Name Solution Period ____

AP Calculus BC

2003 Calculus BC Multiple Choice Exam Part A

1) If $y=\sin(3x)$ then $\frac{dy}{dx}=$ $3\cos(3x)$

(A) $-3\cos(3x)$

(B) $-\cos(3x)$

(C) $-\frac{1}{3}\cos(3x)$

(D) $\cos(3x)$

(E) $3\cos(3x)$

2) $\lim\limits_{x\to 0}\frac{e^x-\cos x-2x}{x^2-2x}$ is

$\frac{e^0-\cos 0-2(0)}{0^2-2(0)}$ $\frac{1-1-0}{0-0}$ $\frac{0}{0}$

by L'Hopital's $=\lim\limits_{x\to 0}\frac{e^x+\sin x-2}{2x-2}$ $\frac{e^0+\sin 0-2}{2(0)-2}=\frac{1+0-2}{0-2}$

$=\frac{-1}{-2}=\frac{1}{2}$

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) *nonexistent*

3) $\int(3x+1)^5\,dx=$

$u=3x+1$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

$\frac{1}{3}\int u^5\,du$

$\frac{1}{3}\frac{u^6}{6}+C$

$\frac{1}{18}(3x+1)^6+C$

(A) $\frac{(3x+1)^6}{18}+C$

(B) $\frac{(3x+1)^6}{6}+C$

(C) $\frac{(3x+1)^6}{2}+C$

(D) $\frac{(\frac{3x^2}{2}+x)^6}{2}+C$

(E) $(\frac{3x^2}{2}+x)^5+C$

4) For $0 \le t \le 13$ an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $-\frac{4\tan 13}{3}$

(D) $\boxed{-\frac{4}{3\tan 13}}$

(E) $-\frac{3}{4\tan 13}$

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-3\sin t}\Bigg|_{t=13} = \frac{4\cos 13}{-3\sin 13} = \frac{4}{-3\tan 13} = -\frac{4}{3\tan 13}$$

5) Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3

(B) 5

(C) $\boxed{6}$

(D) 10

(E) 12

(x, y)	$y_{n+1} = y_n + h(f'(x))$
$(1, 2)$	$y = 2 + 0.5((1)+(2)) = 3.5$
$(1.5, 3.5)$	$y = 3.5 + 0.5((1.5)+(3.5)) = 6$
$(2, 6)$	$f(2) \approx 6$

6) What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}\,dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $\boxed{p > \frac{1}{2}}$

(D) $p > 1$

(E) There are no values of p for which this integral converges.

$$\lim_{b \to \infty} \int_1^b x^{-2p}\,dx$$

$$\lim_{b \to \infty} \left[\frac{x^{-2p+1}}{-2p+1}\right]_1^b$$

$$\lim_{b \to \infty} \left(\frac{1}{-2p+1}\right) b^{-2p+1} - \frac{1}{-2p+1}$$

∞^{-2p+1} diverges unless exponent is negative

$-2p+1 < 0$

$-2p < -1$

$2p > 1$

$p > 1/2$

7) The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

$\frac{dx}{dt}$ & $\frac{dy}{dt} = 0$

$\frac{dx}{dt} = 3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0, \underline{\underline{t = 2}}$

$\frac{dy}{dt} = 6t^2 - 6t - 12 = 0$

$6(t^2 - t - 2) = 0$

$6(t+1)(t-2)$

$t = -1 \quad \underline{\underline{t = 2}}$

(A) -1 *only*

(B) 0 *only*

$\boxed{\text{(C)}}$ 2 *only*

(D) -1 *and* 2 *only*

(E) -1, 0, *and* 2

8) $\int x^2 \cos\left(x^3\right) dx =$

u-sub: $u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$x^2 dx = \frac{1}{3} du$

$\frac{1}{3} \int \cos u \, du$

$\frac{1}{3} \sin u + C$

$\frac{1}{3} \sin(x^3) + C$

(A) $-\frac{1}{3}\sin\left(x^3\right) + C$

$\boxed{\text{(B)}}$ $\frac{1}{3}\sin\left(x^3\right) + C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right) + C$

(D) $\frac{x^3}{3}\sin\left(x^3\right) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

9) If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

$f'(x) = \frac{1}{x + 4 + e^{-3x}} (1 - 3e^{-3x})$

$f'(0) = \frac{1}{0 + 4 + e^0} (1 - 3e^0)$

$= \frac{1}{5}(1 - 3)$

$= \frac{-2}{5}$

$\boxed{\text{(A)}}$ $-\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

(E) *nonexistent*

10) What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$? $= \sum_{n=1}^{\infty} \frac{2 \cdot 2^n}{3^n} = \sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^n$ geometric, converges $\left(\frac{2}{3}<1\right)$

to $\frac{a}{1-r}$

$\frac{2(2/3)}{1-2/3} = \frac{4/3}{1/3} = 4$

(A) 1
(B) 2
(C) 4
(D) 6
(E) The series diverges.

11) The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$? $= x^2 \cdot \frac{1}{1-x^2}$

$\sum_{n=0}^{\infty} (x^2)^n$

$x^2 \cdot \sum_{n=0}^{\infty} x^{2n} = x^2(1+x^2+x^4+\cdots)$

$= x^2+x^4+x^6+\cdots$

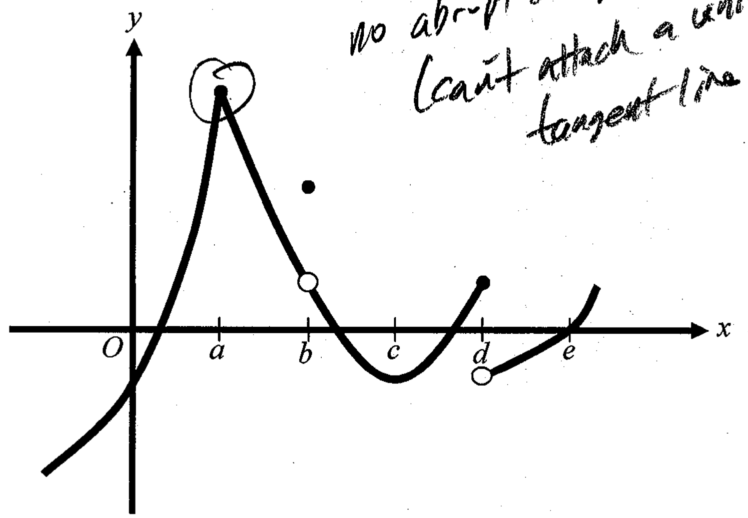
(A) $1+x^2+x^4+x^6+x^8+\cdots$
(B) $x^2+x^3+x^4+x^5+\cdots$
(C) $x^2+2x^3+3x^4+4x^5+\cdots$
(D) $x^2+x^4+x^6+x^8+\cdots$
(E) $x^2-x^4+x^6-x^8+\cdots$

12) The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

$\frac{dV}{dt} = k\sqrt{V}$

(A) $V(t)=k\sqrt{t}$
(B) $V(t)=k\sqrt{V}$
(C) $\frac{dV}{dt}=k\sqrt{t}$
(D) $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$
(E) $\frac{dV}{dt}=k\sqrt{V}$

no abrupt changes
(can't attach a unique
tangent line here)

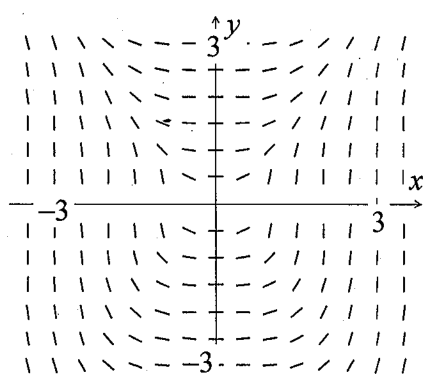


13) The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) **a** (B) **b** (C) **c** (D) **d** (E) **e**

$\frac{dy}{dx}$ −

$\frac{dy}{dx}$ +



$\frac{dy}{dx}$ −

$\frac{dy}{dx} = +$

14) Shown above is a slope field for which of the following differential equations?

	quadrant I	II	III	IV
(A) $\frac{dy}{dx}=\frac{x}{y}$	+	−	+	−
(B) $\frac{dy}{dx}=\frac{x^2}{y^2}$	+	+	+	+
(C) $\frac{dy}{dx}=\frac{x^3}{y}$	+	−	+	−
(D) $\frac{dy}{dx}=\frac{x^2}{y}$	+	+	−	−
(E) $\frac{dy}{dx}=\frac{x^3}{y^2}$	+	−	−	+
graph	+	−	−	+

15) The length of a curve from $x=1$ to $x=4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

$[f'(x)]^2 = 9x^4$, $f'(x) = 3x^2$

(A) $y=3+3x^2$

$\boxed{\text{(B)}}$ $y=5+x^3$

(C) $y=6+x^3$

(D) $y=6-x^3$

(E) $y=\frac{16}{5}+x+\frac{9}{5}x^5$

$f(x) = \int 3x^2 dx = x^3 + c$

$6 = (1)^3 + c$ $c=5$

$f(x) = x^3+5$

16) If the line tangent to the graph of the function f at the point $(1,7)$ passes through the point $(-2,-2)$, then $f'(1)$ is $= \frac{y_2-y_1}{x_2-x_1} = \frac{7-(-2)}{1-(-2)} = \frac{9}{3} = 3$

(A) –5

(B) 1

$\boxed{\text{(C)}}$ 3

(D) 7

(E) undefined

17) A curve C is defined by the parametric equations $x=t^2-4t+1$ and $y=t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3,8)$?

$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t-4}$

at $(-3,8)$

$y=8$

$t^3=8$

$t=2$

$\boxed{\text{(A)}}$ $x=-3$

(B) $x=2$

(C) $y=8$

(D) $y=-\frac{27}{10}(x+3)+8$

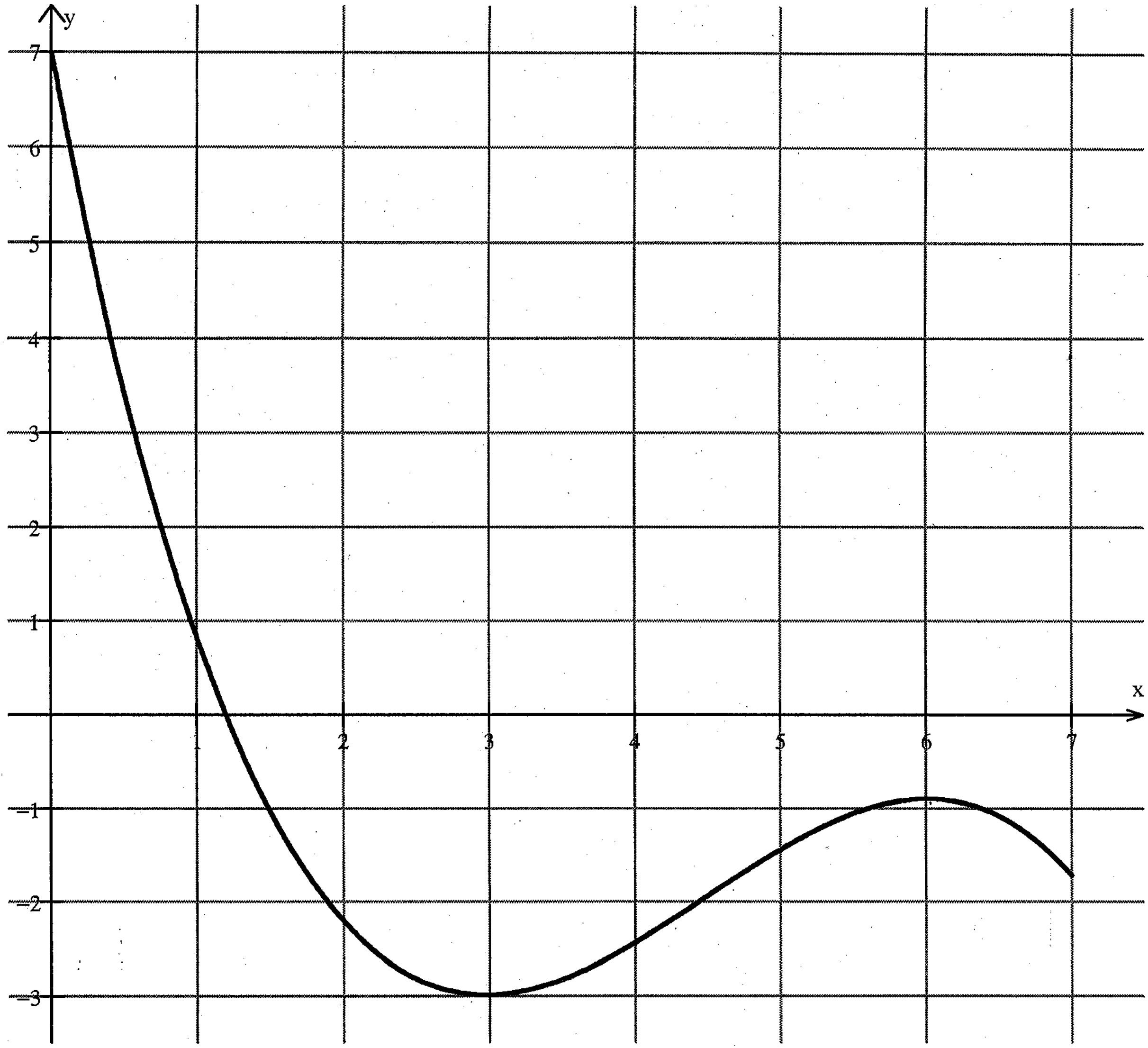
(E) $y=12(x+3)+8$

@ $t=2$

$m = \frac{3(2)^2}{2(2)-4} = \frac{12}{0}$ ∞

(vertical tangent)

$x=-3$



Graph of f

18) The graph of the function f shown in the figure above has horizontal tangents at $x=3$ and $x=6$. If $g(x)=\int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0

(B) −1

(C) −2

(D) −3

(E) −6

$g(x) = \frac{d}{dx}\int_0^{2x} f(t)dt = f(2x)\cdot 2$ ← chain rule

$g'(3) = 2f(2\cdot 3) = 2f(6) = 2(-0.9?) = -1.8$

closest is C

−2

19) A curve has slope $2x+3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

(A) $y=5x-3$
(B) $y=x^2+1$
(C) $y=x^2+3x$
$\boxed{\text{(D)}}$ $y=x^2+3x-2$
(E) $y=2x^2+3x-3$

$f'(x)=2x+3$
$f(x)=\int(2x+3)dx = x^2+3x+c$
$2=(1)^2+3(1)+c$
$2=4+c, \; c=-2$
$f(x)=x^2+3x-2$

20) A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$.

Which of the following is an expression for $f(x)$?

(A) $-3x\sin(x)+3x^2$
(B) $-\cos(x^2)+1$
(C) $-x^2\cos(x)+x^2$
$\boxed{\text{(D)}}$ $x^2e^x-x^3-x^2$
(E) $e^{x^2}-x^2-1$

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots\frac{x^n}{n!}$
Starts here but $\cdot x^2$
$x^2e^x = \underbrace{x^2+x^3}_{\text{remove these}}+\frac{x^4}{2!}+\frac{x^5}{3!}\cdots$
$x^2e^x-x^2-x^3$

21) The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt}=0.6M(1-\frac{M}{200})$, where t is the time in years and $M(0)=50$. What is $\lim_{t\to\infty} M(t)$?

carrying capacity, L

(A) 50
$\boxed{\text{(B)}}$ 200
(C) 500
(D) 1000
(E) 2000

general form:
$\frac{dP}{dt}=kP(1-\frac{P}{L})$
so $L=200$

22) What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

direct comparison w/ $\frac{n}{n^p} = \frac{1}{n^{p-1}}$

p-series, converges if $p-1>1$

$p>2$

(A) $p>0$
(B) $p \geq 1$
(C) $p>1$
(D) $p \geq 2$
(E) $p>2$

23) $\int x\sin(6x)\,dx =$

by parts: $u=x$ $\quad dv=\sin(6x)dx$

$\frac{du}{dx}=1$ $\quad \int dv = \int \sin(6x)dx$

$du=dx$ $\quad v=\frac{1}{6}(-\cos(6x))$

$uv-\int v\,du$

$-\frac{1}{6}x\cos(6x)+\frac{1}{6}\int \cos(6x)dx$

$-\frac{1}{6}x\cos(6x)+\frac{1}{36}\sin(6x)+c$

(A) $-x\cos(6x)+\sin(6x)+C$
(B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$
(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(E) $6x\cos(6x)-\sin(6x)+C$

24) Which of the following series diverge?

$$I.\ \sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$$

$$II.\ \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$$

$$III.\ \sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

24) I) $\sum_{n=0}^{\infty} \left(\frac{\sin 2}{\pi}\right)^n$ geometric $\frac{-1 \le \sin 2 \le 1}{\pi} < 1$ so converges

II) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ p-series w/ $p = 1/3$ diverges

III) $\sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$ n^{th} term test $\lim_{n\to\infty} \frac{e^n}{e^n+1} = 1 \neq 0$ diverges

x	2	5	10	14
$f(x)$	12	28	34	30

25) The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

(A) 296
(B) 312
(C) 343
$\boxed{\text{(D)}}$ 374
(E) 390

interval	x_i	$f(x_i) \cdot \Delta x$	$= area$
[2,5]	5	$28 \cdot 3$	= 84
[5,10]	10	$34 \cdot 5$	= 170
[10,14]	14	$30 \cdot 4$	= 120
			374

$28 \times 3 = 84$

$34 \cdot 10 = 340/2$

170

26) $\int \frac{2x}{(x+2)(x+1)} dx =$

(A) $\ln|x+2| + \ln|x+1| + C$
(B) $\ln|x+2| + \ln|x+1| - 3x + C$
(C) $-4\ln|x+2| + 2\ln|x+1| + C$
$\boxed{\text{(D)}}$ $4\ln|x+2| - 2\ln|x+1| + C$
(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

partial fraction expansion:

$\frac{2x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$A(x+1) + B(x+2) = 2x$

$Ax + A + Bx + 2B = 2x$

$(A+B)x + (A+2B) = (2)x + (0)$

$\begin{cases} A+B=2 \\ A+2B=0 \end{cases}$

$\cancel{A}+B=2$
$-\cancel{A}-2B=0$
$-B=2$

$B=-2$, $A-2=2$
$A=4$

$4\int \frac{1}{x+2}dx - 2\int \frac{1}{x+1}dx$

$4\ln|x+2| - 2\ln|x+1| + C$

chain rule req'd

27) $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)dt\right) =$ $\ln((x^3)^2+1)\cdot 3x^2$

$3x^2 \ln(x^6+1)$

(A) $\frac{2x^3}{x^6+1}$

(B) $\frac{3x^2}{x^6+1}$

(C) $\ln(x^6+1)$

(D) $2x^3\ln(x^6+1)$

$\boxed{\text{(E)}}$ $3x^2\ln(x^6+1)$

28) What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x=0$? (Maclaurin)

$f(0) + f'(0)x + \underline{\frac{f''(0)}{2!}}x^2$

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) 1

$\boxed{\text{(D)}}$ 3

(E) 6

$f(x) = (1+x)^{-2}$

$f'(x) = -2(1+x)^{-3}$

$f''(x) = 6(1+x)^{-4} = \frac{6}{(1+x)^4}$

$f''(0) = \frac{6}{(1+0)^4} = 6$

$\frac{6}{2!} = \frac{6}{2} = 3$

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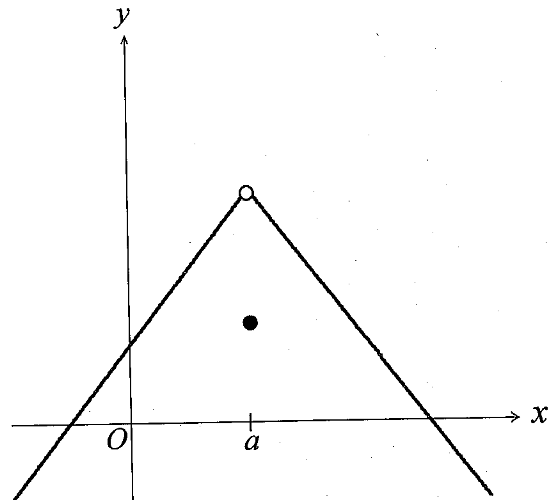
2003 Calculus BC Multiple Choice Exam Part B



Graph of f

76) The graph of the function f is shown above. Which of the following statements must be false?

(A) $f(a)$ exists true
(B) $f(x)$ is defined for $0<x<a$ true
(C) f is not continuous at $x=a$ true
(D) $\lim_{x\to a} f(x)$ exists true
$\boxed{\text{(E)}}$ $\lim_{x\to a} f'(x)$ exists false (no derivative where there is discontinuity)

77) Let $P(x)=3x^2-5x^3+7x^4+3x^5$ be the fifth-degree Taylor polynomial for the function f about $x=0$. What is the value of $f'''(0)$?

$P_5(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \ldots$

$\boxed{\text{(A)}}$ -30
(B) -15
(C) -5
(D) $-\frac{5}{6}$
(E) $-\frac{1}{6}$

$\frac{f'''(0)}{3!} = -5$

$f'''(0) = (-5)(3 \cdot 2) = -30$

78) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

$A = \pi r^2$

$\left(C = 2\pi r = 20\pi \quad r = \frac{20\pi}{2\pi} = 10 \right) \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(10)(0.2) = 4\pi$

(A) $0.04\pi\ m^2/\text{sec}$
(B) $0.4\pi\ m^2/\text{sec}$
$\boxed{\text{(C)}}$ $4\pi\ m^2/\text{sec}$
(D) $20\pi\ m^2/\text{sec}$
(E) $100\pi\ m^2/\text{sec}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–1	6	5	3	–2
1	3	–3	–1	2
3	1	–2	2	3

79) The table above gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

$h'(x) = f'(g(x)) \cdot g'(x)$ Chain Rule

$h'(1) = f'(g(1)) \cdot g'(1)$
$= f'(-1) \cdot g'(1)$
$= (5) \cdot (2)$
$= 10$

(A) 5
(B) 6
(C) 9
$\boxed{\text{(D)}}$ 10
(E) 12

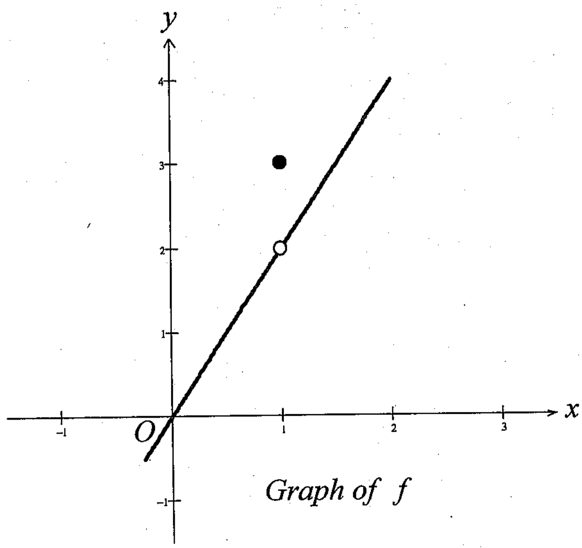
80) Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

net change in $f = \int_a^b f'(x)dx$

$= \int_7^{14} \frac{100e^{-0.1t}}{2-e^{-3t}} dt$ math 9

$= 124.994$

$\boxed{\text{(A)}}$ 125
(B) 100
(C) 88
(D) 50
(E) 12



Graph of f

81) The graph of the function f is shown in the figure above. The value of $\lim_{x\to 1} \sin(f(x))$ is $= \sin(2) = 0.909297$

$\boxed{\text{(A)}}$ 0.909
(B) 0.841
(C) 0.141
(D) −0.416
(E) nonexistent

82) The rate of change (derivative of altitude) of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing? when derivative < 0 graph $r(t) = a'(t) = t^3 - 4t^2 + 6$

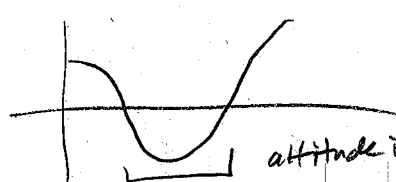
$\boxed{\text{(A)}}$ $\int_{1.572}^{3.514} r(t)\,dt$ ← it is already a derivative

(B) $\int_0^8 r(t)\,dt$

(C) $\int_0^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_0^{2.667} r'(t)\,dt$



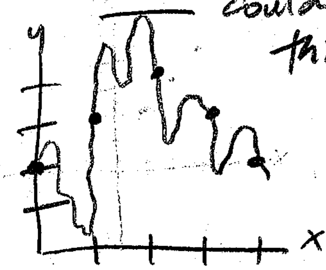
altitude is decreasing $1.57199 < t < 3.51414$

(net) change in altitude $= \int_a^b a'(t)\,dt = \int_{1.572}^{3.514} r(t)\,dt$

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

83) The function f is continuous and differentiable on the closed interval $[0,4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

Could be this...



(A) The minimum value of f on $[0,4]$ is 2.
(B) The maximum value of f on $[0,4]$ is 4.
(C) $f(x) > 0$ for $0 < x < 4$
(D) $f'(x) < 0$ for $2 < x < 4$
$\boxed{\text{(E)}}$ There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

Mean Value (Rolle's) Theorem

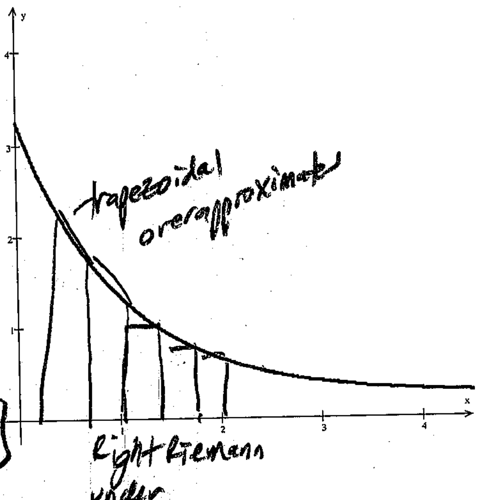
84) A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

$\vec{r}(t) = \langle t^2, \sin(4t) \rangle$
$\vec{v}(t) = \langle 2t, 4\cos(4t) \rangle$
$\vec{v}(3) = \langle 6, 4\cos 12 \rangle$
Speed $= |\vec{v}(t)| = \sqrt{6^2 + (4\cos(12))^2} = 6.884$

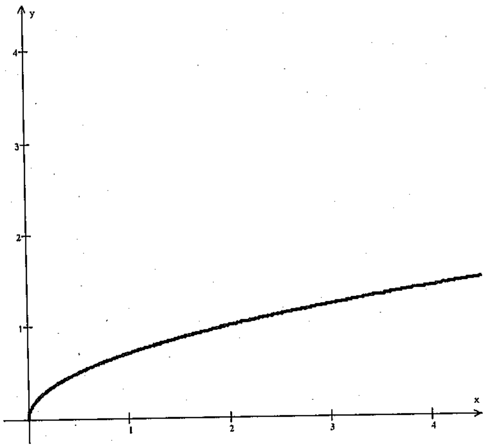
(A) 2.909
(B) 3.062
$\boxed{\text{(C)}}$ 6.884
(D) 9.016
(E) 47.393

85) If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

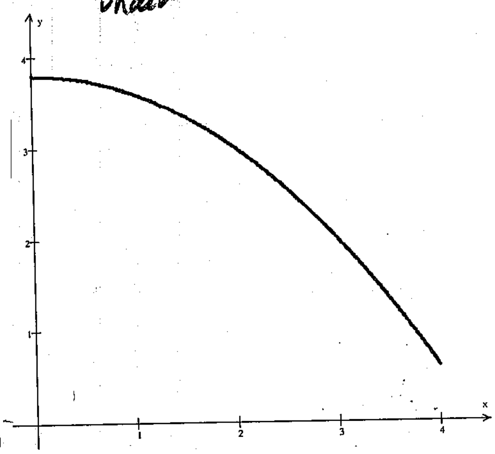
(A)



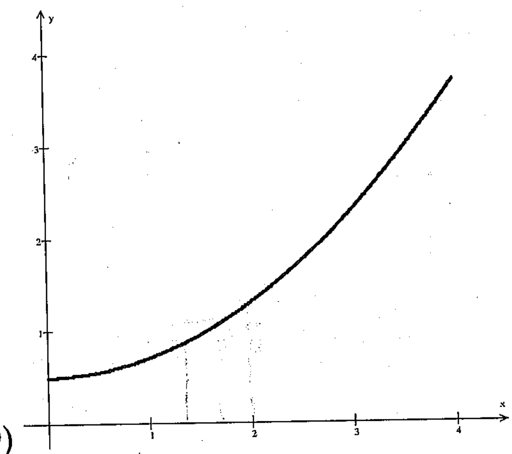
(B)



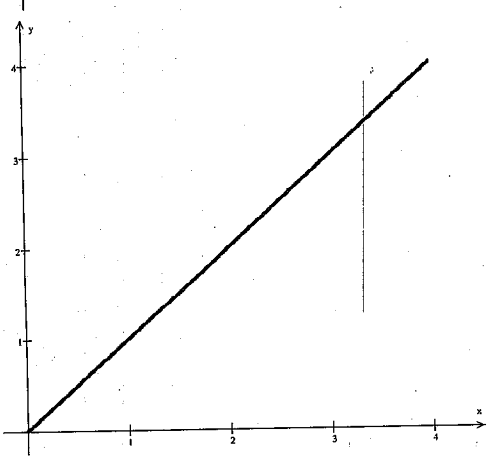
(C)



(D)



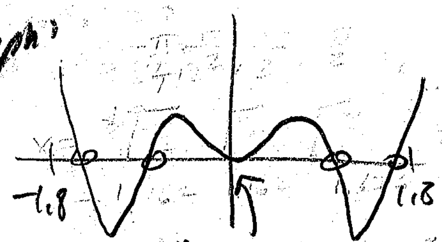
(E)



86) Let f be the function with derivative defined by $f'(x)=\sin\left(x^3\right)$ on the interval $-1.8<x<1.8$. How many points of inflection does the graph of f have on this interval?

$f''(x) = \cos(x^3)\cdot 3x^2$

graph:



f'' doesn't change sign

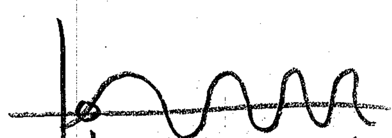
4

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

87) A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t)=\cos\left(2-t^2\right)$. The position of the particle is 3 at time $t=0$. What is the position of the particle when its velocity is first equal to 0?

$\cos(2-t^2)=0$

graph



$t = 0.65513638$

$x(t) = \int \cos(2-t^2)\,dt$

$\int_0^{0.655137} \cos(2-t^2)\,dt = x(0.655137) - x(0)$

math 9

$-.1835406 = x(0.655137) - 3$

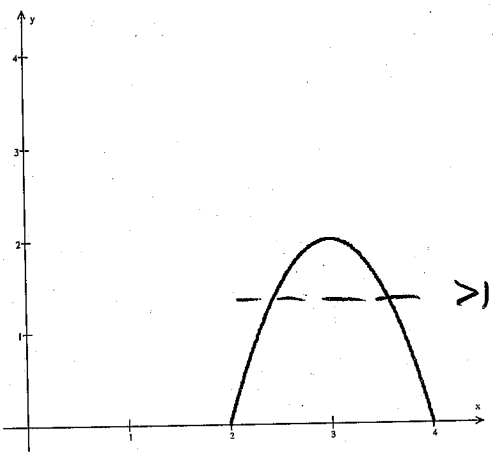
$x(0.655137) = -.1835406 + 3$

$= 2.81645$

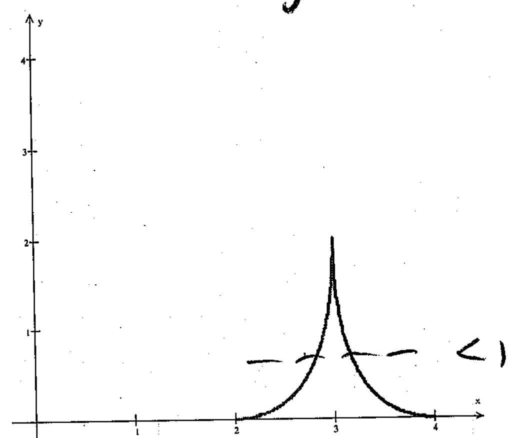
(A) 0.411
(B) 1.310
(C) 2.816
(D) 3.091
(E) 3.411

88) On the closed interval $[2,4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

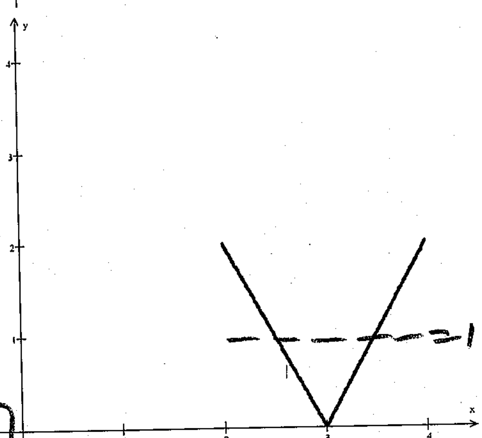
avg value



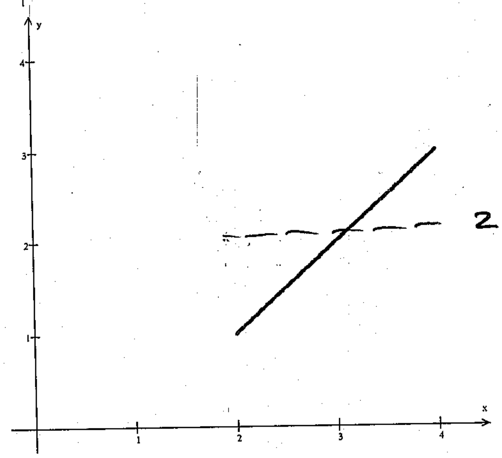
(A)



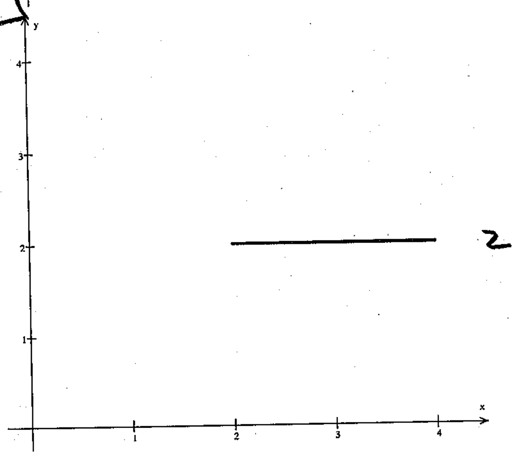
(B)



(C)

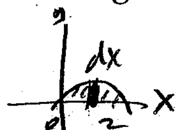
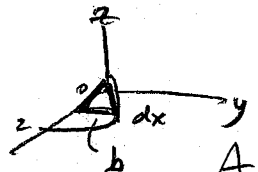
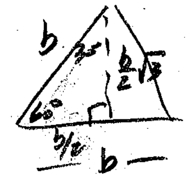


(D)



(E)

89) The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is an equilateral triangle. What is the volume of this solid?

(A) 1.333
(B) 1.067
(C) 0.577
$\boxed{\text{(D)}}$ 0.462
(E) 0.267

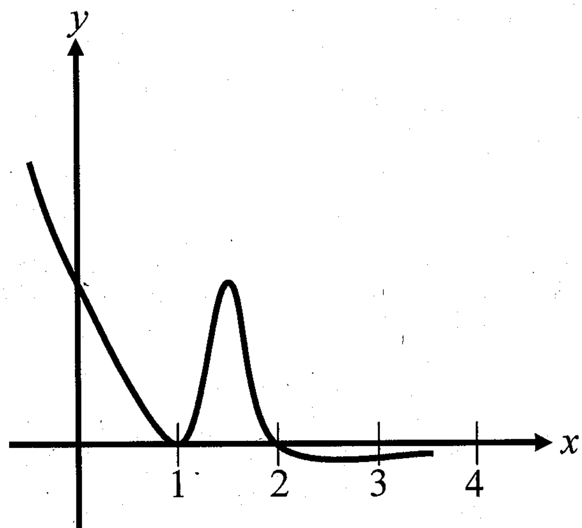
$2x - x^2 = 0$
$x(2-x) = 0$
$x = 0 \quad x = 2$

$A_{cross} =$

$A_{cross} = \frac{1}{2} b\left(\frac{b}{2}\sqrt{3}\right)$
$= \frac{\sqrt{3}}{4} b^2$
$b = (2x - x^2)$

$\text{volume} = \int_a^b A_{cross}\, dx = \int_0^2 \frac{\sqrt{3}}{4}(2x - x^2)^2\, dx$

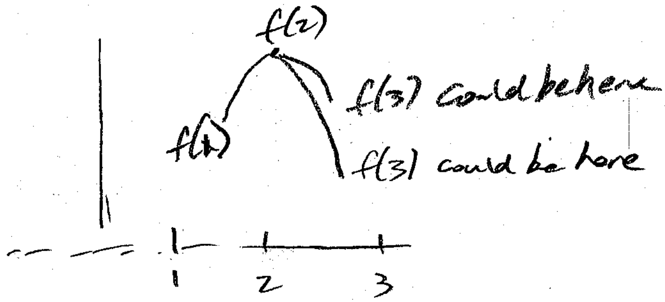
math 9
0.46188



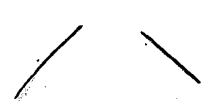
90) The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$ $(0,1)$ $f'(x) > 0$ increasing, so FALSE
II. $f(2) > f(1)$ $(1,2)$ $f'(x) > 0$ increasing, so TRUE
III. $f(1) > f(3)$ $(1,3)$ $f'(x)$ changes sign: can't say must be TRUE, so FALSE

(A) I only
$\boxed{\text{(B) II}}$ only
(C) III only
(D) I and II only
(E) II and III only



$f'(x)$



91) The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters
$\boxed{\text{(B)}}$ 10.263 meters
(C) 34.125 meters
(D) 54.889 meters
(E) 89.005 meters

$v'(t) = 24 + 36t^{1/2} - 32t$

max v' when $a = 0$

$a'(t) = 18t^{-1/2} - 32 = 0$

$\frac{18}{\sqrt{t}} = 32$

$\sqrt{t} = \frac{18}{32}$

$t = \left(\frac{18}{32}\right)^2 = .316406$

$h(.316406) = 10.263$

92) Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4] ?

(A) 0.456
(B) 1.244
$\boxed{\text{(C)}}$ 2.164
(D) 2.342
(E) 2.452

avg of $f'(x)$ $\quad$ $\text{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$f'(x) = 1 + \frac{1}{x}$ instantaneous rate of change $\quad$ avg rate of change of f $= \frac{1}{4-1}\int_1^4 \left(1+\frac{1}{x}\right)dx$

$1 + \frac{1}{x} = \frac{1}{3}\int_1^4 \left(1+\frac{1}{x}\right)dx$

math 9

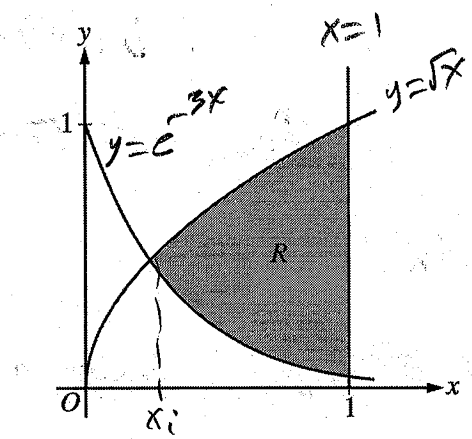
$1 + \frac{1}{x} = 1.46209812$

$\frac{1}{x} = .46209812$

$x = 2.164$

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



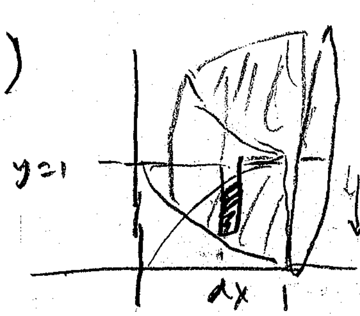
1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

---

a) intersection: $e^{-3x} = \sqrt{x}$ (in calc) at $x_i = .23873413$

$\int_{.23873413}^{1} (\sqrt{x} - e^{-3x})\,dx \overset{\text{math 9}}{=} \boxed{0.443}$

b)



$r = 1 - \sqrt{x}$
$R = 1 - e^{-3x}$

disc method: $V = \int \pi R^2 dx - \int \pi r^2 dx$

$= \pi \int (R^2 - r^2)\,dx$

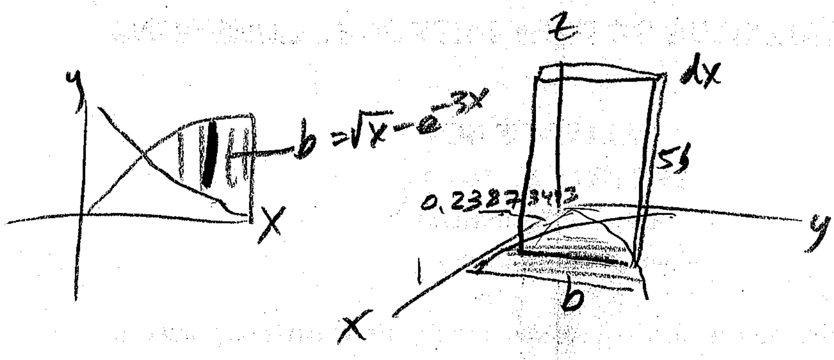
$= \pi \int_{.23873413}^{1} \left[(1 - e^{-3x})^2 - (1 - \sqrt{x})^2\right] dx$

math 9 $= \boxed{1.424}$

(c) $\longrightarrow$

GO ON TO THE NEXT PAGE.

(c)



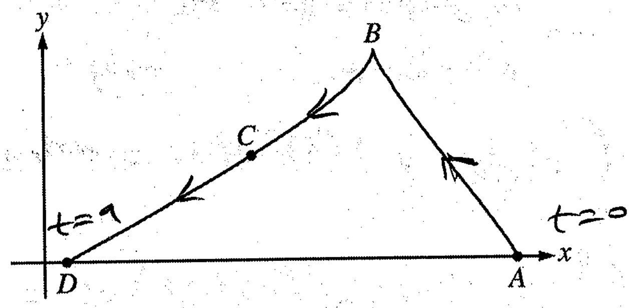
$A_{cross} = b(5b) = 5b^2$

$b = \sqrt{x} - e^{-3x}$

$A_{cross} = 5(\sqrt{x} - e^{-3x})^2$

$V = \int A_{cross}\,dx$

$= \int_{.23873413}^{1} 5(\sqrt{x} - e^{-3x})^2\,dx$ (math 9) $= \boxed{1.554}$



2. A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t, where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

(c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

(a) At point C, $\frac{dy}{dt}$ is not positive because y(t) is decreasing along path at C. $\frac{dx}{dt}$ is not positive because x(t) is decresing along the path at C.

(b) $\frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\pi\frac{\sqrt{t+1}}{2}\right) = 0$ or undefined.

either $\cos\left(\frac{\pi t}{6}\right) = 0$, $\frac{\pi t}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$, $t = 3, 9$

or $\sin\left(\pi\frac{\sqrt{t+1}}{2}\right) = 0$, $\pi\frac{\sqrt{t+1}}{2} = 0, \pi$, $t = -1$, $\sqrt{t+1} = 2$, $t+1 = 4$, $t = 3$

$\boxed{t=3}$ is $0 \le t \le 9$

(c) $y = \frac{5}{9}x - 2$

$\frac{dy}{dx} = \frac{5}{9} = \frac{dy/dt}{dx/dt}$

at $t=8$: $\frac{dx}{dt} = -9\cos\left(\frac{\pi 8}{6}\right)\sin\left(\frac{\pi\sqrt{8+1}}{2}\right)$

$\frac{dy}{dt} = -9\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{3\pi}{2}\right) = -9\left(-\frac{1}{2}\right)(-1) = -\frac{9}{2}$

so $\frac{dy/dt}{-9/2} = \frac{5}{9}$, $\frac{dy}{dt} = \frac{5}{9}\left(-\frac{9}{2}\right) = -\frac{5}{2}$ $\therefore \boxed{\vec{v}(t) = \langle -\frac{9}{2}, -\frac{5}{2}\rangle}$

speed $= |\vec{v}(t)| = \sqrt{\left(-\frac{9}{2}\right)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{\frac{81}{4} + \frac{25}{4}} = \boxed{\sqrt{\frac{106}{4}}}$

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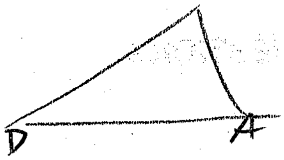
(d) → (on back)

d)

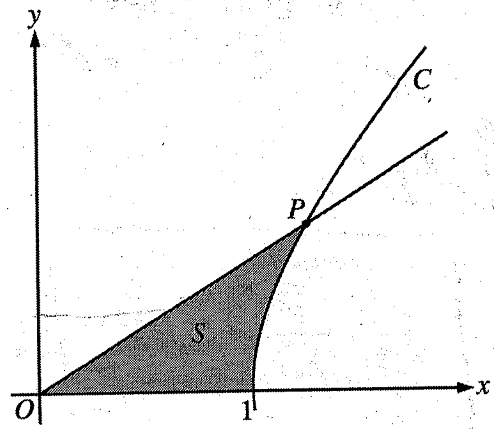


these points have same y, so distance is determined by the net accumulation of change in x only!

$$\int_0^9 x'(t)\,dx = x(9)-x(0) = \text{distance apart}$$

$$\int_0^9 \left(-9\cos\left(\frac{\pi t}{6}\right)\sin\left(\pi\sqrt{\frac{t+1}{2}}\right)\right)dt = \text{distance apart}$$

(math 9) $= \boxed{-39,255}$



3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

(a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

(b) Set up and evaluate an integral expression with respect to y that gives the area of S.

(c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

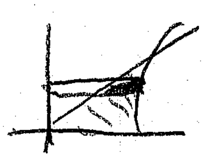
(d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.

(a) intersection: $\sqrt{1+y^2} = \frac{5}{3}y$ $\quad x = \frac{5}{3}\left(\frac{3}{4}\right) = \frac{5}{4}$ $\quad \boxed{P\left(\frac{5}{4}, \frac{3}{4}\right)}$ $\quad \frac{dx}{dy} = \frac{1}{2}(1+y^2)^{-1/2}(2y)$

$1+y^2 = \frac{25}{9}y^2$ $\quad = \frac{y}{\sqrt{1+y^2}} = \frac{3/4}{\sqrt{1+(3/4)^2}}$

$\frac{16}{9}y^2 = 1$, $y^2 = \frac{9}{16}$, $y = \pm\frac{3}{4} = \frac{3}{4}$, $\quad = .6 = \boxed{\frac{3}{5}}$

END OF PART A OF SECTION II

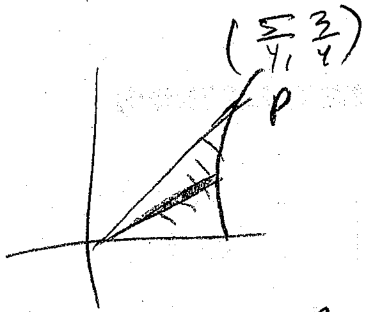
(b)  area $= \boxed{\int_0^{3/4} \left(\sqrt{1+y^2} - \frac{5}{3}y\right) dy} = \boxed{0.347}$ math 9

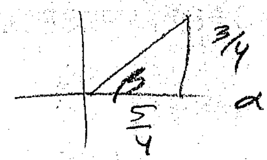
(c) $x^2 - y^2 = 1 \Rightarrow (r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$ $\quad \boxed{r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}}$

(d) $\rightarrow$ (on back)

(d)





$\tan\beta = \frac{3/4}{5/4} = \frac{3}{5}$

$\alpha = 0$

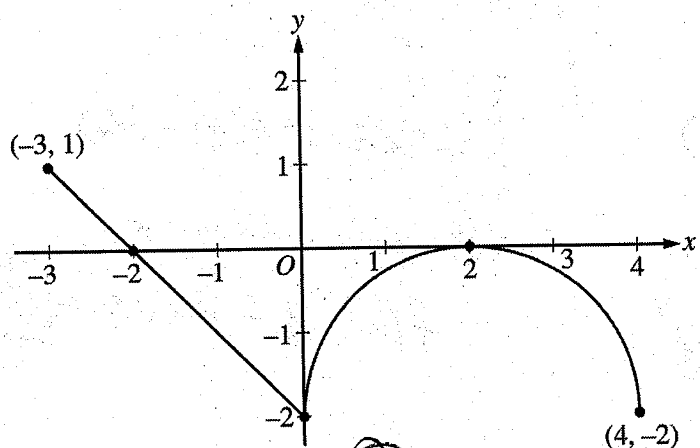
$\beta = \tan^{-1}\left(\frac{3}{5}\right)$

$$\text{area} = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

$$= \boxed{\frac{1}{2}\int_{0}^{\tan^{-1}(3/5)} \frac{1}{\cos^2\theta - \sin^2\theta} \, d\theta}$$

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

(a) f is increasing where $f'(x) > 0$, which is true for $\boxed{-3 \le x < -2,}$

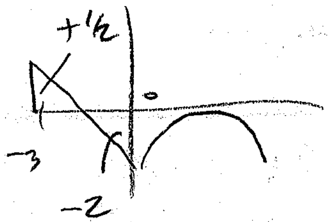
(b) f has an inflection point where $f''(x)$ changes sign, which means where $f'(x)$ is changing from increasing to decreasing (or vice versa). This occurs at $\boxed{x = 0}$ and $\boxed{x = 2}$.

(c) $f'(0) = -2$ is the slope of the tangent line, so $\boxed{(y-3) = -2(x-0)}$

(d) → (on back)

(d) $\int_a^b f'(x)dx = f(b)-f(a)$, we have point $(0,3)$ on f
$f(0)$

so for $f(-3)$: $\int_{-3}^{0} f'(x)dx = f(0) - f(-3)$



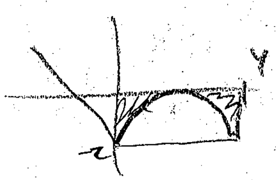
$\frac{1}{2} - 2 = 3 - f(-3)$

$f(-3) = 3 + 2 - \frac{1}{2} = 4.5 = \boxed{\frac{9}{2}}$

for $f(4)$: $\int_0^4 f'(x)dx = f(4) - f(0)$

$-(8-2\pi) = f(4) - 3$

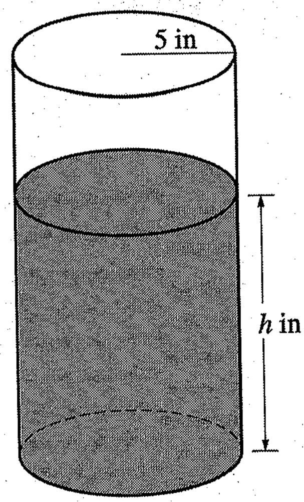
$f(4) = 3 - 8 + 2\pi = \boxed{-5+2\pi}$



$A = 8 - \frac{1}{2}\pi(2)^2$

$8 - \frac{1}{2}\pi 4$

$-(8-2\pi)$



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

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6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

   for all real numbers x.

   (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

   (b) Show that $1 - \dfrac{1}{3!}$ approximates $f(1)$ with error less than $\dfrac{1}{100}$.

   (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

END OF EXAMINATION

5 a) related rates: $V = \pi r^2 h$

so since r is fixed at $r=5$

$$V = 25\pi h$$

then $\frac{d}{dt}[V] = \frac{d}{dt}[25\pi h]$

$1 \cdot \frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt}$ if $\frac{dV}{dt} = -5\pi\sqrt{h}$

then $-5\pi\sqrt{h} = 25\pi \frac{dh}{dt}$

so $\boxed{\frac{dh}{dt} = -\frac{\sqrt{h}}{5}}$

b) $h=17$ at $t=0$ solve DE:

$\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

Separable: $\frac{1}{\sqrt{h}}dh = -\frac{1}{5}dt$

$\int h^{-1/2}dh = -\int \frac{1}{5}dt$

$2h^{1/2} = -\frac{1}{5}t + C$

$2\sqrt{h} = -\frac{1}{5}t + C$ so $h=17$, $t=0$

$2\sqrt{17} = -\frac{1}{5}(0) + C$, $C = 2\sqrt{17}$

$2\sqrt{h} = -\frac{1}{5}t + 2\sqrt{17}$

$\sqrt{h} = -\frac{1}{10}t + \sqrt{17}$

$\boxed{h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2}$

c) empty when $h=0$:

$\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$

$-\frac{1}{10}t + \sqrt{17} = 0$

$\frac{1}{10}t = \sqrt{17}$

$\boxed{t = 10\sqrt{17} \text{ seconds}}$

6a) $f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots \frac{(-1)^n x^{2n}}{(2n+1)!}$

$f'(x) = -\frac{2}{3!}x + \frac{4}{5!}x^3 - \frac{6}{7!}x^5 + \frac{(-1)^n (2n) x^{2n-1}}{(2n+1)!}$

$\boxed{f'(0) = 0}$

$f''(x) = -\frac{2}{3!} + \frac{4 \cdot 3}{5!}x^2 - \frac{6 \cdot 5}{7!}x^4 + \frac{(-1)^n (2n)(2n-1) x^{2n-2}}{(2n+1)!}$

$f''(0) = -\frac{2}{3!} = -\frac{2}{3 \cdot 2} = \boxed{-\frac{1}{3}}$ at $x=0$ $f'(x)=0$, $f''(x)<0$ concave down

f has a local $\boxed{\text{maximum}}$ at $x=0$
because $f'(0)=0$ and $f''(0)<0$, so the curve is concave down.

b) This series is an alternating series: $1 - \frac{1}{3!}x^2 + \frac{1}{5!}x^4 \ldots$
and the remainder for alternating series is $\le$ the first omitted term: $|\text{error}| \le |\frac{1}{5!}x^4|$
for $f(1)$, $x=1$ so $|\text{error}| \le |\frac{1}{5!}(1)^4| = .0083$
therefore $1 - \frac{1}{3!}$ approximates $f(1)$ with error $< \frac{1}{100}$ (.01)

c) $y = f(x)$ solution to $xy' + y = \cos x$?

$xy' + y \stackrel{?}{=} \cos x$

$x\left(-\frac{2}{3!}x + \frac{4}{5!}x^3 - \frac{6}{7!}x^5 + \ldots\right) + \left(1 - \frac{1}{3!}x^2 + \frac{1}{5!}x^4 - \frac{1}{7!}x^6 + \ldots\right)$

$-\frac{2}{3!}x^2 + \frac{4}{5!}x^4 - \frac{6}{7!}x^6 + \ldots + \left(1 - \frac{1}{3!}x^2 + \frac{1}{5!}x^4 - \frac{1}{7!}x^6 + \ldots\right)$

$1 - \left(\frac{2}{3!} + \frac{1}{3!}\right)x^2 + \left(\frac{4}{5!} + \frac{1}{5!}\right)x^4 - \left(\frac{6}{7!} + \frac{1}{7!}\right)x^6 + \ldots$

$1 - \frac{3}{3!}x^2 + \frac{5}{5!}x^4 - \frac{7}{7!}x^6 + \ldots$

$1 - \frac{3}{3 \cdot 2!}x^2 + \frac{5}{5 \cdot 4!}x^4 - \frac{7}{7 \cdot 6!}x^6 + \ldots$

$1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$

memorized form B.

$\cos x$ ✓