1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

---

2. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

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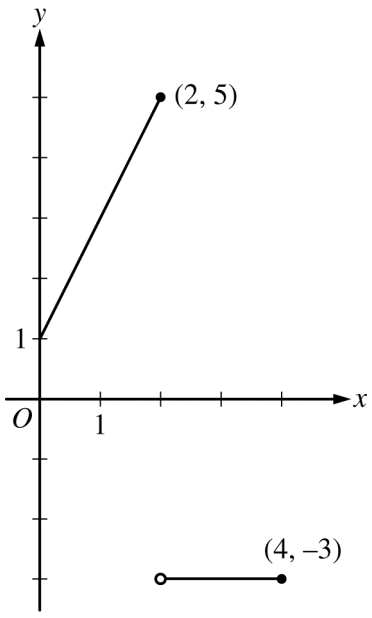

Graph of f

3. The graph of f is shown above for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

GO ON TO THE NEXT PAGE.

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right) dx$

---

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

GO ON TO THE NEXT PAGE.

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2 - 3\right)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$

(B) $\int_{-2}^{13} u^5 \, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

---

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1 - x^2}}$

(B) $\frac{xy}{\sqrt{1 - x^2}}$

(C) $\frac{y}{1 + x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1 + x^2}$

GO ON TO THE NEXT PAGE.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

---

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

GO ON TO THE NEXT PAGE.

10. $\int_1^4 t^{-3/2}\,dt =$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

---

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

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12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

(C) $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

(E) $(-1, -1)$ is a point of inflection of the graph of f.

---

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

14. Let k be a positive constant. Which of the following is a logistic differential equation?

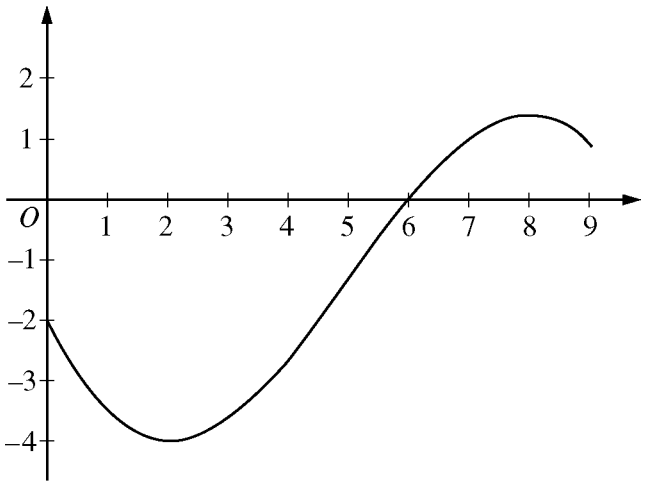
(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1 - t)$

(D) $\frac{dy}{dt} = ky(1 - t)$

(E) $\frac{dy}{dt} = ky(1 - y)$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) 1 (C) $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

---

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$

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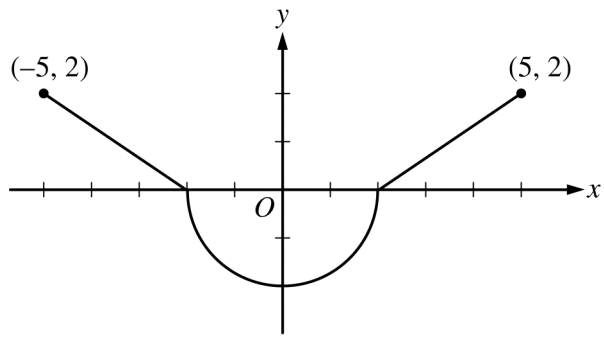

Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2},\dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\dfrac{2}{3}\right)$

(E) There are no such points.

---

20. $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

(A) $\ln(8)$ (B) $\ln\left(\dfrac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

GO ON TO THE NEXT PAGE.

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x-5}$ (D) $y = \dfrac{5x}{1-x}$ (E) $y = \dfrac{20x^2 - x}{1+4x^2}$

---

22. The power series $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

---

24. Let f be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \int 4x^3 \cos x\,dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

25. $\int_1^{\infty} xe^{-x^2}\,dx$ is

(A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent

GO ON TO THE NEXT PAGE.

26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

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27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

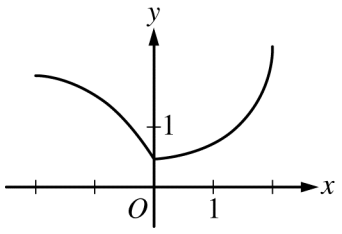
28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \dfrac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) The limit does not exist.

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



Graph of f

76. The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

---

77. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

(A) $(0.831, 7.384)$ only

(B) $(-\infty, 0.831)$ and $(7.384, \infty)$

(C) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(D) $(-0.816, 1.430)$ and $(8.613, \infty)$

(E) $(-\infty, \infty)$

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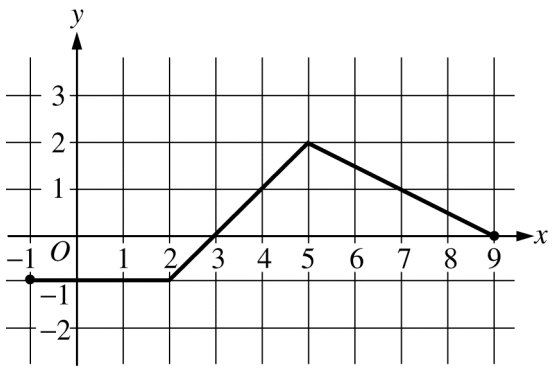

Graph of f

78. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9}(3f(x)+2)\,dx$?

(A) 7.5 (B) 9.5 (C) 27.5 (D) 47 (E) 48.5

---

79. Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(E) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

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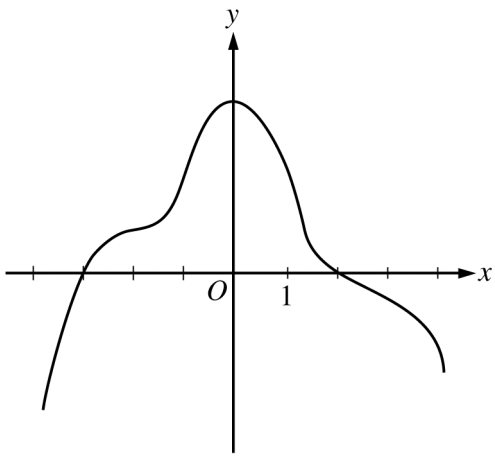

Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

81. Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table above. If $f(x) = f(-x)$, what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

(A) $x = 0$ only

(B) $x = 1$ only

(C) $x = 0$ and $x = 1$

(D) $x = -1$ and $x = 1$

(E) There are no points of inflection on $-2 < x < 2$.

---

82. What is the average value of $y = \sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{2}$?

(A) -0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

83. If the function f is continuous at $x = 3$, which of the following must be true?

(A) $f(3) < \lim_{x \to 3} f(x)$

(B) $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

(C) $f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

(D) The derivative of f at $x = 3$ exists.

(E) The derivative of f is positive for $x < 3$ and negative for $x > 3$.

---

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

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85. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(s) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, what is the rate at which its fuel consumption is changing?

(A) 0.215 mpg per hour

(B) 4.299 mpg per hour

(C) 19.793 mpg per hour

(D) 25.793 mpg per hour

(E) 515.855 mpg per hour

GO ON TO THE NEXT PAGE.

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)dt = 0,$ which of the following could be a table of values for the function f ?

(A)

x	$f(x)$
4	−4
5	−3
7	0

(B)

x	$f(x)$
4	−4
5	−2
7	5

(C)

x	$f(x)$
4	−4
5	6
7	3

(D)

x	$f(x)$
4	0
5	0
7	0

(E)

x	$f(x)$
4	0
5	4
7	6

87. Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3 - x)$, for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.442 (B) 1.029 (C) 1.296 (D) 3.233 (E) 4.071

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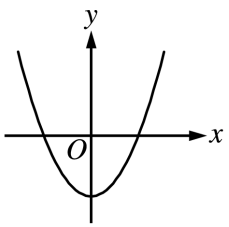
88. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

(A)



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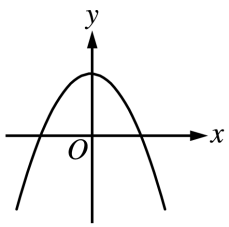

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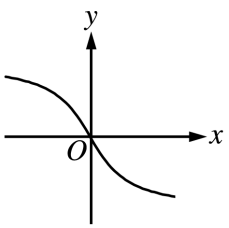

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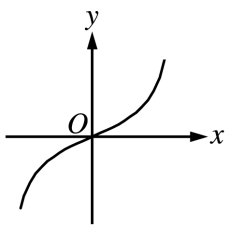

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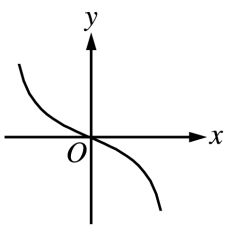

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

---

90. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} n a_n$ diverges.

(E) $\sum_{n=1}^{\infty} \dfrac{a_n}{n}$ converges.

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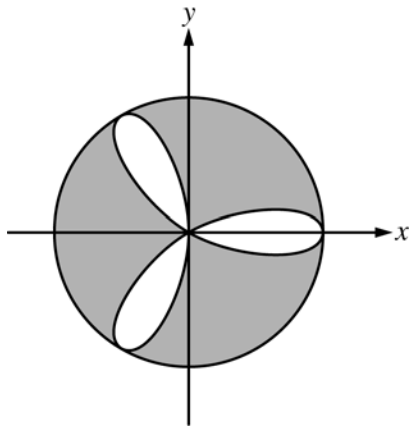

91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

GO ON TO THE NEXT PAGE.

92. The function h is differentiable, and for all values of x, $h(x) = h(2 - x)$. Which of the following statements must be true?

I. $\int_0^2 h(x)\,dx > 0$

II. $h'(1) = 0$

III. $h'(0) = h'(2) = 1$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.