) 7 (E) undefined

slope of tan line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$

$= f'(1)$

---

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$f'(x) = 2x\frac{d}{dx}[e^x] + e^x\frac{d}{dx}[2x] = 2xe^x + 2e^x$

$f''(x) = 2x\frac{d}{dx}[e^x] + e^x\frac{d}{dx}[2x] + \frac{d}{dx}[2e^x] = 2xe^x + 2e^x + 2e^x = 2xe^x + 4e^x = 2e^x(x+2)$

$2e^x(x+2) = 0$ (never zero)

$x+2=0$

$x=-2$

$-\infty$ test -3 -2 test 0 ∞

$2e^{-3}(-3+2)$ $-$ $2e^0(0+2)$ $+$

concave down?

$(-\infty, -2)$

or $x < -2$

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

when $g' < 0$

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

(can't change to + between $x=-2$ & $x=-1$ or there would be another zero)

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

$f'(x) = 2x+3$

$f(x) = \int (2x+3)dx = x^2+3x+C$

$2 = (1)^2+3(1)+C$

$2 = 4+C$

$-2 = C$

$f(x) = y = x^2+3x-2$

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x\to3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

I

$\lim_{x\to3} f(x)$

LH $\lim_{x\to3^-}(x+2)$ RH $\lim_{x\to3^+}(4x-7)$

$(3)+2$ $4(3)-7$

$5 = 5$

yes, exist

True

II

1) $f(3) = (3)+2 = 5$ exists ✓

2) $\lim_{x\to3} f(x)$ exists ✓

3) $f(3) \overset{?}{=} \lim_{x\to3} f(x)$

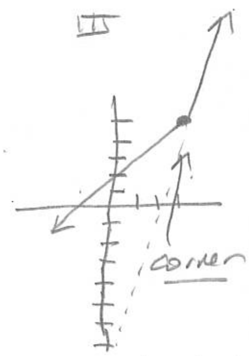
$5 = 5$

yes

True

(continuous)

III



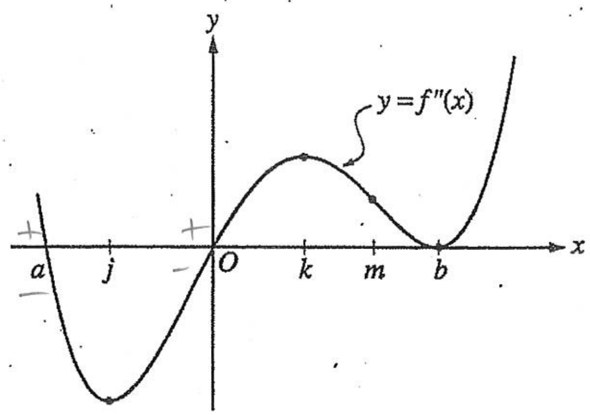
corner

not differentiable

(derivative DNE)

False

I & II



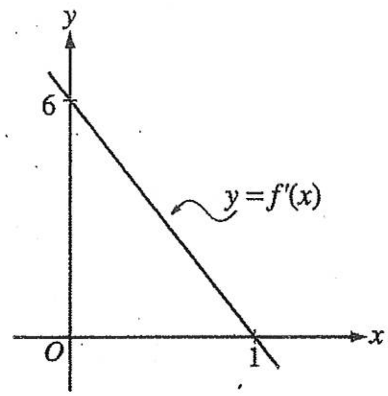
21. The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

$\boxed{\text{(A)}}$ 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

point of inflection when $f''(x)=0$ and $f''(x)$ is changing sign
this occours at $x=a$ and $x=0$

GO ON TO THE NEXT PAGE.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A



22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

$f'(x) = -6x+6$

so $f(x) = \int(-6x+6)\,dx = -3x^2+6x+C$ must go through (0,5)

$5 = -3(0)^2+6(0)+C$, $C=5$

$f(x) = -3x^2+6x+5$ so $f(1) = -3(1)^2+6(1)+5 = 8$

---

23. $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^2} \sin(t^3)\,dt\right) =$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

Fundamental Theorem of Calculus: $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$

let $w = x^2$

the $\dfrac{dw}{dx} = 2x$

so $\dfrac{d}{dw}\displaystyle\int_0^w \sin(t^3)\,dt = \sin(w^3)$

$\dfrac{d}{dx} = \dfrac{d}{dw}\cdot\dfrac{dw}{dx}$ so $= \sin((x^2)^3)\cdot 2x$

$\sin(x^6)\cdot 2x$

GO ON TO THE NEXT PAGE.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5 = 7$

$f(-1) = 4(-1)^3 - 5(-1) + 3$

$= -4 + 5 + 3$

$= 4$

$(y - 4) = 7(x - (-1))$

$y - 4 = 7x + 7$

$y = 7x + 11$

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

$x'(t) = 6t^2 - 42t + 72 = 0$

$6(t^2 - 7t + 12) = 0$

$6(t - 3)(t - 4) = 0$

$t = 3, \ t = 4$

$$\begin{array}{r} 12 \\ 6\overline{)72} \\ \underline{6} \\ 12 \end{array}$$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 $\quad$ $\boxed{\text{(B)}}$ $\frac{4}{9}$ $\quad$ (C) $\frac{7}{9}$ $\quad$ (D) $\frac{6}{7}$ $\quad$ (E) $\frac{5}{3}$

implicit differentiation: $3y^2-2x^2=6-\underline{2xy}$ ← product rule

$$\frac{d}{dx}[3y^2]-\frac{d}{dx}[2x^2]=\frac{d}{dx}[6]-\left((2x)\frac{d}{dx}[y]+(y)\frac{d}{dx}(2x)\right)$$

$$6y\frac{dy}{dx}-4x=0-\left(2x\frac{dy}{dx}+y(2)\right)$$

$$(6y+2x)\frac{dy}{dx}=4x-2y \qquad \frac{dy}{dx}=\left.\frac{4x-2y}{6y+2x}\right|_{(3,2)}=\frac{4(3)-2(2)}{6(2)+2(3)}=\frac{8}{18}=\frac{4}{9}$$

---

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ $\quad$ $\boxed{\text{(B)}}$ $\frac{1}{4}$ $\quad$ (C) $\frac{7}{4}$ $\quad$ (D) 4 $\quad$ (E) 13

theorem: if $g(x)=f^{-1}(x)$ then $g'(x)=\frac{1}{f'(g(x))}$

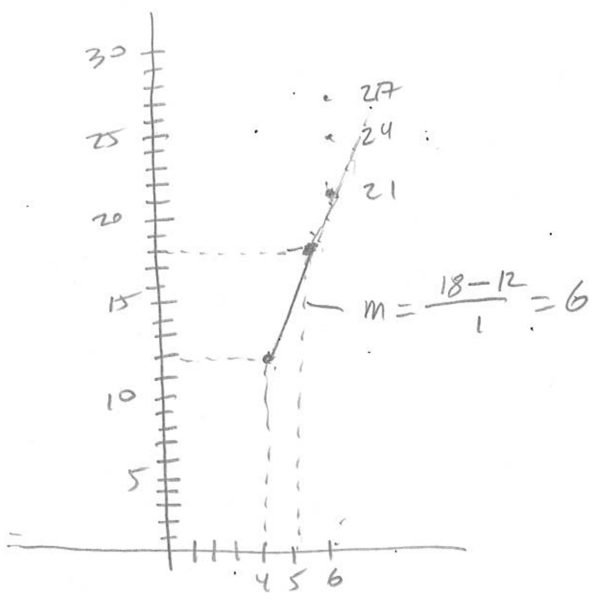
$f'(x)=3x^2+1$

$g'(2)=\frac{1}{f'(g(2))}=\frac{1}{3(1)^2+1}=\frac{1}{4}$

GO ON TO THE NEXT PAGE.

28. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 $\boxed{\text{(E)}}$ 27



So slope from $(5,18)$ to $(6,?)$ must be > 6

$(5,18) \rightarrow (6,21)$

$m = \frac{21-18}{1} = 3$ X

$(5,18) \rightarrow (6,24)$

$m = \frac{24-18}{1} = 6$ X

$(5,18) \rightarrow (6,27)$

$m = \frac{27-18}{1} = 9$ ✓

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

B B B B B B B B B

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

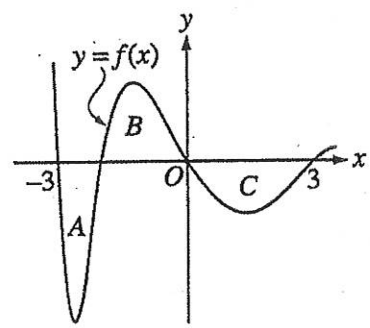
76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

$a(t) = v'(t) = -4.1(0.9)\sin(0.9t)$

$a(4) = -4.1(0.9)\sin(0.9(4)) = 1.6329$

radian mode



77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$?

(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

$\int_{-3}^{3}(f(x)+1)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3}(1)\,dx$

$= [(-2)+(+2)+(-2)] + x\Big]_{-3}^{3} = [-2] + (3-(-3)) = 4$

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ \text{m}^2/\text{sec}$

(B) $0.4\pi\ \text{m}^2/\text{sec}$

$\boxed{\text{(C)}}$ $4\pi\ \text{m}^2/\text{sec}$

(D) $20\pi\ \text{m}^2/\text{sec}$

(E) $100\pi\ \text{m}^2/\text{sec}$

$A = \pi r^2$

implicit WRT time:

$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$

$1\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$C = 2\pi r$

$20\pi = 2\pi r$

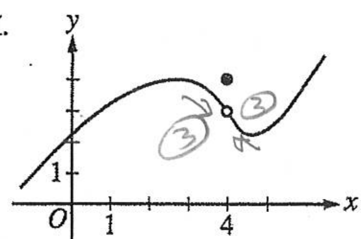
$r = 10\,m$

$\frac{dr}{dt} = 0.2\ \frac{m}{s}$

$\frac{dA}{dt} = 2\pi(10\,m)(0.2\ \frac{m}{s}) = 4\pi\ \frac{m^2}{s}$

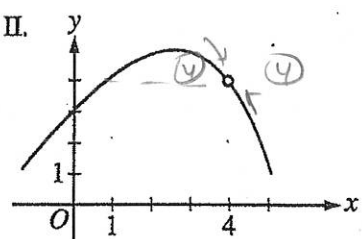
79. For which of the following does $\lim\limits_{x \to 4} f(x)$ exist?

I.



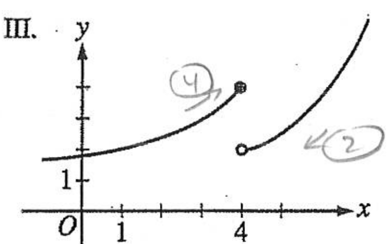
Graph of f

II.



Graph of f

III.



Graph of f

(A) I only

(B) II only

(C) III only

$\boxed{\text{(D)}}$ I and II only

(E) I and III only

$\lim\limits_{x\to 4} f(x) = 3$ exists

$\lim\limits_{x\to 4} f(x) = 4$ exists

LH $\lim\limits_{x\to 4^-} f(x) = 4$ $\neq$ RT $\lim\limits_{x\to 4^+} f(x) = 2$

DNE

GO ON TO THE NEXT PAGE.

80. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$. NO (intermediate value theorem)

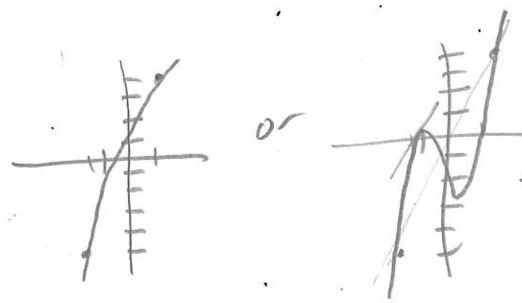
(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$. possibly always increasing ✓

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$. NO (Intermediate Value Theorem)

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$. NO (Mean Value Theorem)

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

NO (maximum must exist somewhere)



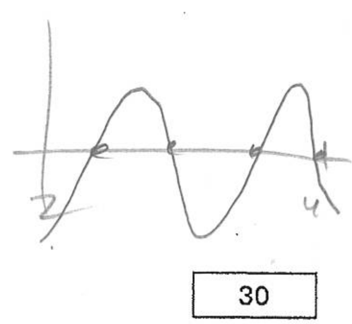
← any number of turns but smooth & continuous

---

81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

Graph this derivative function in calculator and count # times the sign changes:



4 extrema (sign changes)

GO ON TO THE NEXT PAGE.

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

$\boxed{\text{(A)}}$ $\int_{1.572}^{3.514} r(t)\,dt$

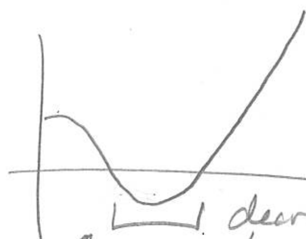
(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

this is the derivative
of "change in altitude"
so "change in altitude"
is integral of this.
decreasing = when $r(t) < 0$
graph $r(t)$



decreasing here
(use calculator 'zero' feature)

$t = 1{,}57199$ $t = 3{,}514137$

So $\int_{1{,}572}^{3.514} r(t)\,dt$

B B B B B B B B B

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

average value of a function $= \frac{1}{b-a}\int_a^b f(x)dx$

So $\frac{1}{3-0}\int_0^3 (e^t + te^t)dt$

MATH9: $\frac{1}{3}(60.25661077) = 20.0855$

---

84. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

$\frac{dT}{dt} = -110e^{-0.4t}$ applies once pizza is in 75°F ambient ~~temp~~.

At $t=0$, pizza starts at 350°F so

$T(5) = T(0) + \int_0^5 (-110e^{-0.4t}dt)$

(MATH9)

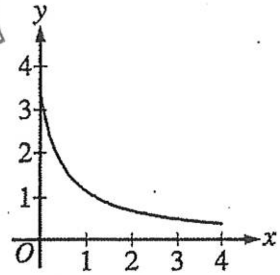
$= 350 + (-237.7827971) = 112.2172$°F

GO ON TO THE NEXT PAGE.

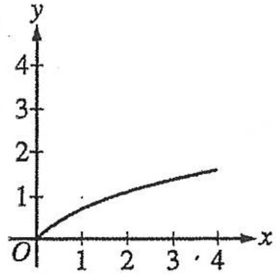
85. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

4 curve shapes..

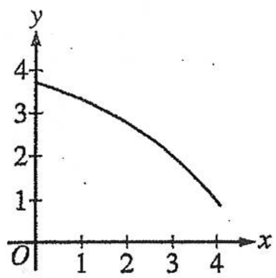
(A)



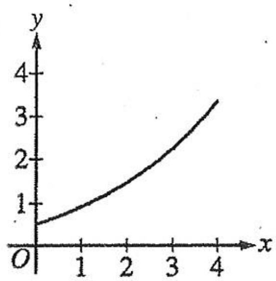
(B)



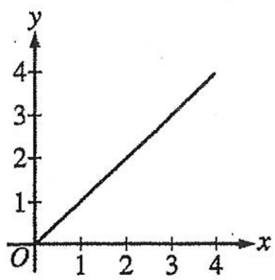
(C)

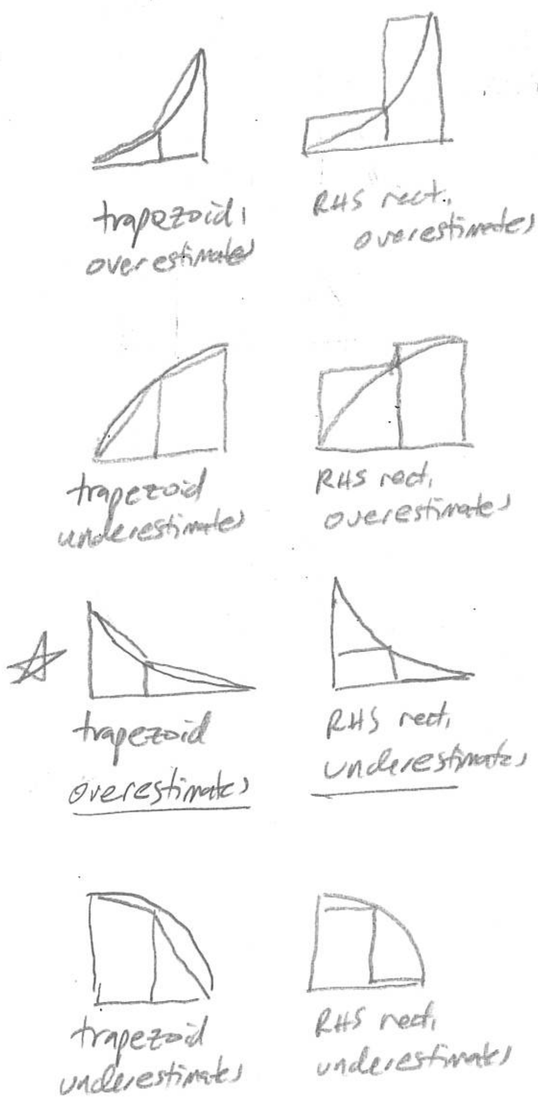


(D)



(E)





GO ON TO THE NEXT PAGE.

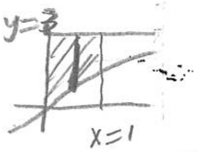
B B B B B B B B B

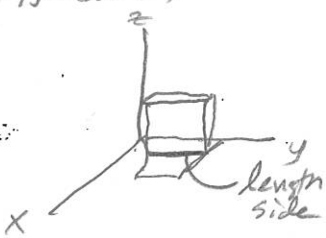
86. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 $\boxed{\text{(B)}}$ 6.612 (C) 8.046 (D) 8.755 (E) 20.773

graph $y=3, y=\tan^{-1}(x)$

base





resulting crossectional area is

$A_{square} = (3-\tan^{-1}(x))^2$

so $V = \int_0^1 (3-\tan^{-1}(x))^2\,dx = 6.6123$ (math 9)

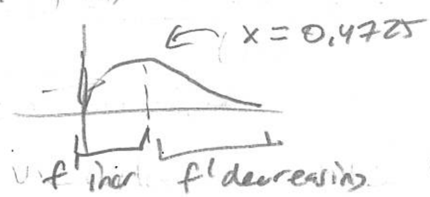
length of cross-section side length is set by $3-\tan^{-1}x$

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 $\boxed{\text{(B)}}$ 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

inflection when $f''(x)=0$ which is when $f'(x)$ changes from increasing to decreasing

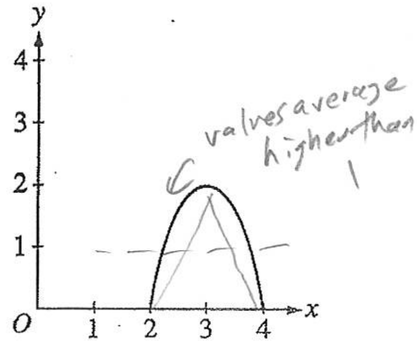
graph $f'(x)$: $f'(x)$



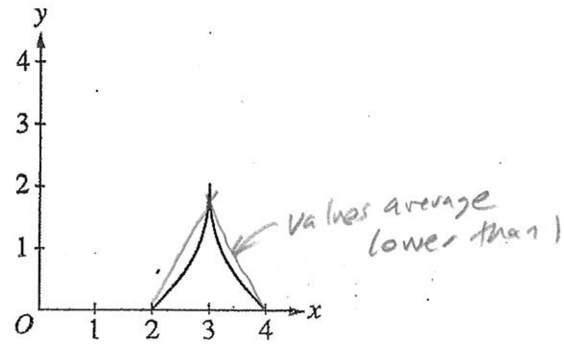
GO ON TO THE NEXT PAGE.

88. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$? ← average value from [2, 4]

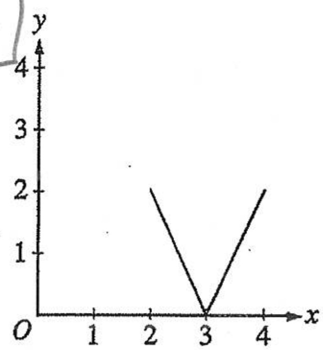
(A)



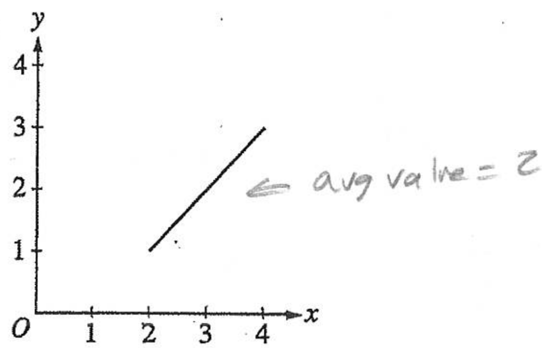
(B)



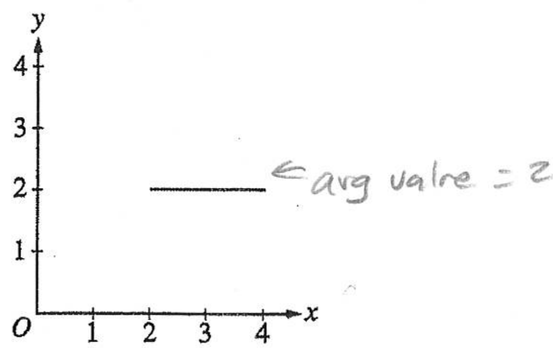
(C)



(D)



(E)



89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

$\boxed{\text{(D)}}$ $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

$g(x) = xf(x)$
$g(2) = 2f(2) = 2(3) = 6$
so $(y-6) = m(x-2)$
↑ set by $g'(2)$

$g(x) = xf(x)$
$g'(x)$ requires product rule
$g'(x) = x\,f'(x) + f(x)(1)$
$g'(2) = 2\,f'(2) + f(2) = 2(-5) + 3 = -7$
so $(y-6) = -7(x-2)$

---

90. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

+2, +3, +4

$\boxed{\text{(B)}}$

x	$f(x)$
2	7
3	11
4	14
5	16

+4, +3, +2

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

−4, −3, −2

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

−2, −3, −4

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

−3, −3, −3

$f'(x)+$ increasing
$f''(x)-$ concave down

GO ON TO THE NEXT PAGE.

B B B B B B B B B

91. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 $\boxed{\text{(E)}}$ 3.346

$v = \int a\,dt = \int \ln(1+2^t)\,dt = v(2) - v(1)$

so $v(2) = v(1) + \int_1^2 \ln(1+2^t)\,dt$
(math 9)

$= 2 + 1.346313534 = 3.3463$

---

92. Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

$\boxed{\text{(D)}}$ $1.772 \le x \le 2.507$

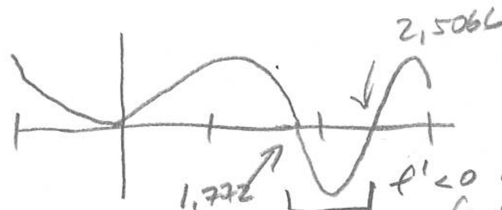
(E) $2.802 \le x \le 3$

by Fund. Thm of Calculus:

$g'(x) = \sin(x^2)$

$g(x)$ decreasing when $g'(x) < 0$

graph $\sin(x^2)$ in calculator:



$f' < 0$ here, using calculator zero features! $(1{,}772,\ 2{,}5066)$

END OF SECTION I

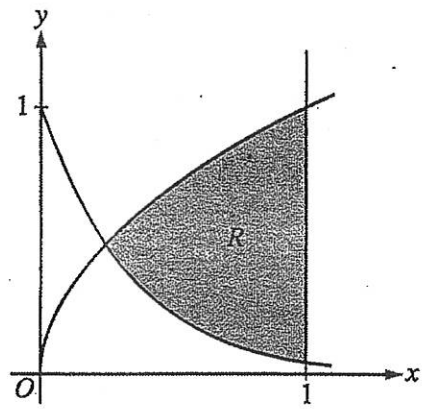
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

GO ON TO THE NEXT PAGE.

2. A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

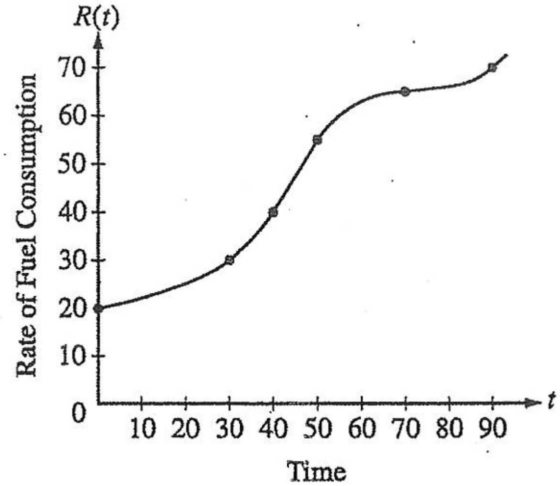
At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

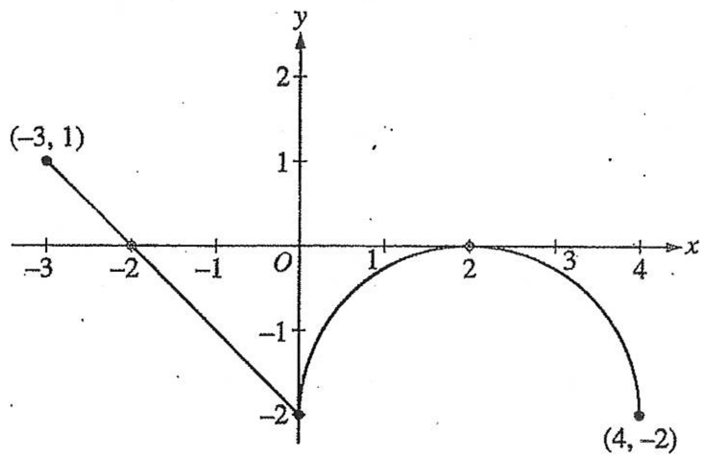
(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

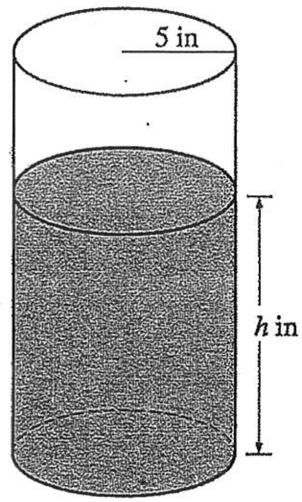
4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

---

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

   (a) Is f continuous at $x = 3$? Explain why or why not.

   (b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

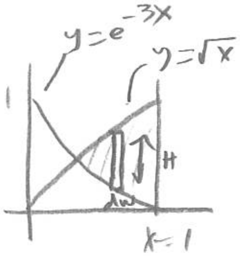
   (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

   where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

END OF EXAM

(1) (a)

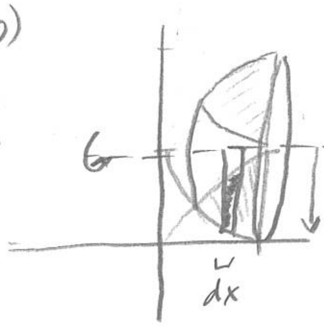


intersection: when $e^{-3x}=\sqrt{x}$ can't solve exactly,
use calculator: at $(0.23873413, 0.48860427)$

$A=\int H \cdot dw \quad H=\sqrt{x}-e^{-3x}$
$dw=dx$

$A=\int_{0.23873413}^{1}(\sqrt{x}-e^{-3x})dx$ (math 9) $=\boxed{0.4426299\ u^2}$

(b)



$r=1-\sqrt{x}$
$R=1-e^{-3x}$

using disc method

$V=\int \pi R^2 dh - \int \pi r^2 dh$

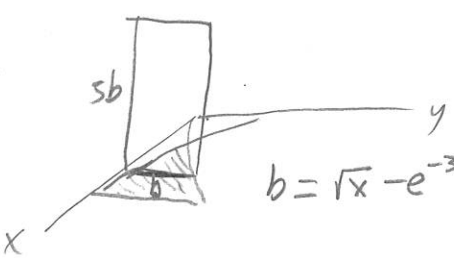
$=\int_{0.23873413}^{1}\pi(1-e^{-3x})^2dx - \int_{0.23873413}^{1}\pi(1-\sqrt{x})^2dx$

$=\pi(.50809696)-\pi(.0549642305)$

$=.4531327\pi$

$=\boxed{1.423558\ u^3}$

(c)



$b=\sqrt{x}-e^{-3x}$

cross-sectional area:

$A_{cross}=b(5b)=5b^2$
$=5(\sqrt{x}-e^{-3x})^2$

So $V=\int A_{cross}\,dx$

$=\int_{0.23873413}^{1}5(\sqrt{x}-e^{-3x})^2dx$

$=\boxed{1.55435\ u^3}$

② (a) $v(t) = -(t+1)\sin\left(\frac{1}{2}t^2\right)$ accel. is deriv of velocity

product rule req'd

$a(t) = -(t+1)\frac{d}{dt}\left[\sin\left(\frac{1}{2}t^2\right)\right] + \sin\left(\frac{1}{2}t^2\right)\frac{d}{dt}\left[-(t+1)\right]$

$= -(t+1)\cos\left(\frac{1}{2}t^2\right)\cdot(t) + \sin\left(\frac{1}{2}t^2\right)(-1)$

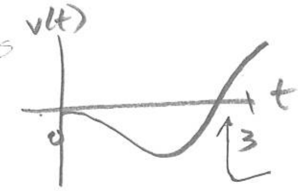
$a(2) = 1.58758359$ (calculator — make sure in radian mode)

$v(2) = -2.72789$

speed is negative at $t=2$ (velocity is negative)
and because acceleration is positive the numerical value of v is getting more positive, so the currently negative is getting closer to zero.
Therefore, the speed of the particle is decreasing, because speed is the magnitude of velocity.

(b) The particle changes direction when the velocity changes sign.

graphing $v(t)$:



using calculator zero feature
zero at $\boxed{t = 2.5066283}$

Could also do this without graphing ... Solve for when $v=0$

$-(t+1)\sin\left(\frac{1}{2}t^2\right) = 0$

$\cancel{t=-1}$ not possible

$\sin\left(\frac{1}{2}t^2\right) = 0 \qquad \theta = \frac{1}{2}t^2$

$\sin(\theta) = 0$ when $\theta = 0$ or $\theta = \pi$

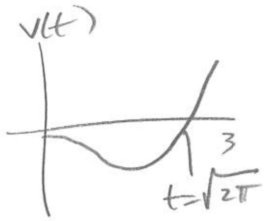
$\frac{1}{2}t^2 = 0 \qquad \frac{1}{2}t^2 = \pi$

$t = 0 \qquad t^2 = 2\pi$

$t = \pm\sqrt{2\pi}$

$\boxed{t = \sqrt{2\pi}}$

(c)



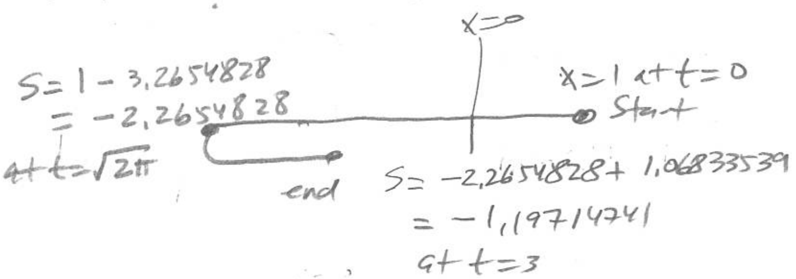
total distance $= \int_a^b |v(t)|\,dt$ we need to negate any regions where velocity is negative

$= -\int_0^{\sqrt{2\pi}} \left(-(t+1)\sin\left(\frac{1}{2}t^2\right)\right)dt + \int_{\sqrt{2\pi}}^{3} \left(-(t+1)\sin\left(\frac{1}{2}t^2\right)\right)dt$

$= -(-3.2654828) + 1.06833539$

$= \boxed{4.33381819}$

(d) The problem states the particle starts at $x=1$ when $t=0$, and in part (c) we showed the particle first moves left 3.2654828 from $t=0$ to $t=\sqrt{2\pi}$, then moves right 1.06833539 from $t=\sqrt{2\pi}$ to $t=3$:



So the greatest distance between the particle and the origin is $\boxed{2.2654828}$ in the negative direction (at $t=\sqrt{2\pi}$).

(3) (a) $R'(45) \approx \frac{R(50)-R(40)}{50-40} = \frac{55-40}{10} = \boxed{1.5 \text{ (gallons/min)/min}}$

(b) $R''(45) = \boxed{0}$ because if $R'(x)$ has a maximum at $x=45$ $R''(x)$ must be zer at $x=45$.

(c)

interval	x	f(x)		width		Area = f(x)·width
[0,30]	0	20	·	30	=	600
[30,40]	30	30	·	10	=	300
[40,50]	40	40	·	10	=	400
[50,70]	50	55	·	20	=	1100
[70,90]	70	65	·	20	=	1300 +

Sum = $\boxed{3700}$

This approximation is $\boxed{\text{less}}$ than the true value of $\int_0^{90} R(t)\,dt$ because the left end values in each interval are the lowest values within each interval.

(d) $\int_0^b R(t)\,dt$ represents to total accumulated amount of fuel consumed from $t=0$ until $t=b$ minutes. Units would be gallons of fuel consumed.

$\frac{1}{b}\int_0^b R(t)\,dt$ represents the average rate of fuel consumption from $t=0$ until $t=b$ minutes. Units would be gallons/min.

(4) (a) f is increasing when f' is positive. On the graph of $f'(x)$ provided this occurs from $\boxed{[-3,-2)}$ or $\boxed{-3 \le x < -2}$.

(b) Inflection points occur when $f'(x)$ changes from decreasing to increasing or increasing to decreasing. On the graph, this occurs at $\boxed{x=0}$ and $\boxed{x=2}$

(c) at $(0,3)$ $x=0$ and $f'(0)=-2$ is the slope
so the tangent line equation is $\boxed{(y-3)=-2(x-0)}$
or
$y-3=-2x$
$\boxed{y=-2x+3}$.

(d) using the given point $(0,3)$

for $f(-3)$: $\int_{-3}^{0} f'(x)dx = f(0)-f(-3)$

so $f(-3)=f(0)-\int_{-3}^{0} f'(x)dx$

in this region, $f'(x)=-x-2$

so $f(-3)=3-\int_{-3}^{0}(-x-2)dx = 3-\left[-\frac{1}{2}x^2-2x\right]_{-3}^{0}$

$=3-\left[(0)-\left(-\frac{1}{2}(-3)^2-2(-3)\right)\right]$

$=3-\left[0-\left(-\frac{9}{2}+6\right)\right]=3+\frac{3}{2}=\boxed{\frac{9}{2}}$

for $f(4)$: $\int_{0}^{4} f'(x)dx = f(4)-f(0)$

so $f(4)=f(0)+\int_{0}^{4} f'(x)dx$

in this region: $\int_{0}^{4} f'(x)dx =$ 

this area (negative)

$=A_{rectangle}-A_{semicircle}$

$=(2)(4)-\frac{1}{2}\pi(2)^2$

$=8-2\pi$ (but negative)

$f(4)=3+(-(8-2\pi))$

$=3-8+2\pi$

$=\boxed{2\pi-5}$

(5)

(a)
$$V = \pi r^2 h$$
$$V = 25\pi h$$
$$\frac{d}{dt}[V] = \frac{d}{dt}[25\pi h]$$
$$1 \cdot \frac{dV}{dt} = 25\pi \frac{dh}{dt}$$
$$(-5\pi\sqrt{h}) = 25\pi \frac{dh}{dt}$$
so $$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = \frac{-\sqrt{h}}{5}$$

(b) $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$ is a separable DE
$$-\frac{1}{\sqrt{h}}\,dh = \frac{1}{5}\,dt$$
$$-\int h^{-1/2}\,dh = \int \frac{1}{5}\,dt$$
$$-\left(\frac{h^{1/2}}{1/2}\right) = \frac{1}{5}t + C$$
$$-2\sqrt{h} = \frac{1}{5}t + C$$
if $h(0) = 17$
$$-2\sqrt{17} = \frac{1}{5}(0) + C, \quad C = -2\sqrt{17}$$
$$-2\sqrt{h} = \frac{1}{5}t - 2\sqrt{17}$$
$$\sqrt{h} = -\frac{1}{10}t + \sqrt{17}$$
$$\boxed{h(t) = \left(-\frac{1}{10}t + \sqrt{17}\right)^2}$$

(c) solve $h(t) = 0$
$$\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$$
when $\frac{1}{10}t = \sqrt{17}$
$$\boxed{t = 10\sqrt{17} \text{ seconds}}$$

(6) (a) 1) $f(3)$ exist? $f(3) = \sqrt{3+1} = 2$ ✓

2) $\lim_{x \to 3} f(x)$ exist?

LH RH

$\lim_{x \to 3^-} \sqrt{x+1}$ $\lim_{x \to 3^+} 5-x$

$\sqrt{3+1}$ $5-3$ so $\lim_{x \to 3} f(x) = 2$ ✓

$2 = 2$

3) $f(3) \stackrel{?}{=} \lim_{x \to 3} f(x)$

$2 = 2$ so $\boxed{\text{yes}}$ $f(x)$ is continuous at $x = 3$.

(b)
$$f_{avg} = \frac{1}{5-0}\int_0^5 f(x)\,dx = \frac{1}{5-0}\left[\int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx\right]$$

$$= \frac{1}{5}\left(\left[\frac{2}{3}(x+1)^{3/2}\right]_0^3 + \left[5x - \frac{1}{2}x^2\right]_3^5\right)$$

$$= \frac{1}{5}\left(\left[\frac{2}{3}(\sqrt{4})^3 - \frac{2}{3}(1)\right] + \left[\left(25 - \frac{25}{2}\right) - \left(15 - \frac{9}{2}\right)\right]\right)$$

$$= \frac{1}{5}\left(\frac{16}{3} - \frac{2}{3} + 10 - 8\right) = \frac{1}{5}\left(\frac{14}{3} + 2\right) = \frac{1}{5}\left(\frac{14}{3} + \frac{6}{3}\right) = \frac{1}{5}\frac{20}{3} = \boxed{\frac{4}{3}}$$

(c) To be differentiable at $x = 3$, $g(x)$ must be continuous at $x = 3$

so $k\sqrt{x+1} = mx + 2$ at $x = 3$

$k\sqrt{3+1} = m(3) + 2$

$2k = 3m + 2$

for the function to not have an abrupt change, the derivatives on each side must also be equal:

LH: $g'(x) = k\,\frac{1}{2}(x+1)^{-1/2}(1) = \frac{k}{2\sqrt{x+1}}$

RH: $g'(x) = m$

at $x=3$: $\frac{k}{4} = m$

so $\begin{cases} 2k = 3m + 2 \\ \frac{k}{4} = m \end{cases}$ substituting

(a system)

$2k = 3\left(\frac{k}{4}\right) + 2$

$8k = 3k + 8$

$5k = 8$ $\boxed{k = \frac{8}{5}}$

and $m = \frac{\left(\frac{8}{5}\right)}{4} = \frac{8}{20} = \boxed{\frac{2}{5} = m}$