Name______________________Period ____

AP Calculus BC

2003 Calculus BC Multiple Choice Exam Part A

1) If $y = \sin(3x)$ then $\dfrac{dy}{dx} =$

(A) $-3\cos(3x)$
(B) $-\cos(3x)$
(C) $-\dfrac{1}{3}\cos(3x)$
(D) $\cos(3x)$
(E) $3\cos(3x)$

2) $\lim\limits_{x\to 0} \dfrac{e^x - \cos x - 2x}{x^2 - 2x}$ is

(A) $-\dfrac{1}{2}$
(B) 0
(C) $\dfrac{1}{2}$
(D) 1
(E) *nonexistent*

3) $\int (3x+1)^5 \, dx =$

(A) $\dfrac{(3x+1)^6}{18} + C$
(B) $\dfrac{(3x+1)^6}{6} + C$
(C) $\dfrac{(3x+1)^6}{2} + C$
(D) $\dfrac{(\frac{3x^2}{2} + x)^6}{2} + C$
(E) $(\dfrac{3x^2}{2} + x)^5 + C$

4) For $0 \le t \le 13$ an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $-\frac{4\tan 13}{3}$

(D) $-\frac{4}{3\tan 13}$

(E) $-\frac{3}{4\tan 13}$

5) Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3

(B) 5

(C) 6

(D) 10

(E) 12

6) What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}\,dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $p > \frac{1}{2}$

(D) $p > 1$

(E) There are no values of p for which this integral converges.

7) The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) -1 *only*

(B) 0 *only*

(C) 2 *only*

(D) -1 *and* 2 *only*

(E) -1, 0, *and* 2

8) $\int x^2 \cos\left(x^3\right) dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right) + C$

(B) $\frac{1}{3}\sin\left(x^3\right) + C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right) + C$

(D) $\frac{x^3}{3}\sin\left(x^3\right) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

9) If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

(E) *nonexistent*

10) What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

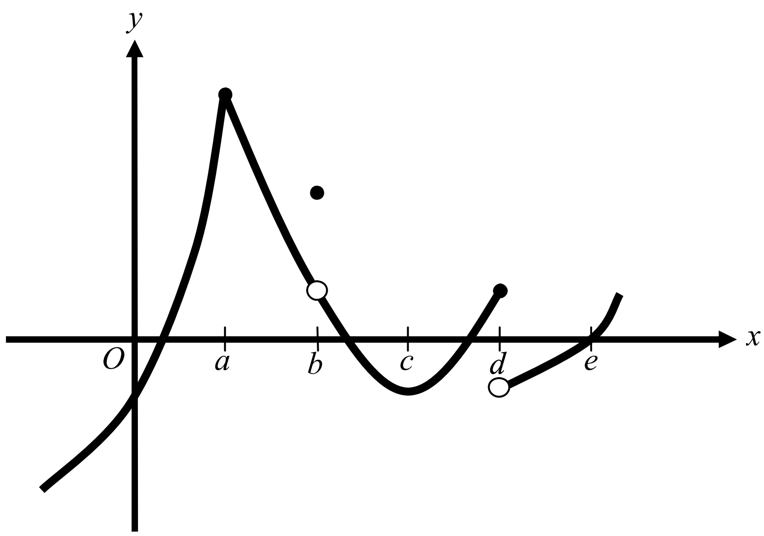
(A) 1
(B) 2
(C) 4
(D) 6
(E) The series diverges.

11) The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1+x^2+x^4+x^6+x^8+\cdots$
(B) $x^2+x^3+x^4+x^5+\cdots$
(C) $x^2+2x^3+3x^4+4x^5+\cdots$
(D) $x^2+x^4+x^6+x^8+\cdots$
(E) $x^2-x^4+x^6-x^8+\cdots$

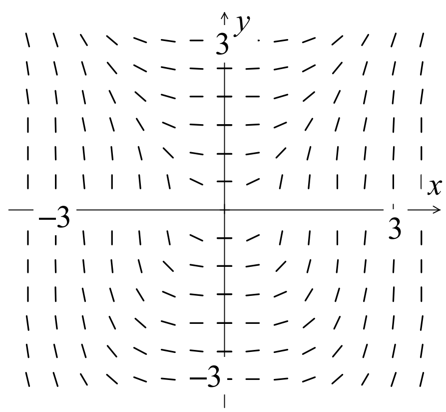
12) The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t)=k\sqrt{t}$
(B) $V(t)=k\sqrt{V}$
(C) $\frac{dV}{dt}=k\sqrt{t}$
(D) $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$
(E) $\frac{dV}{dt}=k\sqrt{V}$



13) The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) **a** (B) **b** (C) **c** (D) **d** (E) **e**



14) Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

15) The length of a curve from $x=1$ to $x=4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx.$ If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

(A) $y=3+3x^2$

(B) $y=5+x^3$

(C) $y=6+x^3$

(D) $y=6-x^3$

(E) $y=\frac{16}{5}+x+\frac{9}{5}x^5$

16) If the line tangent to the graph of the function f at the point $(1,7)$ passes through the point $(-2,-2)$, then $f'(1)$ is

(A) -5

(B) 1

(C) 3

(D) 7

(E) undefined

17) A curve C is defined by the parametric equations $x=t^2-4t+1$ and $y=t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3,8)$?

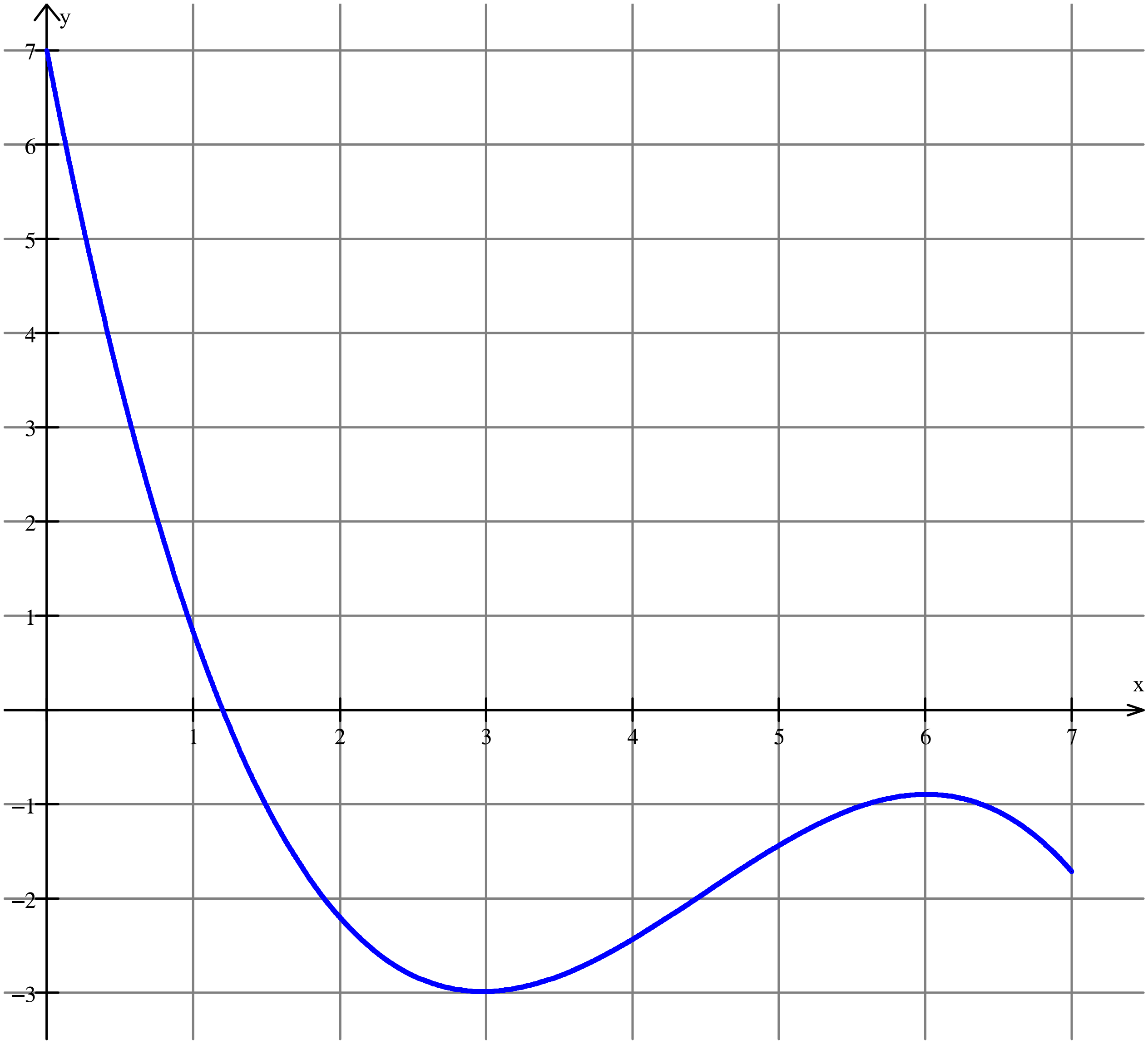
(A) $x=-3$

(B) $x=2$

(C) $y=8$

(D) $y=-\frac{27}{10}(x+3)+8$

(E) $y=12(x+3)+8$



Graph of f

18) The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)dt$, what is the value of $g'(3)$?

(A) 0
(B) −1
(C) −2
(D) −3
(E) −6

19) A curve has slope $2x+3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

(A) $y=5x-3$

(B) $y=x^2+1$

(C) $y=x^2+3x$

(D) $y=x^2+3x-2$

(E) $y=2x^2+3x-3$

20) A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$.

Which of the following is an expression for $f(x)$?

(A) $-3x\sin(x)+3x^2$

(B) $-\cos(x^2)+1$

(C) $-x^2\cos(x)+x^2$

(D) $x^2e^x-x^3-x^2$

(E) $e^{x^2}-x^2-1$

21) The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt}=0.6M(1-\frac{M}{200})$, where t is the time in years and $M(0)=50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50

(B) 200

(C) 500

(D) 1000

(E) 2000

22) What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

(A) $p > 0$
(B) $p \geq 1$
(C) $p > 1$
(D) $p \geq 2$
(E) $p > 2$

23) $\int x\sin(6x)\,dx =$

(A) $-x\cos(6x)+\sin(6x)+C$
(B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$
(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(E) $6x\cos(6x)-\sin(6x)+C$

24) Which of the following series diverge?

$$I.\ \sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$$

$$II.\ \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$$

$$III.\ \sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

x	2	5	10	14
$f(x)$	12	28	34	30

25) The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

(A) 296
(B) 312
(C) 343
(D) 374
(E) 390

26) $\int \frac{2x}{(x+2)(x+1)} dx =$

(A) $\ln|x+2| + \ln|x+1| + C$
(B) $\ln|x+2| + \ln|x+1| - 3x + C$
(C) $-4\ln|x+2| + 2\ln|x+1| + C$
(D) $4\ln|x+2| - 2\ln|x+1| + C$
(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

27) $\dfrac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)dt\right) =$

(A) $\dfrac{2x^3}{x^6+1}$

(B) $\dfrac{3x^2}{x^6+1}$

(C) $\ln(x^6+1)$

(D) $2x^3\ln(x^6+1)$

(E) $3x^2\ln(x^6+1)$

28) What is the coefficient of x^2 in the Taylor series for $\dfrac{1}{(1+x)^2}$ about $x=0$?

(A) $\dfrac{1}{6}$

(B) $\dfrac{1}{3}$

(C) 1

(D) 3

(E) 6

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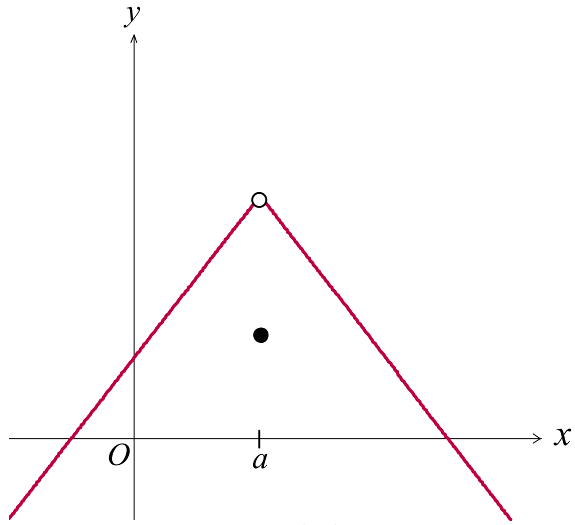
2003 Calculus BC Multiple Choice Exam Part B



Graph of f

76) The graph of the function f is shown above. Which of the following statements must be false?

(A) $f(a)$ exists

(B) $f(x)$ is defined for $0 < x < a$

(C) f is not continuous at $x = a$

(D) $\lim_{x\to a} f(x)$ exists

(E) $\lim_{x\to a} f'(x)$ exists

77) Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) -30

(B) -15

(C) -5

(D) $-\frac{5}{6}$

(E) $-\frac{1}{6}$

78) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

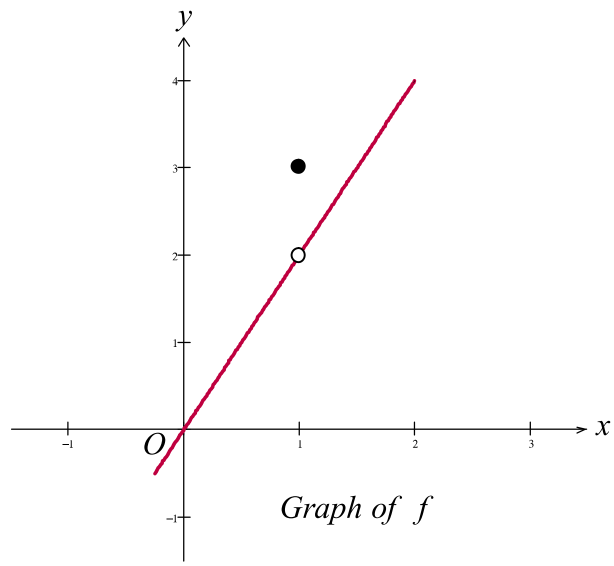
(A) $0.04\pi\ m^2/\text{sec}$
(B) $0.4\pi\ m^2/\text{sec}$
(C) $4\pi\ m^2/\text{sec}$
(D) $20\pi\ m^2/\text{sec}$
(E) $100\pi\ m^2/\text{sec}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

79) The table above gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$
(A) 5
(B) 6
(C) 9
(D) 10
(E) 12

80) Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125
(B) 100
(C) 88
(D) 50
(E) 12



81) The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1} \sin(f(x))$ is

(A) 0.909
(B) 0.841
(C) 0.141
(D) -0.416
(E) nonexistent

82) The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$
(B) $\int_{0}^{8} r(t)\,dt$
(C) $\int_{0}^{2.667} r(t)\,dt$
(D) $\int_{1.572}^{3.514} r'(t)\,dt$
(E) $\int_{0}^{2.667} r'(t)\,dt$

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

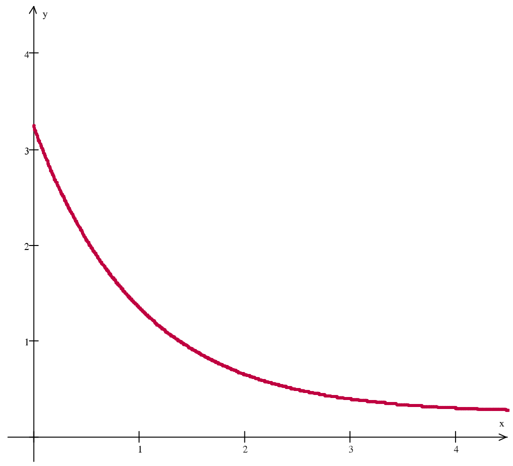
83) The function f is continuous and differentiable on the closed interval $[0,4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

(A) The minimum value of f on $[0,4]$ is 2.
(B) The maximum value of f on $[0,4]$ is 4.
(C) $f(x)>0$ for $0<x<4$
(D) $f'(x)<0$ for $2<x<4$
(E) There exists c, with $0<c<4$, for which $f'(c)=0$.

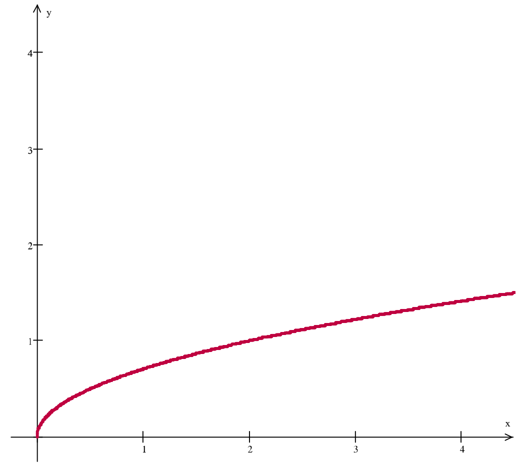
84) A particle moves in the xy-plane so that its position at any time t is given by $x(t)=t^2$ and $y(t)=\sin(4t)$. What is the speed of the particle when $t=3$?

(A) 2.909
(B) 3.062
(C) 6.884
(D) 9.016
(E) 47.393

85) If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?



(A)



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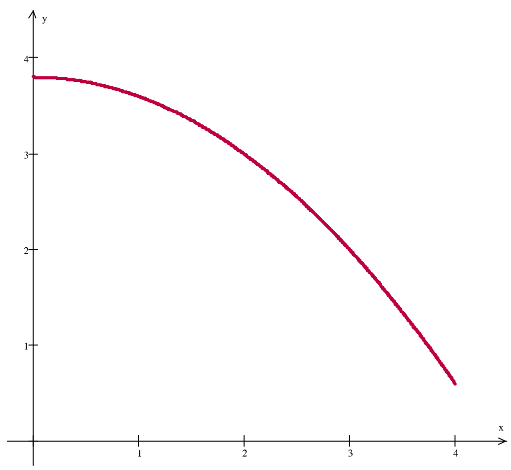

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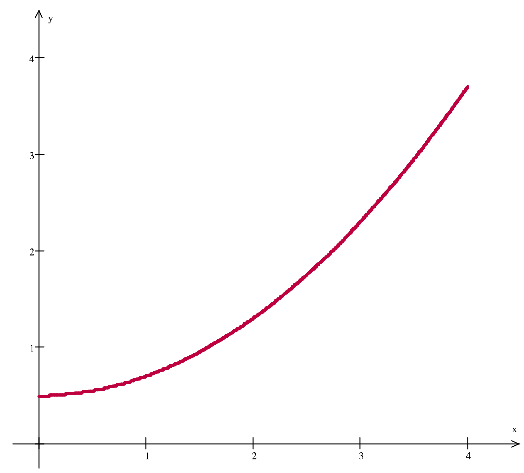

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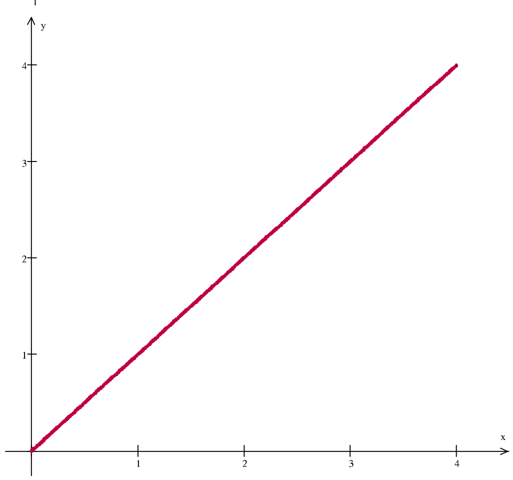


(E)

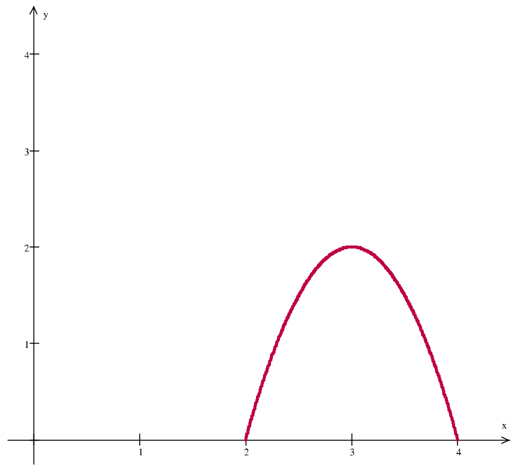
86) Let f be the function with derivative defined by $f'(x) = \sin\left(x^3\right)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

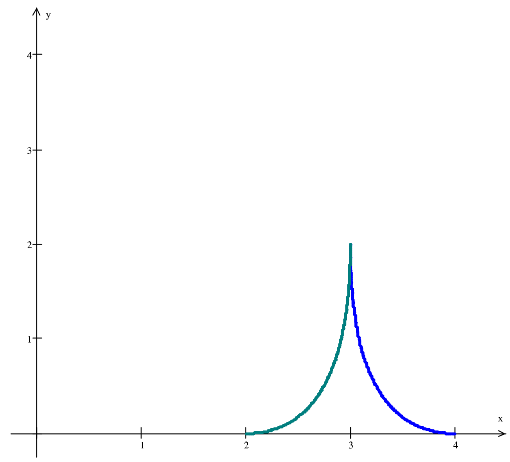
87) A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

(A) 0.411
(B) 1.310
(C) 2.816
(D) 3.091
(E) 3.411

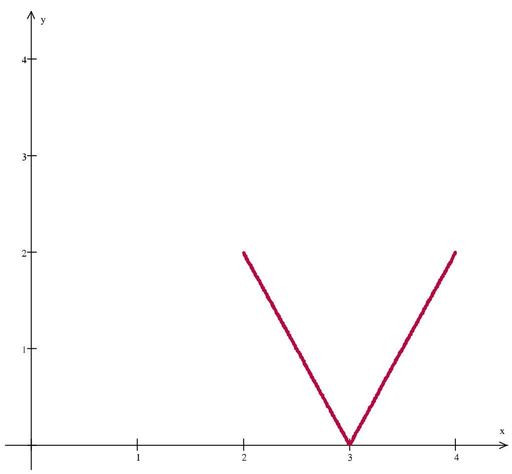
88) On the closed interval $[2,4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?



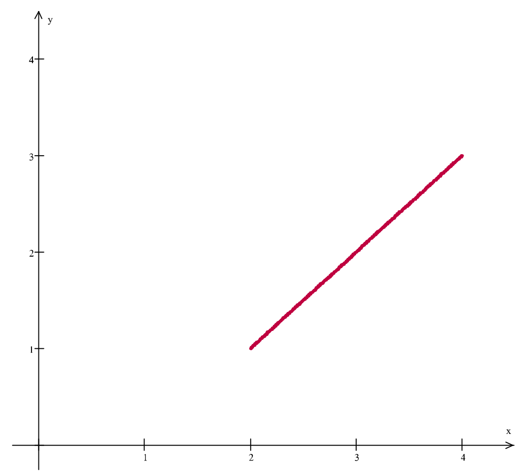
(A)



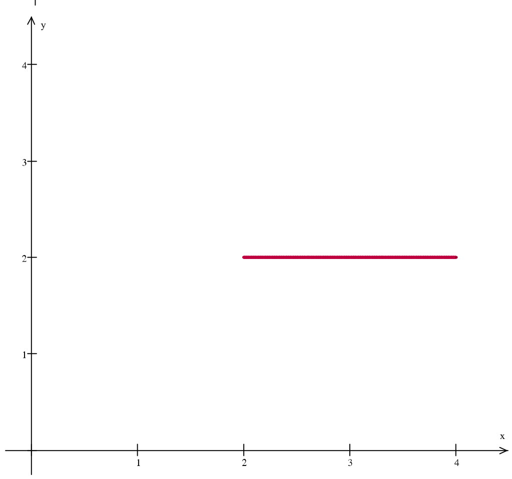
(B)



(C)



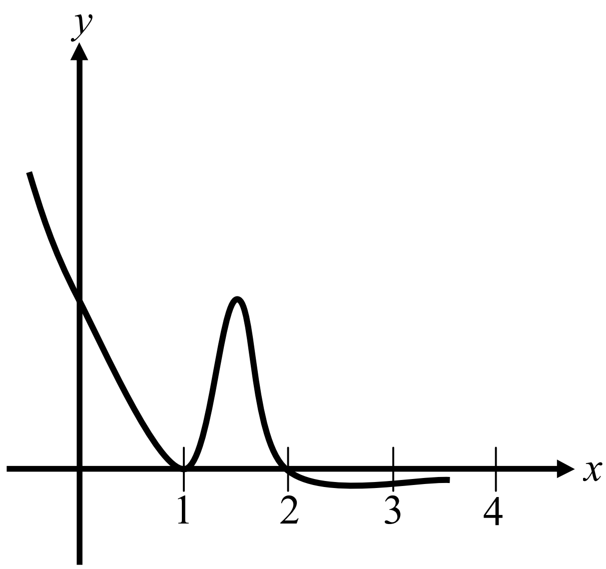
(D)



(E)

89) The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is an equilateral triangle. What is the volume of this solid?

(A) 1.333
(B) 1.067
(C) 0.577
(D) 0.462
(E) 0.267



90) The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

91) The height $h,$ in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2.$ What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters
(B) 10.263 meters
(C) 34.125 meters
(D) 54.889 meters
(E) 89.005 meters

92) Let f be the function defined by $f(x) = x + \ln x.$ What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

(A) 0.456
(B) 1.244
(C) 2.164
(D) 2.342
(E) 2.452